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FUNCTIONAL RELATIONSHIPS BETWEEN DENUDATION, RELIEF, AND UPLIFT IN LARGE MID-LATITUDE DRAINAGE BASINS

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ABSTRACT. The mean denudation rate in mid-latitude river basins is directly proportional to mean basin relief. Mean annual precipitation has no noticeable effect upon the denudation rate. Without any uplift, the relief is reduced to 10 percent of its initial value in 11 m.y.; with isostatic compensation, this time increases to at least 18.5 m.y. Establishment of a steady state relief (Hack, 1960) would require a constant rate of uplift for more than 20 m.y. and thus is unlikely to occur. Oscillating rates of uplift cause corresponding oscillations of relief. The quantitative relationship between uplift, relief, and denudation may permit order-of-magnitude estimates of current rates of uplift.

INTRODUCTION

The influence of the available relief upon the rate of denudation is a fact familiar even to the casual observer who compares the rushing power of a mountain torrent with the quiet flow of a lowland brook. The greater the relief, the steeper are, as a rule, the slopes, and the more rapid is the downhill transport of waste and thus the downwearing, or denudation, of the land. Equally obvious is the fact that uplift, by itself, tends to increase the relief and thus exerts an indirect influence upon the rate of denudation, too.

This paper attempts to shed light on the more difficult question of the quantitative relationship between these variables. Even the measurement of their numerical values is beset with problems. Relief can be determined easily enough from topographic maps; the investigator merely has to decide whether to define it as the elevation difference within a standard areal unit (for example, 1.0 km^2), within a drainage basin of a given order, or between an individual interfluve and the adjacent valley bottom. Much harder to assess is the rate of denudation, partly because it is very slow compared to the human time scale, and partly because it is so sensitive to environmental change that the instrumentation used to measure the denudation on slopes can interfere with, and thus modify, the process itself. Similarly, only the most rapid rates of present uplift—as for example the glacio-isostatic uplift of northern Sweden—can be measured by direct observation. The much more common slower rates of uplift have to be inferred from indirect geological evidence which usually provides an indication of the mean rate of uplift over a longer span of geologic time.

In order to avoid the pitfalls of direct denudation measurement on slopes, the rate of denudation is usually derived from the determination of the sediment transport of streams. All rock material that passes through a particular cross section of a stream originates from the portion of the drainage basin that lies upstream from the cross section, and conversely all rock material that is transported out of the drainage basin segment will pass through this cross section, barring any transport across the perimeter of the basin by wind or groundwater. Sediment load measurements yield a value for the total weight of rock material removed from the drainage basin per unit time, which can be converted into the mean denudation rate through division of the weight by the basin area, and through further conversion from weight to volume, and thereby to thickness, of rock material removed. Apart from the fact that this method supplies only an average rate of denudation for the entire drainage basin and does not distinguish between rates of stream erosion, slope denudation, and summit lowering, it also is based on the tacit assumption that the rate of rock waste production, and of transport of that waste, within the drainage basin is during the short period of measurement in equilibrium with the rate at which this rock material is removed from the basin. This assumption is not necessarily justified. Nevertheless, the method of determining denudation by way of sediment yield remains the best one to date.

Among previous studies of sediment transport by streams those of Schumm (1956, 1963) have paid special attention to the effect of relief upon sediment load and thus upon average denudation rates. Working mainly with small drainage basins, Schumm has consistently obtained a quantitative relationship in which the rate of denudation increases exponentially with increasing relief. Ruxton and McDougall (1967), in a study of denudation rates on the slopes of the Hydrographer's Volcano in New Guinea, show a linear relationship between relief and denudation. This does not, however, invalidate the findings of Schumm; Ruxton and McDougall define relief as depth of valley incision into the slopes of the volcano and measure denudation as the volume of material removed by this same valley incision. Thus the local relief, in their study, is perceived as a function of denudation rather than vice versa.

Schumm's stream load data are derived, for the most part, from the sediment accumulation rates of small reservoirs and thus include dissolved load only to the extent to which it may have precipitated out of solution in the reservoir. Possibly such precipitation amounts only to a small part of the actual dissolved load, so that the load measured represents mainly the suspended load and the bedload. In that case, the exponential relationship between measured sediment load and relief does not necessarily reflect an exponential relationship between denudation and relief. The share of dissolved load in percent of total load tends to decrease with increasing relief, since with increasing relief also the surface runoff increases at the expense of infiltration, and a higher proportion of surface runoff causes more intensive erosion of particles by surface wash and by the higher flood discharges of streams. The increase of solid load with increasing relief may thus well be exponential, but this relationship is probably in part offset by a relative decrease of the dissolved load, so that the total load would increase less rapidly with increasing relief than Schumm's curves suggest.

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and denudation. For example, climate exerts an effect upon denudation both directly through the action of temperature and precipitation and indirectly through the vegetation cover it supports. Peltier (1950) has tried to show diagrammatically the relative intensity of denudational processes as a function of mean annual temperature and mean annual precipitation. Since the processes are not shown quantitatively, and since the magnitude and frequency of climatic events (Wolman and Miller, 1960) has greater bearing on the intensity of morphological processes than mean annual values, Peltier's diagrams cannot give a sufficiently detailed description of the effects of climate. Langbein and Schumm (1958) have presented a study of the quantitative relationship between sediment load and annual precipitation, according to which the maximum sediment yield occurs in areas of 250 to 300 mm (10-12 in.) annual rainfall, and the load decreases rapidly with both higher and lower preciptation. The seasonal distribution of rainfall is not considered, and load measurements are confined to the solid load. The latter's decrease with higher rainfall may be at least in part compensated by an accompanying increase of the unmeasured dissolved load.

When the range of relief values is great, the morphologically effective factors of climate themselves vary as functions of the relief and thus implicitly enter into the denudation/relief relationship. Areas of high relief encompass a great range of altitudes and hence may reach through several altitudinal climatic zones. Above the timberline, processes of weathering and denudation tend to be more intensive than below it. The sediment yield of drainage basins that include, because of their high relief, sizable areas above the timberline could thus be disproportionately higher than the sediment yield from lower-relief basins whose divides are forest-covered. Of course, this effect is one of altitude rather than relief: an upland that lies entirely above the timberline would be subject to more intensive denudation than a basin that has the same relief but lies below the timberline.

Lithologic differences create additional problems. The presence of rocks of varying resistance to weathering causes variations in the denudation rate. This is particularly important in the case of small drainage basins that lie on only one or two types of rock. With increasing size, the basins usually contain a greater variety of rock types whose different resistances are likely to balance out so that the effect of lithologic differences upon the total sediment load becomes neutralized.

Another cause of different total sediment yield from basins having the same relief lies in the varying proportion of stream corrasion as compared to hillslope denudation. In a largely flat plateau landscape dissected by wide-spaced streams that flow in deep, narrow gorges, the main part of the sediment load of these streams is derived from the corrasion of their beds and from the sides of their gorges—little or nothing from the interfluves. By contrast, in a densely dissected landscape in which the streams have ceased to cut down, most of the sediment yield is derived from the denudation of the slopes and from the lowering of the interfluves. Presence or absence of recent uptlift, and the latter's rate, will also influence the amount of load produced by corrasion of the stream beds.

Finally, the effects of man's works on the land surface must not be overlooked. Removal of the natural vegetation and disturbance of the natural soil structure for purposes of agriculture or large-scale construction projects tend to increase the detrital sediment yield from the slopes and interfluves. Covering the natural, permeable soil surface with impermeable pavements, roofs of buildings, *et cetera* protects the surface from denudation, increases surface runoff at the expense of infiltration, heightens the flood discharge of streams, causes greater frequency of stream floods, and hence encourages stream erosion. On the other hand, canalization of streams and artificial paving of their beds and banks interferes with natural processes of erosion and corrasion. The works of man have not been widespread long enough to have caused more than very local changes in the relief. However, the sediment yields measured in inhabited areas reflect these changes; the current rate of denudation computed from these sediment yields is thus not necessarily representative of the rate of denudation that existed in the same areas over the last several thousand years.

Mindful of the complex interplay of all these factors, an attempt will be made here to investigate the relationship between denudation, relief, and uplift, and to assess its significance for the morphological development of landscapes on the basis of available data from twenty river basins that range in area from several hundred square kilometers to over 100,000 km^2. The large size of these basins goes far to insure that lithologic differences may be neglected. All basins lie in the middle latitudes of the northern hemisphere, between about 32°N (Flint River, Georgia) and 52°N (River Thames, England). Half the basins are located in the western United States, the other half in the humid eastern United States and in western Europe. Mean annual precipitation ranges from less than 250 mm (west of the Rocky Mountains) to more than 2500 mm (in the Alps), but none of the basins selected has the pronounced seasonality of precipitation that characterizes the monsoonal, mediterranean, and low-latitude subhumid and semiarid types of climate. Thus the data permit also some assessment of the possible influence of annual precipitation upon denudation rates.

THE DATA AND THEIR EVALUATION

The mean rates of denudation of the twenty river basins are listed in table 1. They had to be taken from several sources, in order to insure that a wide range of both relief values and denudation rates would be included. There are some differences, from source to source, in the method of computing the denudation rates. For example, Leopold, Wolman, and Miller (1964, p. 76) consider only the dissolved and suspended loads, Judson and Ritter (1964) estimate the bedload at 10 percent of the total detrital load, whereas Corbel (1959) uses specific values for the bedload. Corbel determines mean annual runoff from a river basin by

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Colorado River above San Saba
River Thames above London,
Delaware River above Trenton
Canadian River above Amarillo
Little Colorado River above
Juniata River above Newport,
Green River above Green Rive
Escalante River, Utah
Dirty Devil River, Utah
Bighorn River above Thermo
Colorado River above Cisco, U
Wind River above Dubois, Wy
Animas River above Farmingto
Sarine, Switzerland
Rhine above Lake Constance,
Isére above Grenoble, France
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subtracting evaporation (either measured or estimated) from precipitation, and then multiplies the sampled load per unit of discharge by this runoff figure, in order to obtain the total annual load. Leopold, Wolman, and Miller, on the other hand, estimate the total annual load as the product of sampled load times measured mean annual discharge of the river, without resort to precipitation and evaporation. For conversion of the weight of the sediment load into amounts of mean denudational lowering of the land surface, Corbel assumes a mean specific weight of 2.5; Judson and Ritter use 2.64. Corbel's value of 2.5 appears more suitable for the large river basins considered here, which contain a variety of rock types including relatively light sedimentary rocks.

TABLE 1

River basin	Mean relief h (m)	Mean sine of slope	Mean denudation rate d (mm/1000 yrs)
Flint River above Montezuma, Ga.	89	0.033	28
Colorado River above San Saba, Tex.	102	0.020	16
River Thames above London, England	159	0.022	16
Delaware River above Trenton, N.J.	299	0.052	42
Canadian River above Amarillo, Tex.	353	0.071	52
Little Colorado River above Woodruff, Ariz.	392	0.056	31
Juniata River above Newport, Pa.	490	0.089	41
Green River above Green River, Utah	644	0.113	82
Escalante River, Utah	842	0.130	135
Dirty Devil River, Utah	912	0.106	177
Bighorn River above Thermopolis, Wyo.	1004	0.138	109
Colorado River above Cisco, Utah	1040	0.134	124
Wind River above Dubois, Wyo.	1091	0.132	115
Animas River above Farmington, N. Mex.	1273	0.150	195
Sarine, Switzerland	1395	0.287	210
Rhine above Lake Constance, Switzerland	1994	0.410	321
Isére above Grenoble, France	2046	n.m.	287
Reuss above Lake Lucerne, Switzerland	2320	0.345	300
Kander, Switzerland	2428	0.401	430
Rhone above Lake Geneva, Switzerland	2869	0.361	418

n.m. = not measured.

Denudation rates after Corbel (1959), Louis (1961), Wolman and Miller (1961), Leopold, Wolman, and Miller (1964), Judson and Ritter (1964), Iorns, Hembree, and Oakland (1965).

The solid material removed from the surface by mechanical denudational processes is soil rather than rock and normally has a specific weight of less than 2.0 because of greater pore volume and expansion of mineral grains by chemical weathering. However, it is safe to assume that for long periods of denudational surface lowering the soil thickness at any locality remains approximately constant, since the processes of weathering and of removal tend toward a state of dynamic equilibrium (Gilbert, 1877; Jahn, 1954; Ahnert, 1954, 1967). The lowering of the soil surface thus tends to be equal to the lowering of the bedrock surface underneath by weathering and by partial removal of mineral matter in solution. As Judson and

Ritter (1964) have pointed out, solution of bedrock, which contributes to the dissolved sediment load, should be interpreted as a part of the complex process of denudational surface lowering. Consequently, whenever dissolved load is considered together with the detrital load, the density of the bedrock rather than that of the soil must be used for estimating the average rate of lowering of the land surface.

For the denudation rates in table 1, the values of Corbel were accepted directly as he gives them, in millimeters of surface lowering per 1000 yrs. Additional data listed from various sources by Louis (1961), also in mm/1000 yrs, match well with those of Corbel and were also included as given. The other sources indicate the amounts removed in tons per square mile. These data were converted into mm/1000 yrs using Corbel's assumed mean specific weight of 2.5.

The mean relief for each river basin was determined as the average of the relief measured in regularly spaced sample areas, each sample area being a 20 x 20 km square. The number of sample areas for each basin varies according to the size of the basin, and their spacing was set with the aid of geographical or of grid coordinates on the maps used (scale at or near 1:250,000). The large size of each sample area virtually guarantees that the area includes both valley bottom and interfluves and thus permits determination of the relief. Since there is no systematic relationship between map coordinates and drainage pattern, the regular spacing of sample areas is as adequate as a random spacing would have been.

Schumm (1956, 1963) has expressed preference for the use of the relief ratio (total basin relief divided by basin length) instead of the relief by itself. The relief ratio is a dimensionless number and as such useful for comparing the relief of drainage basins of varying size. It serves well for the very small basins Schumm has studied, because the morphological character of the terrain does not change much within these basins. However, the relief ratio is too coarse a measure for the large basins listed in table 1. For example, the Canadian River above Amarillo, Texas, and the Colorado River above Cisco, Utah, have very nearly the same relief ratio (0.0094 and 0.0097, respectively) but differ greatly in their mean denudation rates, because mountainous terrain, with high local relief and steep slopes, occupies only a small part of the Canadian River basin but a very large part of the Colorado River basin. This difference, so important for the sediment load to be expected in these two rivers, expresses itself clearly in the different mean relief values of the two basins in table 1 and demonstrates that, for the purpose of the present study, the mean basin relief as defined is a more suitable measure.

Also determined for each sample square was the mean slope according to the method of Wentworth (1930), by counting the contours that cross two 20 km traverses, one north-south, the other east-west. The traverses bisect one another in the center of the square. The mean slope (tangent) of each drainage basin is the average of the mean slope values of the sample squares. The tangent data have been converted to the corresponding sines and are so listed in table 1, because the force of

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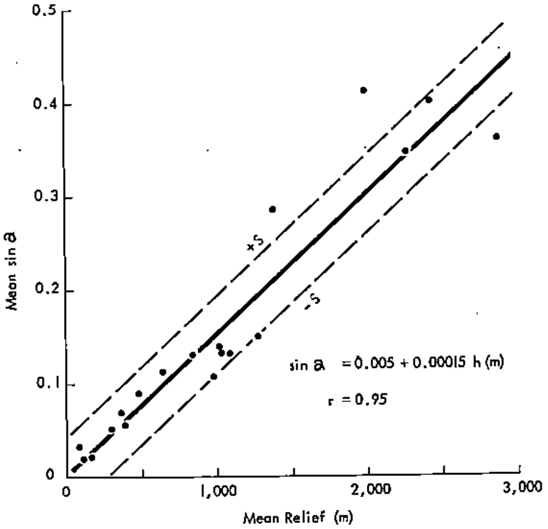

Fig. 1. Relationship between mean sine of slope and mean relief.

gravity on slopes is proportional to the sine rather than to the tangent. In practice, this difference is only significant for steep slopes.

As the first step in the quantitative evaluation of the data, the sine of the mean slope is compared with the mean basin relief h (fig. 1). The relationship appears clearly linear between h = 0 m and h = 1100 m but less clearly so in the few cases of higher relief. The residuals tend to increase with increasing relief for geometrical reasons as shown in figure 2. When the residuals are expressed in percent of the estimated sine, this trend disappears. Presumably, individual observed values deviate from the regression line of figure 1 because of variations in the spacing of valleys. Of the ten river basins located west of long 100°—in the drier part of North America—eight have observed mean sine values that are smaller than those estimated, indicating a tendency toward wider spacing of valleys in areas of drier climate.

Figure 3 shows the correlation between denudation rate and mean basin relief. The distribution of residuals suggests that the relationship between these two parameters is linear. Significantly, neither the sign nor the magnitude of the residuals can be explained by differences in annual rainfall: five of the western American river basins have positive residuals, the other five have negative ones, and the observed denudation rates for the ten basins from the humid eastern United States and western Europe are also equally divided by the regression line of figure 3. The data thus imply that there is no relationship between mean annual precipitation and the rate of denudation.[1]

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[1] For basins of this size, the basin area does not appear to have a significant effect, either; this contrasts with the findings of Langbein and Schumm for very small drainage basins.

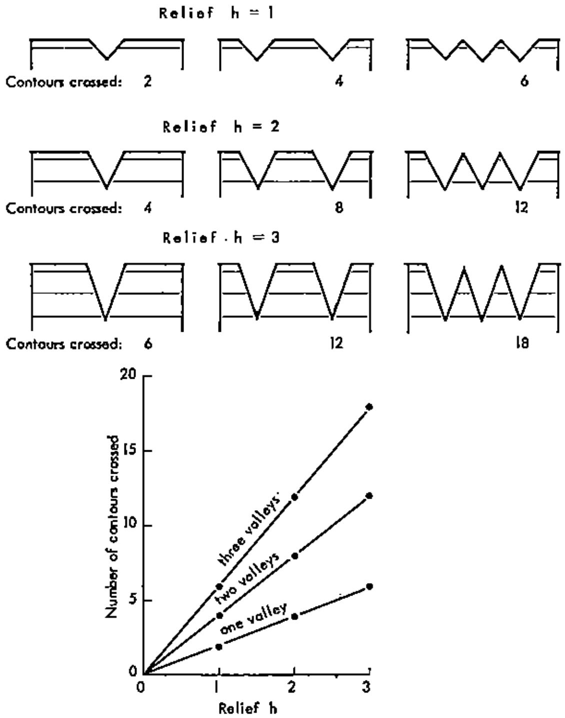
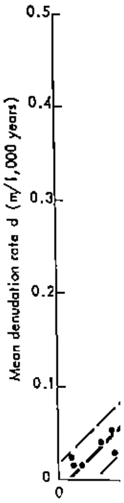


Fig. 2. Relationship between relief, spacing of valleys, and numbers of contours crossed along traverses of equal length (schematic). The number of contours crossed (and with it the mean slope) is proportional to the product of relief and valley density. This explains why on figure 1 larger residuals occur in areas of high relief than in areas of low relief.

A comparison of the denudation rates of Ruxton and McDougall (1967) from the Hydrographer's Volcano in New Guinea with those of figure 3 appears also to confirm the absence of an appreciable effect of mean annual precipitation. The volcano, 1887 m high, lies 20 km from the coast; mean annual precipitation ranges from 2250 mm at sea level to 3000 mm at the summit. The study of Ruxton and McDougall has been confined to the seaward slope of the volcano, so that the height of the mountain is approximately equivalent to the relief within a 20 x 20 km square enclosing their study area. With h = 1887 m in the regression equation of figure 3, one obtains a denudation rate d = 279 mm/1000 yrs; the average of all local denudation rates listed by Ruxton and McDougall is 336 mm/1000 yrs, only 20 percent higher than the estimated value, despite the fact that their method of determining denudation rates is entirely different from that employed here.

It seems, therefore, that the rapid removal of waste by the rare, but intensive runoff over little-protected soil in the dry western United States is just as effective an agent of denudation as the slower, but more con-



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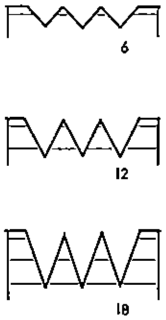

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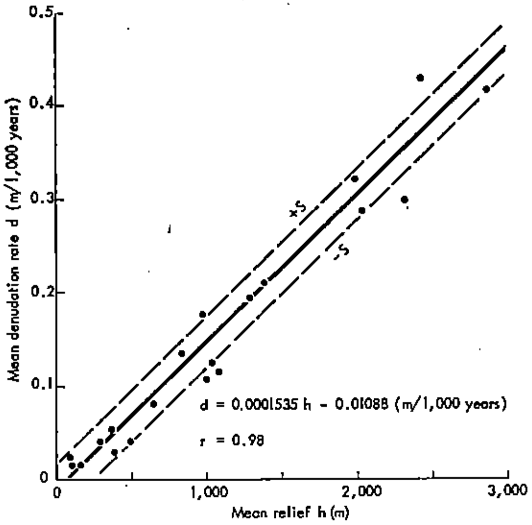

Fig. 3. Relationship between mean denudation rate and mean relief.

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s the slower, but more continuous waste removal processes in the humid eastern United States, in western Europe, or in New Guinea. The difference lies less in the amount removed than in the mode of removal. The rivers of the western United States carry only a small percentage of their total load in solution, whereas the dissolved load of the rivers in humid areas accounts frequently for more than half the total load.

Instead of mean annual precipitation, it is the seasonality of rainfall that affects the denudation rate. Corbel (1959) has shown that areas with strong seasonal variations of precipitation tend to have higher denudation rates than those with less rainfall seasonality. The long dry season of the monsoon climate, the mediterranean climate, and the tropical savanna and steppe climates leaves the ground bare and vulnerable to erosion by the often heavy rains of the wet season. Although the twenty river basins considered here vary greatly in annual precipitation, they lack such pronounced seasonality. For these basins and for the mid-latitude belt of more or less evenly distributed rainfall they represent, relief alone suffices to explain about 96 percent (r^2) of the observed variation in the denudation rates.

DENUDATION AND THE REDUCTION OF RELIEF

The regression equation of figure 3,

$$d = 0.0001535h - 0.01088 \text{ (m/1000 yrs)}$$

suggests that $d = 0$ for $h = 70.88$ m, which may be rounded up to $h = 71$ m. If this value is entered in the regression equation of figure 1, one obtains $\sin \alpha = 0.01565$. Theoretically, this means that no stream load is likely to be transported out of the drainage basin if the mean slope of the basin is less than 0°53′38″. Since there is always a possibility of load removal in solution as long as there is a gradient that allows water to flow out of the area, one must conclude that the small constant minus 0.01088, which determines the magnitude of this lower limit of the mean basin relief, is the result of some slight accidental scatter of the data, rather than of physical necessity. Even if it were accepted as real, one might still argue that, although no material leaves the basin, denudation may continue on the higher points, and the rock waste may be transported to the lower points within the basin itself, so that the relief is further reduced. Because of its smallness and probable factitiousness, the constant may be omitted without serious loss: the equation then reads

$$d = 0.0001535\ h/1000\ \text{yrs}$$

and is independent of the units of height measurement used.

The mean denudation rate d is rather unsatisfactory as a direct measure of relief reduction, that is, of the lowering of the summits relative to the valley bottoms. Where erosional deepening of valleys is nil, the denudation rate on the lower parts of the slopes is probably smaller and on the upper parts probably larger than the mean rate. As a first approximation, one may expect the mean rate to be present halfway between valley bottom and divide and the actual local denudation rate to increase linearly from zero at the valley bottom to twice the mean rate at the summits (fig. 4).

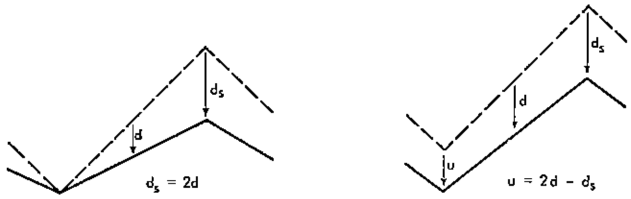

Fig. 4. A. Relationship between mean denudation rate d and rate of summit denudation d_s in the absence of stream incision (schematic).

B. Relationship between mean denudation rate d, summit denudation rate d_s, and rate of stream incision u (schematic).

A further complication arises from the differing amounts of stream erosion in the various basins. Erosion in excess of the removal of material transported to the stream by slope processes adds to the total sediment load and thus to the mean denudation rate of the basin. Some rates plotted in figure 3 are probably higher than they would be in the absence of such erosion, and the same is true for the regression coefficient. How much lower that coefficient would be if there were no significant deepening or lateral shifting of valleys in any of the basins can only be surmised by a consideration of the lower limit of denudation values in figure 3. The four relatively largest negative residuals, expressed in percent of their respective estimated denudation rates, are minus 24, minus 27, minus 36, and minus 38 percent, with a mean of minus 31 percent. If these values may be considered to represent the approximate mean denudation rates that are to be expected in basins without significant

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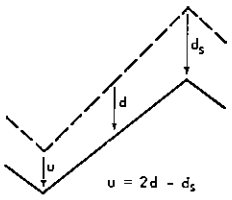

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stream erosion, then it is reasonable to assume that for such basins the regression coefficient of the estimating equation also would be about 31 percent lower than the coefficient of figure 3, so that the equation becomes

$$d = (1 - 0.31)\ 0.0001535\ h/1000\ \text{yrs} = 0.000106\ h/1000\ \text{yrs}.$$

Obviously, stream erosion is only one of many factors to affect the magnitude of mean regional denudation rates. Nevertheless, it probably ranks second only to relief in importance, since it is influenced by a wide range of environmental conditions, including the existing relief. The long-term effect of vertical stream erosion, namely, a lowering of valley bottoms relative to summits, and thus a retardation of relief reduction or even an increase in relief, will be discussed later, in conjunction with the role of uplift.

For the rate of summit denudation $d_s = f(h)$, and thus of relief reduction in the absence of vertical stream cutting, the preceding discussion suggests four possible estimating equations:

1. The rate directly derived from figure 3, $d_s = 0.0001535\ h/1000$ yrs.
2. A rate twice as high, in order to comply with the conditions of figure 4, that is, $d_s = 0.000307\ h/1000$ yrs.
3. The rate obtained as probable lower limit of denudation values in figure 3, $d_s = 0.000106\ h/1000$ yrs.
4. A rate twice as high, in compliance with figure 4A, $d_s = 0.00021\ h/1000$ yrs.

Of these four, the last is the only one that combines the attempt to eliminate the effects of stream erosion with the adjustment suggested by figure 4A and therefore seems the most probable estimating equation for the general case of relief reduction in a fluvial mid-latitude landscape without vertical stream erosion and without pronounced seasonality of precipitation. Its coefficient lies virtually halfway between the extremes represented by the second and third equations above and close to the arithmetic mean of all four coefficients. Over longer time spans, this rate of relief reduction cannot be extrapolated linearly, since the relief h itself declines progressively. Rather, it then becomes $d_s = (1 - 0.99979^n)$ h/n thousand yrs; for 1 m.y., $d_s = 0.189$ h. The time t (in millions of yrs) required to reduce the initial mean basin relief h to a lesser relief h_t is then

$$t = \frac{\log h_t - \log h}{\log (1 - 0.189)}.$$

A relief reduction from h to $h_t = 0.1$ h requires approximately 11 m.y. and from h to $h_t = 0.01$ h, 22 m.y. (dashed curve in fig. 5.)[2] This estimate contains the assumption that downwearing of summits starts immediately, at time zero. If parallel retreat of slopes is assumed instead

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[2] With the other three coefficients, the times required to reduce the relief to 0.1 h are about 15 m.y. (for $d_s = 0.0001535\ h/1000$ yrs), 7.5 m.y. (for $d_s = 0.000307\ h/1000$ yrs), and 22 m.y. (for $d_s = 0.000106\ h/1000$ yrs). For a relief reduction from h to 0.01 h these time requirements double.

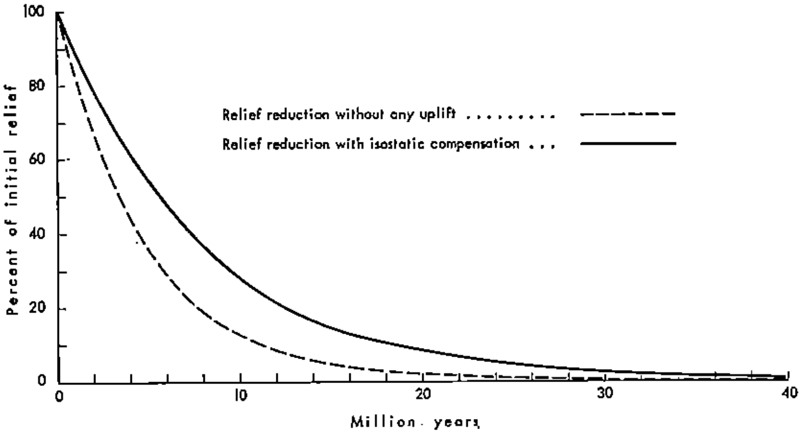

Fig. 5. Time needed for relief reduction (schematic).

of predominant downwearing, there may be an initial period of time in which interfluves and divides are narrowed without being much lowered, so that the total amount of time needed for relief reduction would be even greater.

EFFECT OF ISOSTATIC COMPENSATION

According to the theory of isostasy, the Earth's crust "floats" on the mantle, so that a removal of material from the surface must be compensated by uplift in order to maintain, or rather to reestablish, isostatic equilibrium. Isostatic uplift would probably be discontinuous, setting in after the weight of rock material removed has exceeded a certain threshold value (Schumm, 1963). However, for an estimate of long-term effects one may assume that the adjustment takes place continuously. If the mean density of the removed rock material is 2.5, and that of the upper mantle, in which the crust is partially immersed, is 3.3, the rate of denudation-induced isostatic uplift is

$$u = \frac{2.5\ d}{3.3} = 0.758\ d,$$

where d is the mean rate of denudation. Let it be assumed that the major streams in the drainage basin keep pace with the isostatic adjustment by cutting their channels down at the same rate u at which the land is uplifted, that the summits are worn down at the rate $d_s = 0.189$ h/1,000,000 yrs, and that $d = 0.5\ d_s$. The net rate of relief reduction is then $d_s - u =$ 0.117 h/1,000,000 yrs. At this rate, the relief is reduced to 10 percent of its initial value in 18.5 m.y. and to 1 percent in 37 m.y. (solid curve in fig. 5).

These time estimates constitute a minimum. The time required increases if one takes into account that the stream incision adds to the magnitude of the mean denudation rate d, so that $d > 0.5\ d_s$. If d, in

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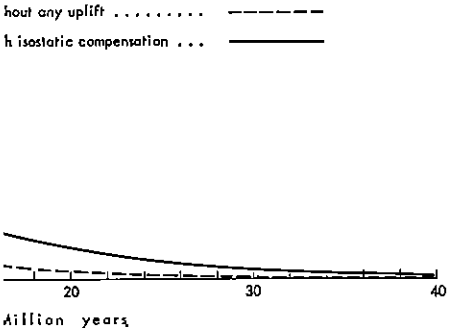

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accord with figure 4B, represents the mean between vertical stream cutting $u = 0.758\ d$ and summit lowering $d_s = 0.189$ h/1,000,000 yrs, then

$$d = \frac{d_s + u}{2} = 0.805\ d_s = 0.152\ h/1{,}000{,}000\ \text{yrs.}$$

The latter rate is only slightly larger than the mean denudation rate indicated by the regression line of figure 3 (which, when converted to the equivalent million-year rate, becomes 0.142 h). From it follows that $u =$ 0.115 h/1,000,000 yrs, and that the net rate of relief reduction $d_s - u =$ 0.074 h/1,000,000 yrs, which means that it would take 30 m.y. to reduce the relief to 10 percent of its initial value, and 60 m.y. to reduce it to 1 percent.

EFFECT OF TECTONIC UPLIFT

In contrast to isostatic compensation, which is a function of the denudation rate, tectonic uplift is independent of the latter. Instead of merely retarding the reduction of the relief, it can create relief and thus indirectly influences the denudation rate.

Tectonic uplift rarely takes place at a constant rate over long periods of time, and the rate is rarely uniform over large areas. Rather, it tends to occur in discontinuous phases of varying intensity, depending upon the cause, type, and magnitude of the crustal stresses that bring it about. To avoid these spatially and temporally varying circumstances, it is necessary to start a general consideration of the effect of tectonic uplift with certain simplifying assumptions, namely, that (1) uplift is continuous and occurs at a constant rate, (2) the streams cut down at the same rate u at which the land is uplifted, (3) at the beginning of uplift, the mean basin relief h is zero, and (4) summit denudation sets in as soon as there is relief.

Except for the independence of u from the denudation rate, the quantitative relationships resemble those found for isostatic compensation. In either case, the rate of relief change per time unit depends on the difference between uplift and summit denudation: $h_2 - h_1 = u - d_s$, where h_1 is the relief at the beginning, h_2 the relief at the end of the time unit. Initially, d_s is smaller than u, so that the relief increases. This causes in turn a progressive increase of d_s, until $d_s = u$; from then on, there is no further change of the relief h as long as the rate of uplift u (and with it the identical rate of stream incision) remains constant. When summits and stream beds are lowered at identical rates, one must expect that intermediate surface points are lowered at the same rate also, so that $d_s = u = d$. Since $d_s = 0.0021$ h/1000 yrs, the mean denudation rate d in this case is higher than the $d = 0.0001535$ h/1,000 yrs estimated by the regression equation of figure 3. This makes sense because figure 3 presumably includes data from river basins that are not being uplifted.

Equality of d_s, u, and d denotes Gilbert's (1877) state of "equality of action", which is identical with the "steady state" of Hack (1960); uplift and downwearing are in dynamic equilibrium.

The mean basin relief h_s at which this steady state is reached can be found from

$$d_s = 0.189\, h_s/1{,}000{,}000 \text{ yrs} = u/(1{,}000{,}000 \text{ yrs})$$

$$h_s = \frac{u}{0.189} = 5.29\, u.$$

Since d_s approaches u asymptotically, the two become truly equal only in infinity. For practical purposes, one may set the beginning of the steady state at the point in time when $d_s = 0.99$ u. This happens, according to

$$(1 - 0.189)^{t_s} = (1 - 0.99),$$

$$t_s = \frac{\log 0.01}{\log 0.811} = 22 \text{ m.y.}$$

after the beginning of uplift, that is, after the same time span that was required to reduce the mean relief of a drainage basin without any uplift to 1 percent of its initial value.

The steady state with constant relief can be reached only if u remains constant for at least 22 m.y. and if there is no change in the quantitative relationship $d_s = f(h)$ during that time. Such stringent conditions are not likely to exist in nature. Uplift rates generally vary within much shorter time spans. Climatic changes that affect the seasonal distribution of rainfall and runoff and thus the relationship $d_s = f(h)$ occur also more frequently, as the repeated shifts of climatic zones during the Pleistocene indicate. Finally, the successive exposure of rocks of varying resistance may also vary the rate of summit denudation.

Nevertheless, these changes are often cyclic. If geologically short periods of increasing rate of uplift alternate with periods of decreasing uplift and periods of tectonic rest, the relief will oscillate around the postulated steady state value. The amplitude of these oscillations depends upon the amplitude of the fluctuations in the rate of uplift and upon the lengths of the periods of increasing uplift, decreasing uplift, and rest. Similar oscillations can be expected if alternating climatic changes vary the relationship $d_s = f(h)$, or if alternating layers of weak and resistant rocks become successively exposed as the land surface is worn down.

Even if a steady state relief is never obtained, as, for example, in cases of progressively increasing and then abruptly ending uplift, the relationships between relief and uplift ($h = f(u)$) and between denudation and relief ($d_s = f(h)$ act as a causal linkage between uplift and denudation, via relief, so that there always exists a tendency toward establishing a dynamic equilibrium between the rate at which the land is raised by crustal movement and the rate at which it is worn down, with the latter striving to match the former. The adjustment of denudation to uplift requires time, and this time lag differs for the several denudational processes involved (Ahnert, 1954, 1967). First to reach equilibrium are the trunk streams in the margins of the uplifted area. By headward erosion, the adjustment is transmitted upstream and into their tributaries. Erosion which this steady state is reached can be

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of the streams steepens the lower parts of the valley side slopes and thus accelerates the transport of rock waste from the slopes to the streams. The steepening of the valley sides, and with it the acceleration of waste movement, extends progressively farther upslope until it reaches the interfluves and summits. Faster removal of waste means that the waste cover becomes thinner, and that the bedrock underneath, now less protected from atmospheric influences, weathers more rapidly. Eventually the rate of waste production by weathering equals the new higher rate of waste removal, and summits, slopes, and stream beds are being lowered at the same rate at which the land is uplifted. The total lag time from the beginning of the uplift to the eventual adjustment of the rate of summit lowering is likely to be longer for large uplifted areas, with large drainage basins, than for small ones, because of the time needed to transmit the adjustment of stream courses throughout the area. This fact has been deliberately neglected in the preceding discussion by the assumption that the rate of stream incision immediately equals the rate of uplift along the entire stream course. The time $t_s = 22$ m.y. represents therefore only the minimum needed to reach the steady state. Once the streams are adjusted, however, the additional adjustment of denudation and weathering on slopes and summits is a local process independent of the size of the uplifted area.

SOME PRACTICAL APPLICATIONS

The relationships between uplift, relief, and denudation discussed in the preceding section may be used as a means of order-of-magnitude estimates of the processes involved, over time spans that are too long to permit direct observation or simple extrapolation from present conditions. For example, the rates of intensive uplift cited by Gilluly (1949), in the vicinity of 10 m/1000 yrs, can occur only for a short time in any one area. If they were constant over greater lengths of geological time, they would produce a steady state relief h_s in the absurd order of 53,000 m, considerably more than the average thickness of the entire crust. Similarly, the even higher rates of glacio-isostatic uplift in the central areas of Pleistocene ice sheets are bound to be of short duration; in fact, near the periphery of these former continental glaciers the glacio-isostatic uplift has already been completed.

For slower, long-continuing uplift, the Colorado Plateau may offer a suitable example. During the Eocene, the area of the plateau became covered with thick sediments that were being deposited at very low elevations. Deposition ceased at the end of the Eocene, and degradation began to set in. Since then, an estimated 3000 m of sediments have been removed; some remnants of the Eocene cover stand now as mountains (for example, Chuska Mtns.) above the general level of the plateau (see Thornbury, 1965, p. 436). On the other hand, the Colorado River has cut its bed deeply into the present plateau surface, by as much as 2000 m (lower Grand Canyon). The total uplift in the 40 m.y. that have elapsed since the end of the Eocene thus probably exceeds 5000 m, and the average rate of uplift 125 m/1,000,000 yrs.

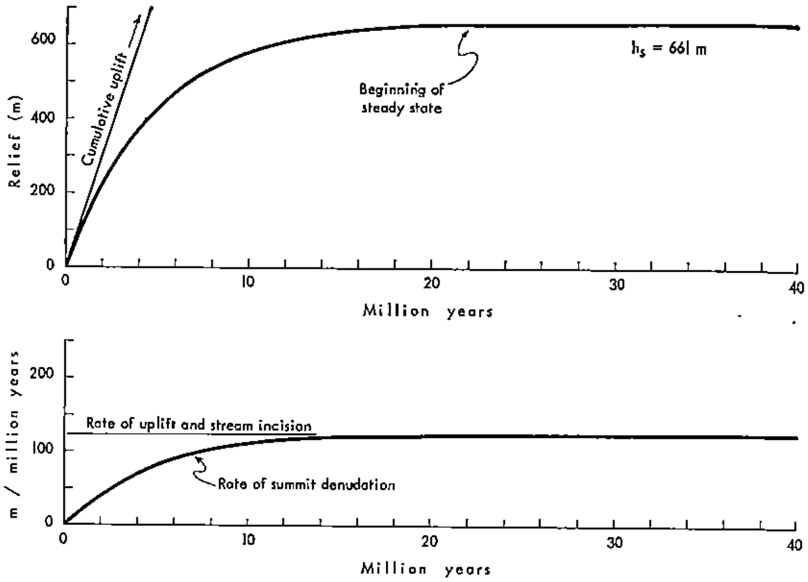

Fig. 6. Relief development with constant rate of uplift.

Using this rate of uplift, one obtains a steady state mean basin relief

$$h_s = 5.29\ u = 661\ \text{m},$$

which compares well with the mean relief per 20 x 20 km square in existence on the Colorado Plateau today. Figure 6 illustrates the relief development with the assumption that the rate of uplift has remained constant and that the rate of vertical erosion by the streams is equal to the rate of uplift. Under these conditions, the Colorado River would have begun erosion of the present Grand Canyon some 16 m.y. ago, during the late Miocene.

Figure 7 attempts to show the effects of variable uplift. The uplift (and stream incision) rate changes cyclically between 25 m/m.y. and 225 m/m.y. Each uplift cycle lasts 8 m.y. Total cumulative uplift over 40 m.y. is still 5000 m, as in figure 6. The relief development contains five cyclic oscillations, in response to the five cycles of uplift, and the rate of summit denudation oscillates in five cycles in response to the variations of relief. Relief increases as long as d_s is smaller than u and decreases during the times in which d_s is larger than u. Consequently, the occurrence of relief maxima and minima lags behind the occurrence of maxima and minima in the rate of uplift: a relief maximum occurs when the summit denudation curve crosses a descending segment of the uplift curve, and a relief minimum occurs when the summit denudation curve crosses an ascending segment of the uplift curve. For purposes of comparison, the relief development curve of figure 6 (constant rate of uplift) has been superimposed on the relief development curve of figure 7 as a dashed line.

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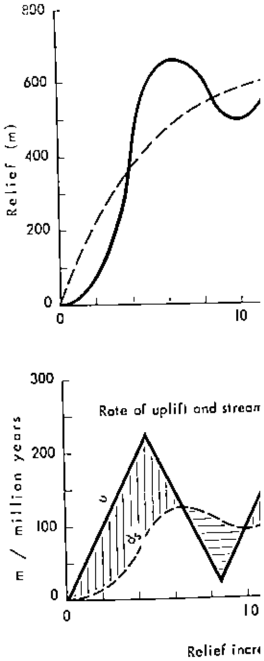

Fig. 7. Relationship bet
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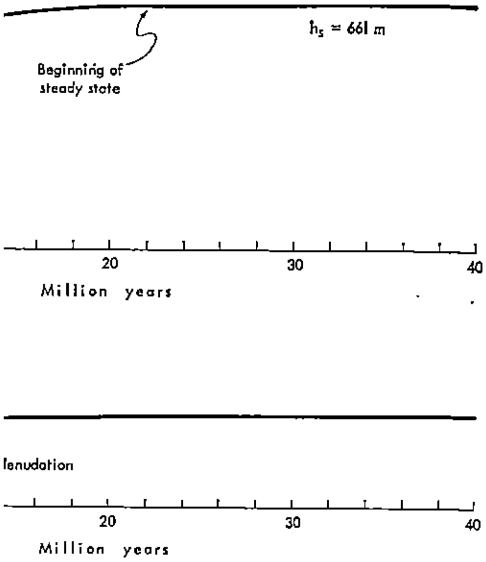

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e mean relief per 20 x 20 km square in ateau today. Figure 6 illustrates the relief tion that the rate of uplift has remained vertical erosion by the streams is equal to se conditions, the Colorado River would sent Grand Canyon some 16 m.y. ago, dur-

w the effects of variable uplift. The uplift nges cyclically between 25 m/m.y. and 225 8 m.y. Total cumulative uplift over 40 m.y. The relief development contains five cyclic five cycles of uplift, and the rate of summit ycles in response to the variations of relief. is smaller than u and decreases during the n u. Consequently, the occurrence of relief ind the occurrence of maxima and minima aximum occurs when the summit denuda- g segment of the uplift curve, and a relief mmit denudation curve crosses an ascend- ve. For purposes of comparison, the relief (constant rate of uplift) has been super- ment curve of figure 7 as a dashed line.

After 22 m.y. (the onset of the steady state in fig. 6), the relief oscillations retain a constant amplitude and elevation range, with maxima of 775 m and minima of 546 m, varying around a mean of 661 m which is identical with the steady state relief of figure 6. Correspondingly, the rate of summit denudation oscillates between 103 m/m.y. and 147 m/m.y. around a mean of 125 m/m.y., which is equal to the steady-state d_s of figure 6 and also equal to the mean rate of uplift in figure 7. Continuation of the cyclic variations of the rate of uplift with the same amplitude, frequency, and mean rate would cause continuation of the relief development cycles and denudation rate cycles within the same parameters as above. This exemplifies a steady rate of relief oscillation to be distinguished from the steady state of constant relief shown in figure 6.

Another possible application of the quantitative relationship between relief and denudation is an order-of-magnitude estimate of the probable rate of present stream incision, and thus possibly of present uplift, in each of the twenty river basins of figure 3. Such an estimate is based on the following premises: (1) The rate of stream incision equals the rate of uplift. (2) Mean denudation rates without stream incision have been estimated earlier to obey the function $d = 0.000106\ h/1000$ yrs: positive deviations of observed denudation rates from this function are assumed to be due to stream incision. (3) The summit denudation rate is $d_s = 0.00021\ h/1000$ yrs. (4) The observed mean denudation rate

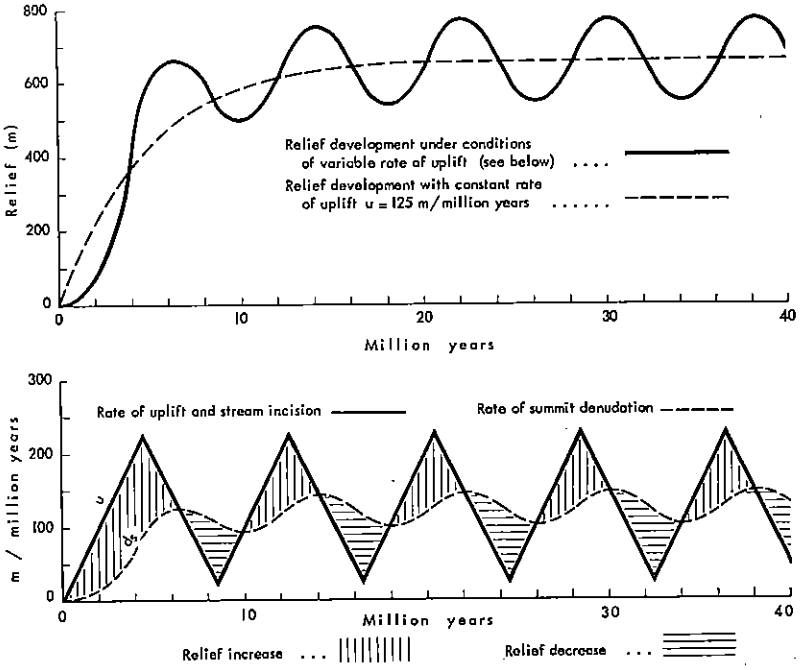

Fig. 7. Relationship between variable uplift, summit denudation, and relief development (schematic).

d (as listed in table 1) represents the mean between the rate of summit denudation d_s and the rate of stream incision (= rate of uplift) u, that is,

$$d = \frac{d_s + u}{2},$$

so that

$$u = 2d - d_s = 2d - 0.00021\ h/1000\ \text{yrs}$$

provides an estimate for the rate of uplift.

Needless to say, these premises are highly debatable. However, they have all been used profitably earlier in this paper, and only their present juxtaposition is partially new. Despite the misgivings that every one of them may evoke, they can be accepted with the understanding that the purpose of following this procedure is to explore its suitability for obtaining a crude idea of the probable rate of uplift.

TABLE 2

River basin	Mean denudation rate d (mm/1000 yrs)	Estimated rate of summit denudation d_s(mm/1000 yrs)	Estimated rate of uplift u (mm/1000 yrs)
Flint River above Montezuma, Ga.	28	19	37
Colorado River above San Saba, Tex.	16	21	11
River Thames above London, England	16	33	— 1
Delaware River above Trenton, N.J.	42	63	21
Canadian River above Amarillo, Tex.	52	74	30
Little Colorado River above Woodruff, Ariz.	31	82	— 20
Juniata River above Newport, Pa.	41	103	— 21
Green River above Green River, Utah	82	135	29
Escalante River, Utah	135	177	93
Dirty Devil River, Utah	177	204	150
Bighorn River above Thermopolis, Wyo.	109	211	7
Colorado River above Cisco, Utah	124	218	30
Wind River above Dubois, Wyo.	115	229	1
Animas River above Farmington, N.M.	195	267	123
Sarine, Switzerland	210	293	127
Rhine above Lake Constance, Switzerland	321	418	224
Isére above Grenoble, France	287	429	145
Reuss above Lake Lucerne, Switzerland	300	486	114
Kander, Switzerland	430	509	351
Rhône above Lake Geneva, Switzerland	418	602	234

Table 2 lists the resulting values of u. They fall clearly into two groups: for eleven basins u varies between 37 and − 21 mm/1000 yrs, and for the nine others it lies above 93 mm/1000 yrs. The first group comprises very heterogeneous climatic and structural environments; any conclusion regarding the differences between their respective estimated values of u would be extremely hazardous. Probably most of them undergo little or no uplift at present, or else the effect of any uplift upon their mean denudation rates remains inconclusive. Three of the nine basins

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of the second group lie on the Colorado Plateau (Escalante River, Dirty Devil River, and Animas River), the other six lie in the Alps. Perhaps by accident, the average u of those three Colorado Plateau basins (122 mm/1000 yrs) coincides almost exactly with the mean long-term uplift rate of 125 mm/1000 yrs that has been estimated for that region above. Only the Little Colorado River basin, the fourth basin on the plateau, casts a shadow over this relationship; however, its negative u may be due to local peculiarities. The six Alpine basins yield the highest average value of estimated uplift, namely, 199 mm/1000 yrs or 199 m/m.y. This figure also fits reasonably well within the rates of uplift that can be deduced from the elevations of erosion surface remnants and former valley floors in the Alps. According to Sölch (1935), erosion surfaces dated as late Miocene occur at elevations above 2000 m, late Pliocene surfaces above 1400 m, and preglacial (Plio-Pleistocene) valley floors around 1100 m, in the northern Alps near the Swiss-Austrian border (including the Alpine part of the Rhine basin). The present major valleys lie generally at about 700 m. Total valley deepening between the upper Miocene and the beginning of the Pleistocene would thus have amounted to more than 900 m in about 12 m.y., or 70 to 80 m/m.y. During the Pleistocene the valleys would have been deepened another 400 m in only 1 m.y. This highest rate is probably in part due to the work of the Pleistocene glaciers. Not only does the average value of $u = 199$ m/m.y. lie between the lower limit of the pre-Pleistocene rate and the upper limit of the Pleistocene, but also the individual values of u for the six Alpine drainage basins all lie between those two limits.

The results suggest that the admittedly very crude and very speculative method of estimating rates of stream incision, and indirectly of uplift, from measurements of relief and of denudation rates has some merit. Since the function $d = 0.000106$ h/1000 yrs (mean denudation rate without stream incision) has a lesser slope than the regression line of figure 3, the majority of the u-values in table 2 were bound to be positive and to increase generally with increasing relief: that in itself, then proves nothing. However, the order-of-magnitude agreement between the values of u and uplift rates estimated by other means in the case of the Colorado Plateau basins and of the Alpine basins is not an inherent consequence of the structure of the estimating procedure and therefore must be ascribed either to some degree of validity of the latter or—which seems less likely—to pure accident.

SUMMARY AND CONCLUSIONS

The preceding study has shown that the rate of denudation is mainly influenced by the available relief, and that it tends to increase linearly in proportion to the latter when removal of rock waste in solution is taken into account along with the removal of solid particles. Differences of mean annual precipitation seem to have no significant influence upon the total denudation rate, although they affect the ratio between removal in solution and removal of particles. Probably the seasonal distribution of precipitation is more important than the annual amount.

The correlation between relief and denudation rates permits order-of-magnitude estimates of several parameters of relief development, provided that certain generalizing assumptions are accepted. According to these estimates 22 m.y. are needed without any crustal movment, and at least 37 m.y. with isostatic compensation, to reduce the relief of an area to 1 percent of its initial value in climates without pronounced seasonality of rainfall. The long time required casts doubt on the possibility of peneplanation under these circumstances. Most known peneplains and inferred peneplain remnants date from the Tertiary, a period of frequent crustal movements over most of the Earth. Intervals of tectonic rest can hardly have been long enough to facilitate peneplanation by the same set of denudational processes active at present in the middle latitudes. Büdel's (1957) explanation of these erosion surfaces as products of more rapid areally effective wash processes under a warm-subhumid climate with seasonal concentration of rainfall offers a plausible alternative; combined with deep chemical weathering, these wash processes require less time to flatten the surface and can reduce the local relief even in the presence of regional uplift.

Time also plays a critical role in the question whether development of a steady state relief (Hack, 1960) is possible. It seems that the rate of uplift has to remain constant for several tens of millions of years before, under mid-latitude conditions, the denudation rate of the summits equals it. This makes attainment of a constant steady state relief very improbable. Periodic increases and decreases in the rate of long-term uplift and a resulting steady state of relief oscillation appear somewhat more likely. However, most uplifts probably are so discontinuous and so variable in magnitude that there is merely a tendency toward establishment of a steady state, that is, of a dynamic equilibrium between uplift and downwearing. This tendency rests directly on the functional relationship between relief and denudation and places an effective limit on the height of mountain ranges; no matter how intensive and long-lasting the uplift, it will eventually be equalled or surpassed by the rate at which the summits are worn down.

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