Estimating exhumation rate and relief evolution by spectral analysis of age–elevation datasets

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ABSTRACT

Extracting independent information on mean exhumation rate and the rate of surface relief change from thermochronometric datasets is essential to improve our understanding of the complex coupling between tectonics and surface erosion, i.e. the time-scale over which landforms react to changes in uplift rate and/or climate. A new method, based on spectral analysis of age–elevation data collected along one-dimensional profiles, is presented that provides independent estimates of the mean exhumation rate and the relative change in surface relief. The results are shown to be independent of the assumed geothermal gradient. The method is applied to an existing age dataset from the Sierra Nevada, California, and provides constraints on the evolution of the present-day relief. The spectral analysis demonstrates how current sampling strategies should be modified to optimize the tectonic and geomorphic information that can be retrieved from a thermochronometric dataset.

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Introduction

Age–elevation relationships have been used widely to constrain the rate of rock exhumation in a range of tectonic environments (Wagner *et al.*, 1977; Morley *et al.*, 1980; Benjamin *et al.*, 1987; Gleadow and Fitzgerald, 1987; House *et al.*, 1997; Batt *et al.*, 2000). In order to avoid complications arising from the thermal perturbation caused by finite-amplitude surface topography (Stüwe *et al.*, 1994), the variation of thermochronometric ages with elevation is usually sampled along near-vertical profiles. It has been shown (House *et al.*, 1998), however, that information pertaining to the evolution of surface landforms through time is also contained in the variation of ages with elevation at a larger length scale. For example, the spatial distribution of (U–Th)/He apatite ages across deeply incised valleys has been used to argue for the antiquity of the present-day drainage system of the Sierra Nevada, California (House *et al.*, 1998, 2001). This low-temperature, chronometric method (Rutherford, 1907; Zeitler *et al.*, 1987) was recently shown to have the potential to constrain the rate of surface landform evolution in response to more or less rapid changes in tectonic or climatic environment (Farley *et al.*, 1996; Wolf *et al.*, 1996). However, the interpretation of age data derived from such a low-temperature (~ 70 °C) thermochronometer is difficult and relies on the knowledge of usually poorly constrained local parameters, notably, the geothermal gradient (House *et al.*, 1998, 2001). Here, a new method to interpret thermochronometric data is proposed that does not suffer from this drawback.

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Spectral analysis

It is well known that the thermal perturbation caused by finite-amplitude surface topography decreases exponentially with depth at a rate that is proportional to the wavelength of topography (Turcotte and Schubert, 1982; Stuwe *et al.*, 1994; Mancktelow and Grasemann, 1997). As illustrated in Fig. 1, for a given isotopic system, there exists a critical wavelength (λ_c) below which topography has little effect on the shape of the corresponding closure temperature isotherm. Thus, the slope of the relationship between age and elevation data collected at a scale smaller than λ_c should provide an accurate estimate of the exhumation rate (Fig. 1a). Conversely, at wavelengths much larger than λ_c, the closure temperature isotherm follows exactly the shape of the topography and there is no variation in age with elevation (Fig. 1b). It is this strong dependency of the slope of age–elevation relationships on the wavelength of the surface topography that is used here to develop a new method of analysing thermochronometric data, which delivers independent estimates of exhumation rate and the rate of change of surface landform. The new method proposed here makes use of the fractal nature of surface topography (Huang and Turcotte, 1989), which allows one to sample the relationship between age and elevation at a wide range of topographic wavelengths by collecting data along one-dimensional (1D) transects. Extracting the relationship between age and elevation is equivalent to determining the so-called 'frequency response function' or 'admittance function' of a system that has elevation as input and age measurements derived from rocks sampled along the profile as output (Jenkins and Watts, 1968). The admittance function can be described in terms of a gain, G, and a phase, F. Both are functions of the wavelength of the input topography, λ. Expressions for $G(\lambda)$ and $F(\lambda)$ can be obtained from classical spectral analysis (Jenkins and Watts, 1968):

$$G(\lambda) = \sqrt{(C_{12}^2 + Q_{12}^2)}/C_{11}, \quad F(\lambda) = \arctan(-Q_{12}/C_{12}), \qquad (1)$$

where C_{12} and Q_{12} are the real and imaginary parts of the cross spectrum obtained from the real and imaginary parts of the smoothed spectral estimators of the input and output signals. These estimators are, in turn, obtained from the Fourier transforms of the windowed input (elevation, z) and output (age, a) signals, R_z, I_z and R_a, I_a (Jenkins and Watts, 1968):

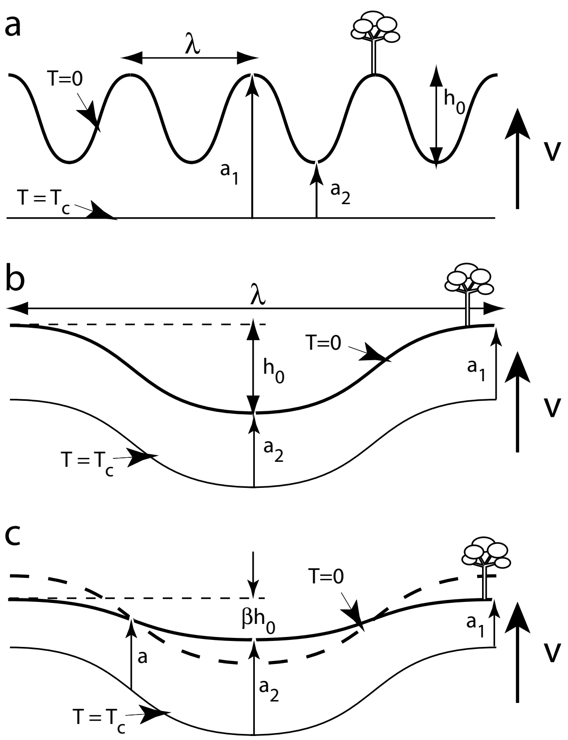

Fig. 1 Age–elevation relationship caused by surface topography. (a) Short-wavelength topography does not affect the geometry of closure temperature isotherm, T_c, and the slope of the age–elevation, *a–h*, relationship gives the inverse of the exhumation rate, v ($da/dh = (a_1 - a_2)/h_0 = 1/v$). (b) Long-wavelength topography strongly affects the shape of the T_c isotherm and age is independent of elevation ($da/dh = 0$). (c) At long topographic wavelengths, any age variation with elevation is indicative of a relative change in relief amplitude since rocks crossed the T_c isotherm.

$$C_{12} = R_z R_a + I_z I_a$$
$$Q_{12} = I_z R_a + R_z I_a. \qquad (2)$$

C_{11} is the power spectrum of the input signal:

$$C_{11} = R_z^2 + I_z^2. \qquad (3)$$

The smoothed spectral estimators are obtained by applying a triangular Bartlett window (see Jenkins and Watts, 1968; p.244) to the elevation and age profiles prior to the calculation of the Fourier transforms. Confidence intervals can be calculated from the smooth spectral estimators (see Jenkins and Watts, 1968; p.437). Efficient methods exist to compute Fourier transforms from regularly spaced data (Jenkins and Watts, 1968). In practice, however, it might be difficult to collect rocks at regular intervals along a linear transect. The simplest way to compute Fourier transforms of irregularly spaced data is to interpolate the data on a regular grid (Press *et al.*, 1992). This might prove inadequate if large gaps exist in the data. In that case, other methods such as Lomb's normalized periodogram must be used (Lomb, 1976).

An example based on synthetic ages

In order to illustrate how the method can be used to interpret thermochronological data, a synthetic age dataset was constructed from the temperature field obtained by solving the heat transport equation numerically. To this effect, a newly developed finite-element solver, Pecube, was used that includes the effects of heat advection by vertical exhumation as well as the thermal perturbation of a finite amplitude, time-varying, isothermal surface topography (Braun, unpubl. data). The computed temperature field is used to derive temperature–time paths, from which an age dataset for the (U–Th)/He chronometer is derived using the procedure and parameters described in Wolf *et al.* (1998) and Farley (2000). A frequency factor (D_0/a^2) of 50.1×10^6 s^{-1} and activation energy (E_a) of 151.5×10^3 kJ mol^{-1} are used. A uniform exhumation rate of 0.3 km Myr^{-1} and a conductive geothermal gradient of 20 °C km^{-1} are assumed. Thermal diffusivity is set at 25 km^2 Myr^{-1} and radioactive heat production is neglected. The 128-km-long topographic profile used in this experiment comes from a 1-km-resolution DEM (GTOPO30) of the South Island, New Zealand. In this way, the input (elevation) signal has a spectral content that is representative of a natural landform (see Fig. 2a). The computed age profile is shown in Fig. 2(a); the computed gain, shown in Fig. 2(b), reveals that, at wavelengths shorter than 8 km, the gain ($\sim$3 Myr km^{-1}) provides a good estimate of the inverse of the imposed exhumation rate (0.3 km Myr^{-1}). This is because, at short wavelengths, the isotherms are not perturbed by topography (see Fig. 1a). At intermediate wavelengths, the gain is smaller than the inverse of the imposed exhumation rate as the finite topography starts to perturb the closure temperature isotherm for (U–Th)/He in apatite. At wavelengths longer than 25 km, the gain tends towards zero as the closure temperature isotherm becomes parallel to the surface topography and thermochronological ages are independent of elevation (see Fig. 1b). Figure 2(c) compares the gain shown in 2(b) with the gain derived from a model experiment in which the surface relief (the amplitude of the surface topography) is artificially increased from half its present-day value over the last 3 Myr of the numerical experiment. At short wavelengths, the results are similar to those of the first experiment, but at large wavelengths, the gain estimates tend asymptotically towards a finite, positive value of $\sim$1.8 Myr km^{-1}. Similarly, in an experiment where surface relief is decreased from twice its present-day value over the last 3 Myr, the predicted gain values (Fig. 2c) tend towards a finite negative value of $\sim$–2.5 Myr km^{-1} at long wavelengths.

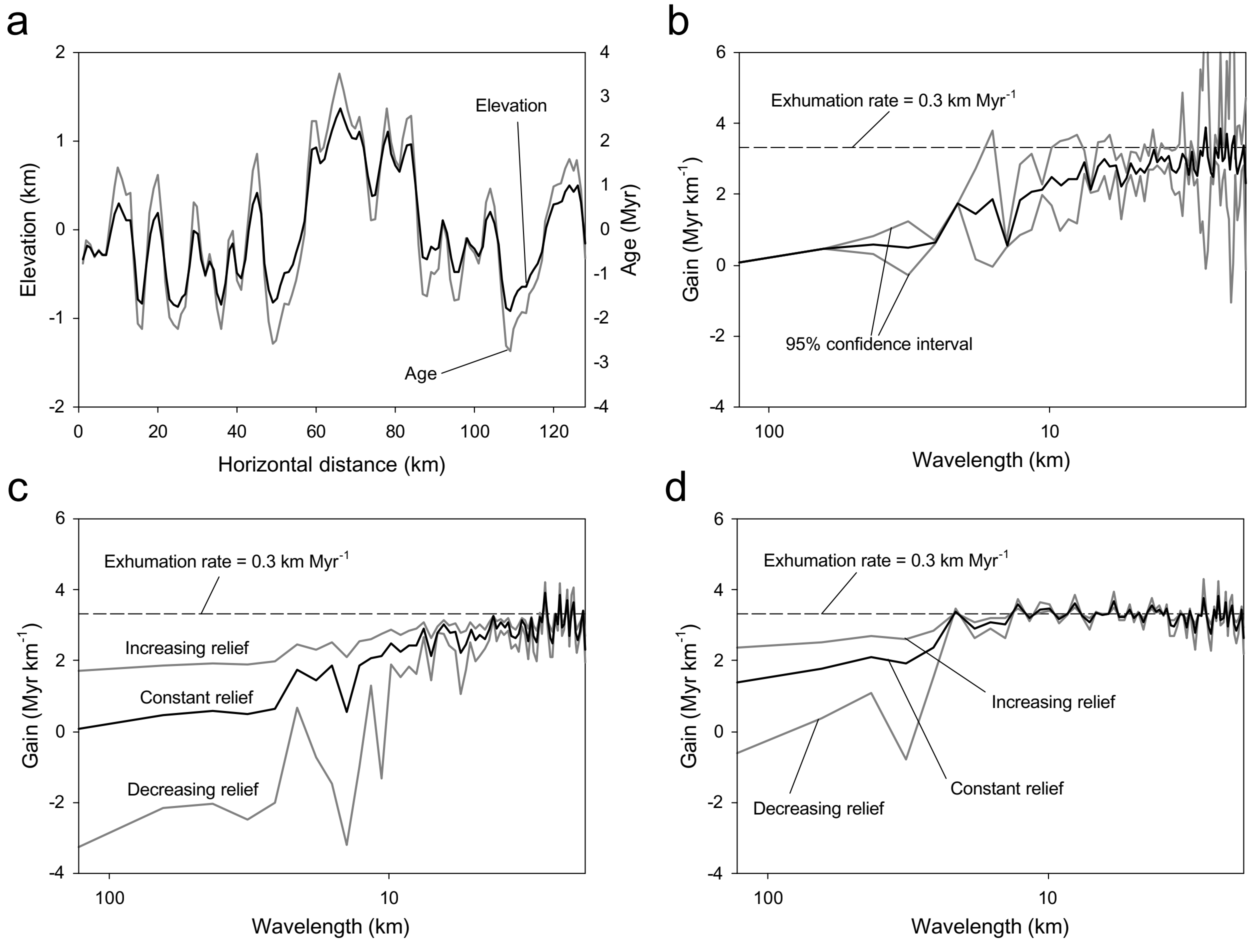

Fig. 2 Synthetic example. (a) Elevation profile extracted from a 1-km DEM of South Island (New Zealand) and predicted (U–Th)/He ages. Mean values of 1.58 km and 10.78 Myr have been subtracted from the elevation and age profiles, respectively. (b) Real part of the gain function between age and elevation and 95% confidence levels. (c) Real part of the gain function between age and elevation calculated by assuming different scenarios for relief evolution. (d) Real part of the gain function for thermochronological ages corresponding to a closure temperature of 300 °C. Because only vertical tectonic transport is considered here, at all wavelengths, variations in age must be in phase or exactly out-of-phase with variations in elevations, i.e. the phase estimates are either 0 or $\pm\ \pi$. Thus, only the real part of the gain is shown.

The computed gain function therefore contains two independent pieces of information: (i) the asymptotic value of the gain at short wavelengths, G_S, provides a direct estimate of the inverse of the mean exhumation rate, and (ii) the asymptotic value of the gain estimate at long wavelengths, G_L, indicates whether, in the recent past, topography has remained constant ($G_L = 0$), has increased ($G_L > 0$) or decreased ($G_L < 0$) in amplitude. As shown in Fig. 1(c), at very long wavelengths, any gradient in age with elevation must be related to recent changes in surface relief, i.e. experienced since the rocks passed through the closure temperature isotherm. If β is defined as the ratio between relief amplitude now and that at the time rocks passed through the closure temperature, the age at the top of a ridge, a_1, is given by (Fig. 1c):

$$a_1 = a - \{[(\beta - 1)h_0/2]/v\}, \qquad (4)$$

where a is the mean age (i.e. for a rock that is exhumed at mean elevation), whereas the age at the bottom of a valley, a_2, is given by:

$$a_2 = a + \{[(\beta - 1)h_0/2]/v\} \qquad (5)$$

and the gradient in age with elevation, da/dz, is equal to:

$$(da/dz)_L = (a_2 - a_1)/h_0 = (\beta - 1/\beta)(1/v). \qquad (6)$$

and, because $G_S = 1/v$, one can write:

$$\beta = 1/(1 - G_L/G_S). \qquad (7)$$

The ratio G_L/G_S provides, therefore, a direct estimate of the relative change in surface relief over the time period defined by the mean age of the rocks. This result is based on the assumption that, when the rocks pass through the closure temperature isotherm, the temperature field is near steady-state (i.e. the isotherms are in phase with the long wavelength topography); if this is not the case, the ratio G_L/G_S must be regarded as a maximum estimate of the relative change in relief. The value of the relief reduction factor, β, computed from the gain values shown in Fig. 2(c) are 0, 2.5 and 0.55, which compares very well with the imposed values of 0, 2 and 0.5, respectively.

Re-interpreting Sierra Nevada age–elevation data

In recent years, several apatite (U–Th)/He age datasets have been collected to constrain the exhumation and relief history of the Sierra Nevada in California (House *et al.*, 1997, 1998, 2001). One of the datasets extends for approximately 180 km across the southern Sierra Nevada Batholith and contains 36 age determinations from rocks collected at or near an elevation of 2000 m (House *et al.*, 1998). This sampling strategy is not well suited to the present analysis method but no better dataset exists at present. The data were interpolated to provide 128 equally spaced samples that were used to compute the spectra and gain. The results are shown in Fig. 3. At short wavelengths, the gain values are not very well constrained, because the data sampling is too coarse and irregular to capture the relationship between age and elevation at wavelengths below 10 km. There exists however, an independent age dataset collected along a very short transect (< 2.5 km) in the Kings Canyons area (House *et al.*, 1997) from which a linear regression between age and elevation yields an exhumation rate of 0.04 km Myr^{-1}. The inverse of this value (25 Myr km^{-1}) is shown in Fig. 3(b) as an horizontal dashed line. Although very noisy, the gain values at short wavelengths are consistent with this estimate. At long wavelengths, the real part of the gain is clearly negative, indicating that relief has decreased over the last 67 Myr. The asymptotic value of the gain at long wavelengths is approximately – 25 Myr km^{-1} which, combined with the estimate of exhumation rate of 0.04 km Myr^{-1} and using (7), yields a value of $\beta = 0.5$ for the predicted relative reduction in relief. This estimate is in good agreement with that obtained by House *et al.* (1998) by interpreting age variations at constant elevation and confirms that, since the end of the Laramide Orogeny some 70–80 Ma, surface relief in the Sierra Nevada has decreased by more than 50%. Contrary to that of House and coworkers, the estimate of relative relief derived here from the admittance function does not rely on any assumption on the value of the past or present geothermal gradient. This example demonstrates the power of spectral analysis to derive accurate and independent estimates of exhumation rate and the rate of change of surface topography from age–elevation datasets.

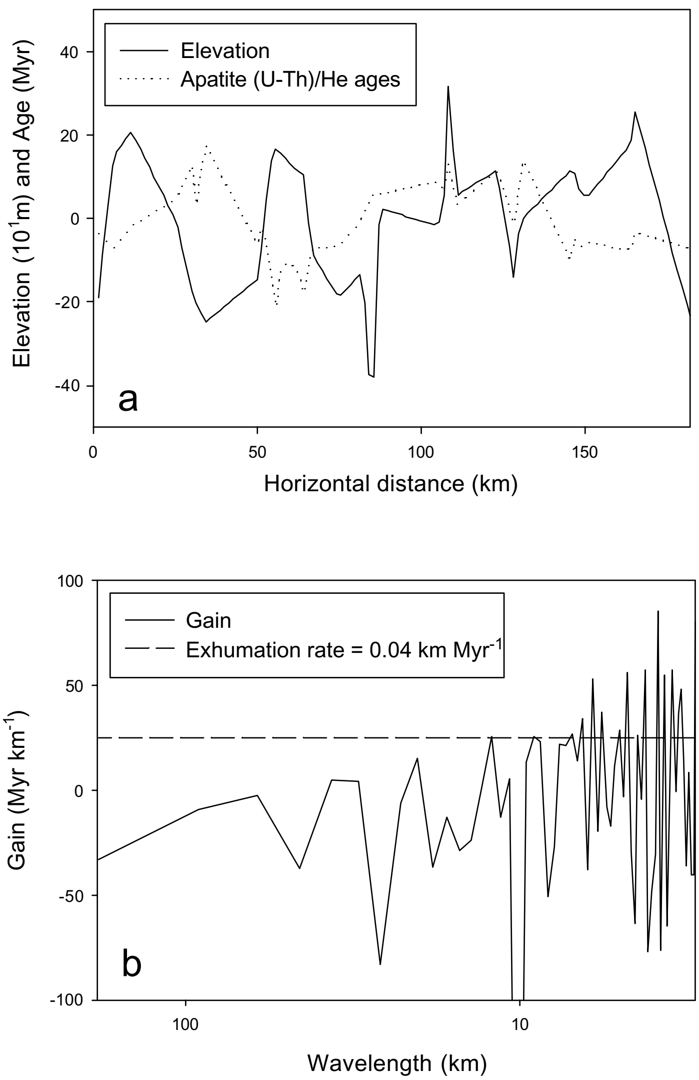

Fig. 3 Application to Sierra Nevada. (a) Interpolated elevation and apatite (U–Th)/He age data from a transect through the Sierra Nevada (House *et al.*, 1998). A mean value of 2 km and 67 Myr has been subtracted from the elevation and age datasets, respectively. (b) Computed gain values.

Discussion

The admittance method described here requires a specific sampling strategy that ensures that elevation and age are measured at the appropriate range of wavelengths. The optimal sampling interval is approximately half an order of magnitude smaller than λ_c, the critical wavelength introduced earlier, and the optimal length of the profile is half an order of magnitude larger than λ_c. The dataset must therefore contain a minimum of 30–50 samples. λ_c is different for each thermochronometric system and depends on the conductive geothermal gradient and the mean exhumation rate (Stuwe *et al.*, 1994). If one assumes that the depth of the thermal anomaly caused by surface topography is, to first order, equal to the wavelength of the topography (Turcotte and Schubert, 1982), then the critical wavelength, λ_c, can be estimated as the ratio of the closure temperature of the thermochronological system, T_c, and the local geothermal gradient, dT/dz (which must take into account the thermal perturbation caused by advection of rocks towards the surface (Batt and Braun, 1997). Attention must therefore be paid to these factors when designing a sampling strategy, with wider spacing for high-temperature thermochronometers, and/or cold, slowly eroding tectonic environments. This is illustrated in Fig. 2(d), where the gain values are derived from thermochronological ages calculated for a higher closure temperature geochronometer such as K–Ar in muscovite ($T_c = 300$ °C). The transition from high- to low-gain values takes place at a longer wavelength. It is important to stress that the spectral method requires stationarity of the relationship between the elevation and age signals along the transect, and, thus, in choosing the location of the transect, one must ensure that geothermal gradient, exhumation rate and relief characteristics are uniform along the transect. It is also clear that the method can only provide estimates of exhumation rate and rate of surface relief change over a time interval that corresponds to the mean age of the samples. As already suggested in Batt and Braun (1997), the use of several thermochronometers is therefore required to invert the age–elevation dataset in areas affected by a complex, multistage tectonic or geomorphic history. Finally, it is worth noting that the method can be extended to investigate the suitability of age–elevation profiles to determining whether landforms are static features or if they are affected by horizontal tectonic transport. If, during exhumation, rocks are also translated horizontally beneath a fixed landform, the resulting age profile will be offset (or out of phase) with respect to the elevation profile. This offset should result in a measurable phase shift in the gain function between age and elevation.

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