The dynamic reference frame of rivers and apparent transience in incision rates

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Description of the model

The model used to generate figures 1 and 2 is based on a simple set of rules. The primary inputs are an uplift rate (U_r) and a streambed elevation history (z_s) that are functions of time. The uplift rate used in our simulations is set to a steady rate of 1 mm yr^{-1}. The streambed elevation history that we choose is a composite of three sine waves meant to mimic Milankovitch-type climate cycles. The equation for the streambed elevation history is:

$$z_s(t) = A_4 \sin\left(\frac{2\pi t}{P_{23}}\right) + A_8 \sin\left(\frac{2\pi t}{P_{41}}\right) + A_{16} \sin\left(\frac{2\pi t}{P_{100}}\right) \quad (1)$$

Where A_4, A_8, and A_{16} are the amplitudes of the sine wave in meters that are set to 4, 8, and 16 respectively; P_{23}, P_{41}, P_{100} set the period of the sine waves and are 23,000 yrs, 41,000 yrs, and 100,000 yrs, respectively, and t is time in years.

The model is set to run for 500 k.y. at 1 yr time-steps following these rules: (1) when an upper inflection point is encountered in the streambed elevation history (e.g. the streambed elevation is rising, then falls) a 'terrace' is generated at the time (t) when the slope of the z_s *curve* is 0. (2) The terrace is then uplifted at the prescribed rate (U_r) and the elevation of the terrace (z_t) at time t is determined by integration:

$$z_t(t) = z_{t0} + \int_0^{t_a} U_r(t)dt \quad (2)$$

where z_{t0} is the formation elevation of the terrace and t_a is the age of the terrace age at time t. The terrace age is defined as:

$$t_a = t - t_0 \quad (3)$$

where t_0 is the time that the terrace formed in years. (3) A terrace is only preserved in the model if it remains higher in elevation than the streambed elevation history throughout the entire model run. In other words, if the streambed elevation ever rises above the elevation of a terrace at time $t + t_0$, the terrace is removed from the model simulation.

For the model run shown in Figure 1, we track the elevation history of a single terrace generated at the first inflection point in the streambed elevation history (ca. 480 kyr BP) (Fig. 1B). The elevation where the terrace forms (z_{t0}) is set as zero on the y-axis and is used as a fixed external reference frame. The terrace is uplifted at a rate of 1 mm yr^{-1} and we track two measurements through time, cumulative uplift (U_Σ):

$$U_\Sigma = z_t - z_{t0} \tag{4}$$

and cumulative incision (I_Σ):

$$I_\Sigma = z_t - z_s \tag{5}$$

We also calculate a time-averaged uplift rate (U_{ra}):

$$U_{ra} = U\, t_a^{-1} \tag{6}$$

and an time-averaged incision rate (I_{ra}):

$$I_{ra} = I\, t_a^{-1} \tag{7}$$

The apparent rates show how measurements of cumulative uplift and cumulative incision calculated from a terrace would evolve with time.

For the model shown in Figure 2 we generate a sequence of 10 terraces following the rules outline above that range in age from 480-60 ka (Table DR1). At the end of the model run the final age (t_{fa}) and elevation (z_{tf}) for each terrace are recorded and used in all subsequent calculations. We then make three different apparent cumulative incision calculations (I_a) using three different assumed streambed elevations histories (z_{sa}):

$$I_a = z_{tf} - z_{sa} \tag{8}$$

In the first set of calculations we use the correct streambed elevation history, such that $z_{sa} = z_s$ for each terrace (Fig. 2A). In the next series of calculations we (incorrectly) assume that the streambed elevation has remained at the same elevation over the duration of terrace formation. We first assume that z_{sa} is 25m higher than the mean elevation at which the terraces formed, which is ~ -10.65 m on the y-axis in Figures 2A &2C, making z_{sa} = 14.35 m. Next we set z_{sa} to 25m lower than the mean elevation at which the terraces formed, making z_{sa} = -35.65 m. Apparent cumulative incision is used to calculate final apparent incision rates (I_{fra}) for each terrace:

$$I_{fra} = I_a \, t_{fa}^{-1} \tag{9}$$

Table DR1 shows all of the apparent cumulative incision and incision rate calculations preformed for this thought experiment.

Relationship between assumed streambed elevation, apparent time-averaged incision rates, and the power-law regression exponent (β) for cumulative incision versus terrace age

Figure DR1 shows a simple graphical example of what happens when different assumed streambed elevations are used to calculate apparent time-averaged incision from terraces. Figure DR1 illustrates the same concepts as Figure 2C. Here three terraces that are 25, 50, and 100 ka are assumed to have uplifted at 1 mm yr^{-1} from the same starting elevation (Fig. DR1A). All three panels in figure DR1 are the same except for that we change the position of zero on the y-axis, which represents the assumed streambed elevation. If the correct streambed elevation is used to calculate cumulative incision, the derived rates (the slope of the gray line in figure DR1 A) are all the same, 1mm yr^{-1}. If the same terraces are used to calculate cumulative incision from an elevation that is higher than the one from which the terraces formed, apparent time-averaged incision rates systematically increase with time (Fig. DR1 B). Conversely, if cumulative incision is calculated from an elevation lower than the actual elevation at which the terraces formed, apparent time-averaged incision rates systemically decrease with time (Fig. DR1 C). This simple exercise highlights the need to use the correct reference frame for the calculation of incision from terraces, otherwise incision rates will be systematically biased. This is the same concept as depicted in Figure 2C.

Figure DR2 shows the same base level curve and modeled terraces as in Figure 2, where 10 terraces ranging in age from ~480 to 50 ka are uplifted at a steady rate of 1 mm yr^{-1}. Just as in Figure 2, zero on the y-axis is referenced to the formation elevation of the first terrace generated in the model and is not used in any subsequent calculations (Fig. DR2 A). In this model we calculate cumulative incision for each uplifted terrace as the difference between its final elevation with respect to a series of static base level from 50 m below to 50 m above the average elevation of terrace formation, which is ~ -11 m on the y-axis (Fig. DR2 A). For each assumed base level we compute the power-law relationship between cumulative incision and terrace age (Fig. DR2 B). The relationship between the offset in assumed base level from the average elevation of terrace formation and deviation of the cumulative incision versus terrace age (e.g. measured interval) power-law exponent (β) is predictable. If the assumed base level is below the average elevation of terrace formation β is always less than 1. Conversely, if it is above the average elevation of terrace formation β is always greater than one. Furthermore, the greater the offset between assumed base level and the average elevation of terrace formation, the more β will deviate from unity (Fig. DR2 B).

Monte Carlo analysis

We use a Monte Carlo analysis to determine the power-law exponent (β) and associated uncertainties from terrace age and cumulative incision data for four published Pleistocene incision records with respect to the modern channel as well as to the lowest Pleistocene terrace (Formento-Trigilio and Pazzaglia, 1998; Frankel and Pazzaglia, 2006; Wegmann and Pazzaglia, 2009; Pederson et al., 2013). We first use the 'raw' data that was measured with respect to the modern channel, so that cumulative incision represents the distance of the terrace strath above the modern river channel and terrace age is the age of the terrace. Next, we use 'corrected' data that is measured with respect to the lowest Pleistocene terrace in the sequence. In this case, 'corrected' cumulative incision (I_c) is the strath elevation (z_{si}) minus the elevation of the strath of the lowest Pleistocene terrace (z_{s0}) such that:

$$I_c = z_{si} - z_{s0} \tag{10}$$

The 'corrected' terrace age (A_c) is the age of the terrace (A_{si}) minus the age of the lowest Pleistocene terrace (A_{s0}):

$$A_c = A_{si} - A_{s0} \tag{11}$$

To determine the power-law exponent and associated 1σ standard error for the four Pleistocene incision records presented in Figure 3 we calculated 10,000 linear fits between the logarithm of cumulative incision and the logarithm of measured interval. For each model iteration a value of measured time and cumulative incision was assigned to each terrace by randomly selecting a data point from a synthetic normal distribution generated using the measured value and standard error of each geochronologic age and measured elevation. The slope and y-intercept of each linear regression was recorded and the mean and 1σ errors from these distributions were used to define the power-law relationship between cumulative incision and measured time interval and to assign uncertainties.

Figure captions:

Figure DR 1: The effect of assumed streambed elevation on incision rates measured from terraces. Three terraces that formed at 25, 50, and 100 ka were uplifted at 1 mm yr^{-1} from the same initial elevation so that they lie at elevations of 25, 50 and 100 m, respectively. The slopes of lines drawn from the age (present-day = 0) and elevation of the streambed represent the calculated incision rates. All figures are scaled the same, only the position of zero on the y-axis that designates the assumed streambed elevation changes (e.g. terrace positions do not change). **A**. When the correct streambed elevation is used to calculate incision, the incision rates are all the same (i.e. lines are parallel). In **B** and **C** apparent incision rates are calculated by assigning a streambed elevation that is higher and lower, respectively, relative to the actual initial elevation at which the terraces formed. These ‘apparent incision’ measurements systematically deviate from the actual incision measurements because adding or subtracting the same amount of elevation to each incision measurement skews the younger, lower terraces more so than the higher, older terraces. **B**, If elevation gain is subtracted from the incision measurements by assuming a streambed elevation that is higher than actual, rates systematically increase with increasing terrace age. This is noted by the steepening of colored lines with increasing terrace age. **C**, If elevation gain is added to the incision measurements by assuming a streambed elevation that is lower than actual, rates systematically decrease with increasing terrace age. This is noted by the decrease in slope of the colored lines with increasing terrace age.

Figure DR2: A, base level history (black line) and the age and formation elevation of 10 terraces (white dots) uplifted at 1 mm yr^{-1}. The final elevation of the uplifted terraces is used to calculate cumulative incision assuming a series of static base level elevations (color ramp) that range from 50 m below to 50 m above the average elevation of terrace formation (~-11m). **B**, power-law regressions through cumulative incision (calculated with respect to an assumed static base level) and terrace age (color ramp).

References:

Formento-Trigilio, M. L., and Pazzaglia, F. J., 1998, Tectonic Geomorphology of the Sierra Nacimiento: Traditional and New Techniques in Assessing Long-Term Landscape Evolution in the Southern Rocky Mountains: The Journal of Geology, v. 106, no. 4, p. 433-454. doi: 10.1086/516034

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Pazzaglia, F. J., and Brandon, M. T., 2001, A Fluvial Record of Long-term Steady-state Uplift and Erosion Across the Cascadia Forearc High, Western Washington State: American Journal of Science, v. 301, no. 4-5, p. 385-431. doi: 10.2475/ajs.301.4-5.385

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Table DR1: Modeled river terrace age and incision measurements shown in Figure 2

terrace number	terrace formation elevation (z_{t0}) (m)	final terrace elevation (z_{tf}) (m)	final terrace age (t_{fa}) (k.y.)	actual cumulative incision (I) (m)[a]	Actual incision rate (I_r) (mm yr^{-1})[b]	apparent cumulative incision (I_a), mean + 25m (m)[c]	apparent cumulative incision rate (I_{fra}), mean + 25m (mm yr^{-1})[d]	apparent cumulative incision (I_a), mean - 25m (m)[e]	apparent cumulative incision rate (I_{fra}), mean - 25m (mm yr^{-1})[f]
1	0.00	478.20	478.22	478.20	1.00	463.85	0.97	513.85	1.07
2	-21.99	431.98	453.97	453.97	1.00	417.63	0.92	467.63	1.03
3	-14.22	371.98	386.20	386.20	1.00	357.63	0.93	407.63	1.06
4	-4.39	358.00	362.40	362.39	1.00	343.65	0.95	393.65	1.09
5	-1.00	271.42	272.42	272.42	1.00	257.07	0.94	307.07	1.13
6	-14.41	182.86	197.27	197.27	1.00	168.51	0.85	218.51	1.11
7	-12.79	166.86	179.66	179.65	1.00	152.51	0.85	202.51	1.13
8	-9.01	147.29	156.30	156.30	1.00	132.94	0.85	182.94	1.17
9	-24.27	84.61	108.88	108.88	1.00	70.26	0.65	120.26	1.10
10	-4.43	62.34	66.77	66.77	1.00	47.99	0.72	97.99	1.47

[a] Difference between the final terrace elevation and the terrace formation elevation

[b] Actual cumulative incision divided by the final terrace age; z

[c] Difference between the final terrace elevation and a static elevation that is 25m higher than the mean terrace formation elevation (-10.65)

[d] Incision rate calculated using incision measurements calculated using inferred static elevation that is 25m higher than the mean terrace formation elevation (-10.65)

[e] Difference between the final terrace elevation and a static elevation that is 25m lower than the mean terrace formation elevation (-10.65)

[f] Incision rate calculated using incision measurements calculated using inferred static elevation that is 25m lower than the mean terrace formation elevation (-10.65)

Table DR2: Real river terrace age and incision measurements

modern channel				lowest terrace						
age (k.y.)	**Uncert. (k.y.)**	**cumulative incision (m)**	**Uncert. (m)**	**age (k.y.)**	**Uncert. (k.y.)**	**cumulative incision (m)**	**Uncert. (m)**	**location**	**reference**	**Landform**
60	10	7	2	-	-	-	-	Jemez River	Formento-Trigilia and Pazzaglia (1998); Frankel and Pazzaglia (2006) for all terraces	strath terrace
140	10	25	10	80	20	18	12	Jemez River		strath terrace
350	50	40	12	290	60	33	14	Jemez River		strath terrace
400	50	70	12	340	60	63	14	Jemez River		strath terrace
610	10	90	12	550	20	83	14	Jemez River		strath terrace
1200	10	190	12	1140	20	183	14	Jemez River		strath terrace
15.2	1.3	11.5	0.1	-	-	-	-	Lee's Ferry	Pederson et al. (2013) for all terraces	top of fill terrace
42.0	3.8	25.6	0.1	26.8	5.1	14.1	0.2	Lee's Ferry		top of fill terrace
91.1	15	41.8	0.1	75.9	16.3	30.3	0.2	Lee's Ferry		top of fill terrace
94.0	9	45.5	0.1	78.8	10.3	34	0.2	Lee's Ferry		top of fill terrace
135.2	12	66.7	0.1	120.0	13.3	55.2	0.2	Lee's Ferry		top of fill terrace
145.2	15	76.5	0.1	130.0	16.3	65	0.2	Lee's Ferry		top of fill terrace
13	2	27	3	-	-	-	-	Bidente River	Wegmann and Pazzaglia (2009) for all terraces	strath terrace
22	2	40	5	9	4	13	8	Bidente River		strath terrace
30	5	48	5	17	7	21	8	Bidente River		strath terrace
140	10	123	10	127	12	96	13	Bidente River		strath terrace
440	25	196	10	427	27	169	13	Bidente River		strath terrace
620	25	318	10	607	27	291	13	Bidente River		strath terrace
800	100	388	10	787	102	361	13	Bidente River		strath terrace
10	1	11	1	-	-	-	-	Musone River	Wegmann and Pazzaglia (2009) for all terraces	strath terrace
27	2	14	1	17	3	3	2	Musone River		strath terrace
40	5	25	4	30	6	14	5	Musone River		strath terrace
160	10	42	22	150	11	31	23	Musone River		strath terrace
450	50	73	15	440	51	62	16	Musone River		strath terrace
775	260	154	5	765	261	143	6	Musone River		strath terrace

Table DR3: Power-law regression exponents and uncertainties for Pleistocene terraces

Location	Reference datum	Citation	Cumulative incision versus measured time interval power-law exponent (β)	± 1σ	timespan of record (k.y.)
Jemez River	modern channel	Formento-Trigilia and Pazzaglia, 1998; Frankel and Pazzaglia, 2006	1.07	0.08	1140
Jemez River	lowest terrace	Formento-Trigilia and Pazzaglia, 1998; Frankel and Pazzaglia, 2006	0.91	0.21	1060
Lee's Ferry	modern channel	Pederson et al., 2013	0.8	0.04	130
Lee's Ferry	lowest terrace	Pederson et al., 2013	0.91	0.13	103
Bidente River	modern channel	Wegmann and Pazzaglia, 2009	0.61	0.03	787
Bidente River	lowest terrace	Wegmann and Pazzaglia, 2009	0.73	0.14	778
Musone River	modern channel	Wegmann and Pazzaglia, 2009	0.57	0.05	765
Musone River	lowest terrace	Wegmann and Pazzaglia, 2009	0.88	0.17	748

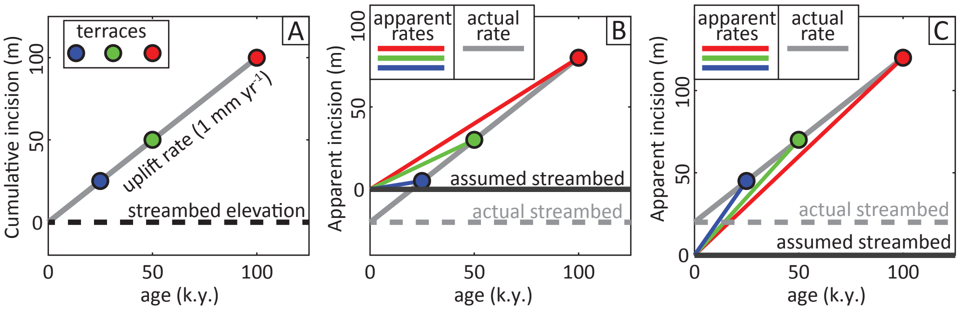

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Figure DR1

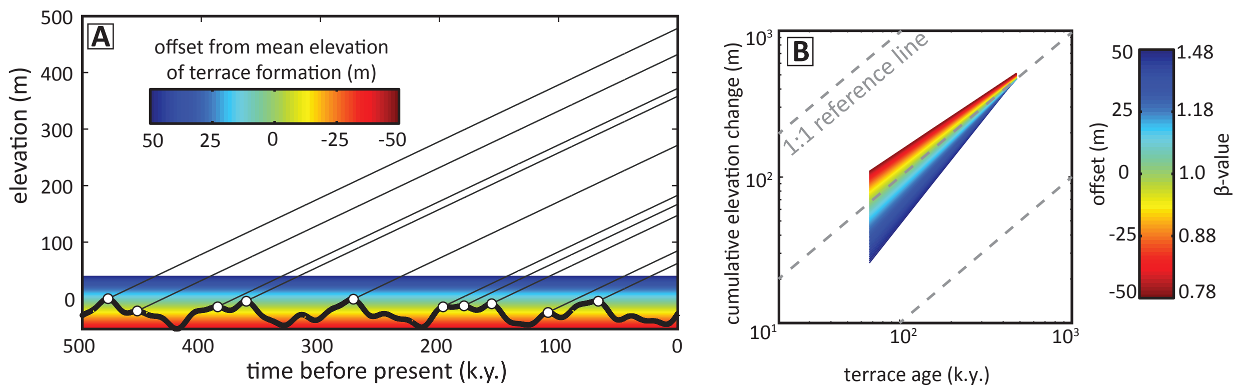

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Figure DR2