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# Modeling fluvial erosion on regional to continental scales

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**Abstract.** The fluvial system is a major concern in modeling landform evolution in response to tectonic deformation. Three stream bed types (bedrock, coarse-bed alluvial, and fine-bed alluvial) differ in factors controlling their occurrence and evolution and in appropriate modeling approaches. Spatial and temporal transitions among bed types occur in response to changes in sediment characteristics and tectonic deformation. Erosion in bedrock channels depends upon the ability to scour or pluck bed material; this detachment capacity is often a power function of drainage area and gradient. Exposure of bedrock in channel beds, due to rapid downcutting or resistant rock, slows the response of headwater catchments to downstream baselevel changes. Sediment routing through alluvial channels must account for supply from slope erosion, transport rates, abrasion, and sorting. In regional landform modeling, implicit rate laws must be developed for sediment production from erosion of sub-grid-scale slopes and small channels.

## Introduction

Since the days of Hutton and Playfair, we have recognized that landscapes are created by erosional/depositional processes acting upon tectonically created surfaces. Late 19th century studies recognized the interactions between tectonics and erosion that occur via isostasy. Until recently this interaction has largely been decoupled in geologic studies, with geomorphologists considering tectonic deformation as an imposed constraint and geodynamicists specifying as boundary conditions the erosional unloading in mountains and the sedimentary loading in basins. However, appreciation of the strong coupling of tectonic and geomorphic processes in the evolution of landscapes at regional to continental scales has been growing. For example, erosion of passive continental margin scarps induces lithospheric flexure that affects relief and drainage patterns over a wide belt [*Gilchrist and Summerfield*, 1990]. The style of deformation in orogenic belts may be influenced by the amount and spatial distribution of erosion [*Dahlen and Suppe*, 1988; *Koons*, 1990; *Isacks*, 1992]. In recognition of the strong interactions between tectonics and topography, several process-based models of regional erosion [*Willgoose et al.*, 1991a,b; *Koons*, 1989; *Chase*, 1992; *Slingerland et al.*, 1994] and basin sedimentation [*Paola*, 1989; *Flemings and Jordan*, 1989; *Jordan and Flemings*, 1990; *Paola et al.*, 1992a; *Slingerland et al.*, 1994] have been developed.

However, most modeling approaches have oversimplified erosion, transport, and depositional processes, particularly in the fluvial system. This paper reviews fluvial processes within the context of modeling long-term landscape evolution over regional scales. The initial discussion (general framework) focuses upon the major channel bed types occurring in natural channels, the factors controlling their occurrence, and their implications for long-term landform evolution. We expand on the work of *Howard* [1980, 1987] and identify several channel types, all of which could occur in a large river system. We argue that a single transport or erosion law cannot suffice. Several field examples illustrate the primary role that bed material type plays in channel evolution. A second section reviews quantitative models for channel evolution and suggests an approach to modeling regional scale landform evolution through explicit treatment of transport and erosion in larger streams coupled with implicit parameterization of slope and low-order channel erosion. Finally, we conclude with a synopsis of remaining uncertainties in modeling of long-term fluvial evolution and crucial research needs.

In nearly every terrestrial landscape, fluvial processes dominate removal of weathering products, their transport, and subsequent deposition at locations that may be separated from the source by thousands of kilometers. By connecting landscapes to their boundaries, rivers provide the primary linkage between tectonic deformation and landscape response. Adequate modeling of the fluvial system requires use of calibratable, mechanistic, transport/erosion laws. Unlike crustal processes that are at least perceived as being driven by large-scale and relatively continuously acting forces, the erosion of landscapes occurs episodically by spatially variable processes that nevertheless create a coherent, integrated network of avenues of transport and concentrated erosion (valleys). Although it is tempting to model the river system with a single rule of transport or incision (e.g., a diffusion equation), such oversimplifications have little bearing upon what is observed.

In modeling sediment deposition in the distal portions of

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drainage basins as fans, deltas, or floodplains (thus leaving a sedimentary record, redistributing mass, and affecting the overall tectonic force balance), treatment of the transport of sediment derived from hillslope erosion is essential. However, modeling of erosion into the underlying bedrock (particularly in headwater areas) is more problematic. In large-scale models it has almost always been treated as resulting from the unsatisfied sediment transport capacity in the conservation of mass equation, which is inappropriate because bedrock incision is not equivalent to erosion of loose sediment. Yet incision is essential to any large-scale landscape model, as it creates relief and is the critical link between tectonics and erosional processes via base level control.

If the planet consisted of one grain size of cohesionless sediment, the rules commonly used to create fluvial systems in landscape models would be acceptable. Inspection of rivers reveals a different picture. Those in steep land typically have significant portions of their beds in exposed bedrock. Even in actively uplifting land and rapidly downcutting rivers, the beds of rivers may be mantled with boulders or gravel. River profiles usually change abruptly where the gravel mantle gives way to sand. These grain size changes exert a primary control on river incision, transport rate, and profile evolution.

## General Framework

Quantitative analysis of long-term stream response to tectonic, lithologic, and climatic variations is complicated by the occurrence of several types of stream channels differing in morphology, dominant processes, and timescales of adjustment. *Howard* [1980, 1987] suggests three major channel types: bedrock, fine-bed alluvial, and coarse-bed threshold. *Knighton* [1987] makes a similar distinction. However, many coarse-bed channels carry appreciable sediment loads, so that identification of an additional type, live bed gravel, is warranted. For purposes of initial discussion, these four channel types can be merged into bedrock and alluvial.

### Conservation Equation for Alluvial Channels

In alluvial channels, conservation of bed sediment mass relates changes in the channel bed surface altitude $y$ to uplift $U$, the spatial divergence of bed sediment transport rate $q_s$, and the influx of sediment from adjacent slopes $q_h$:

$$\frac{\partial y_b}{\partial t} = \frac{\partial y}{\partial t} - U = \frac{1}{\rho_s (1-\eta)} \left\{ \frac{\partial q_s}{\partial x} + \frac{q_h}{W} \right\}, \qquad (1)$$

where $x$ is the downstream direction, $\rho_s$ is bed sediment density, $\eta$ is sediment porosity, $q_s$ is expressed in mass flux per unit channel width $W$, $q_h$ is mass influx per unit channel length of bedload-sized sediment from adjacent slopes on both sides of the channel, $U$ is the tectonic uplift rate, which may be a function of location and time, and $y_b$ is bed altitude referenced to a material coordinate system. In this relationship both $q_s$ and $q_h$ measure sediment only in the grain size range constituting the channel bed. Thus in alluvial channels the important issues are quantifying bed sediment transport rate and sediment contribution from local erosion.

### Erosion of Bedrock Channels

By contrast, bedrock channels, which lack an alluvial bed, occur when stream flow has excess transporting capacity compared to supply rate for all size ranges supplied from upstream and from local slope erosion. Therefore the sediment flux divergence in (1) is related to flow detachment capacity $\mathscr{E}$ rather than to transport capacity. The bed scour $\partial y_b/\partial t$ depends upon intrinsic bedrock erodibility $K_r$, specific discharge $q$, channel gradient $S$, bed sediment flux $q_s$, and possibly the grain size of sediment in transport $d$:

$$\frac{\partial y_b}{\partial t} = \mathscr{E}(K_r, q, q_s, S, d). \qquad (2)$$

The functional form of the detachment capacity is discussed later. Variable aspects of channel morphology, such as width-depth ratio, may also be important.

### Controls on Channel Bed Types

A crucial, but poorly understood, issue in long-term stream response is the factors determining which type of channel will occur in a given physiographic setting. This is addressed below for both alluvial and bedrock channels.

**Role of transport mechanics in determining bed type in alluvial channels.** Although stream channels transport a wide range of grain sizes, only a narrow proportion of this range predominates on the bed and determines the equilibrium channel gradient required to transport the imposed sediment load. Natural channels tend to be dominated either by relatively fine bedload (live bed conditions) or by a coarse, relatively immobile bed (threshold conditions). As we now discuss, this dichotomy is a consequence of the functional form of sediment transport relationships.

In alluvial channels, sediment transport rate in (1) is often expressed as a functional relationship $\mathscr{F}$ between two dimensionless parameters, $\Phi$ (transport number) and $1/\Psi$ (Shield's parameter) [*Einstein*, 1950]:

$$\Phi = \mathscr{F}\left(\frac{1}{\Psi}\right) \qquad (3a)$$

where

$$\Phi = \frac{q_s}{\omega\, d\, \rho_s} \qquad \frac{1}{\Psi} = \frac{\tau}{(\rho_s - \rho_f)\, g\, d}. \qquad (3b)$$

In (3a) and (3b), $\omega$ is the fall velocity of the sediment grains, $\rho_f$ and $\rho_s$ are the fluid and sediment grain densities, $\tau$ is the bed shear stress, and $g$ is the gravitational acceleration. Bedload or total load formulas are commonly expressed as a power function relationship [e.g., *Yalin*, 1977]:

$$\Phi = K_e \left\{ \frac{1}{\Psi} - \frac{1}{\Psi_c} \right\}^p, \qquad (4)$$

where $1/\Psi_c$ is the threshold for transport. In this and subsequent formulas constants of proportionality are indicated by a subscripted $K$ and exponents are indicated by lowercase letters; unless otherwise noted, such constants and exponents are assumed to be spatially and temporally invariant. Because sediment transport rate increases as channel gradient increases (for a given grain size and discharge), channel gradient tends to adjust to an equilibrium value just steep enough to transport sediment load supplied from upstream (in the long-term context considered here, this would be the rate of supply from slope erosion within the drainage basin). This concept of equilibrium, or grade, was explicated by *Mackin* [1948]. The equilibrium gradient can be illustrated via expressing (4) as a functional dependence of gradient on discharge, sediment load, and sediment caliber through the use of equations of steady, uniform flow:

$$\tau = \rho_f \, g R S \tag{5}$$

$$V = K_n \, R^{2/3} \, S^{1/2} \, / \, N_m \tag{6}$$

$$Q = K_p R W V, \tag{7}$$

where $R$ is the hydraulic radius, $V$ is mean velocity, $Q$ is discharge, $N_m$ is Manning's resistance coefficient, $K_n$ is unity in meter seconds and 1.5 in feet seconds, and $K_p$ is a form factor close to unity. Substituting (5)-(7) into (3) and solving for gradient, $S$, gives

$$S = \left\{ \left[ \frac{\Phi}{K_e} \right]^{1/p} + \frac{1}{\Psi_c} \right\}^{10/7} \left\{ (S_s - 1) \, d \left[ \frac{K_n \, K_p}{N_m \, q} \right]^{6/7} \right\}, \tag{8}$$

where $S_s$ is the sediment specific gravity. For a given reach and hence fixed "dominant" water discharge, $q = Q/W$, terms in the second braces are constant, and the first bracket contains two terms, one related to imposed load, $(\Phi/K_e)$, and the second to the threshold of motion $(1/\Psi_c)$. As shown below, the two main bed types correspond to dominance by one of the two terms.

In sand bed streams, $1/\Psi_c$ is generally small compared to the transport parameter $\Phi$ due to the easy mobility of sand, so that the gradient of such fine-bed streams is largely determined by the sediment load supplied by slope erosion, and live bed conditions prevail. In most gravel and boulder bed channels, high discharges barely exceed the critical shear stress, so that transport rates are very low, and the $1/\Psi_c$ term largely controls channel gradient (coarse-bed threshold channels). For coarse, uniform sized grains, $1/\Psi_c$ is essentially a constant, so that gradient is nearly linearly related to grain size.

However, the above analysis begs the question of which channel type will occur in a stream system into which a very wide range of grain sizes is supplied by slope erosion. Equations (4) and (8) are based upon experiments and observation in channels with a narrow size range of bed sediment; transport relationships for mixed sizes of sediment are more complicated due to interactions between sizes in transport, as is discussed further below. Nevertheless, transport relationships of the above form provide a first-order explanation of factors controlling bed type. In natural channels, the bed is composed of grains of a much narrower size range than that of the sediment supplied from slope erosion; a 2-4 $\phi$ range generally encompasses the majority of bed sediment sizes, whereas the range of supply may be 20 $\phi$ or more [e.g., *Howard and Dolan*, 1981, Figure 9]. The reason for lack of sizes significantly finer than the bed is clear: it is transported as wash load.

Few grains much coarser than the dominant size occur on the bed because of limited supply rate. This can be illustrated by assuming that the layer of active transport of the bed is one grain thick, which will be appropriate only for the coarsest bedload. Then the areal concentration $\xi$ (fraction of the bed surface covered by the grains) of sediment of size $d$ is given by the mass flux $q_s$ of grain size $d$ times the ratio of grain cross-sectional area divided by the grain volume, the average particle velocity $v_s$, and the particle density $\rho_s$. For spherical grains this gives

$$\xi = \frac{3 \, q_s}{2 \, \rho_s \, d \, v_s}. \tag{9}$$

The slow rate of motion of large grains tends to increase their concentration on the bed, but this is overbalanced in natural streams by the rapidly declining supply rates from slope erosion for larger sizes. For grain sizes larger than some limiting size $d_c$, the supply of grains will be insufficient to make an appreciable contribution to the sediment bed in comparison to more abundant finer bedload.

In light of the above arguments, *Howard* [1980, 1987] suggests that observed channel gradients and their corresponding grain sizes correspond to a narrow grain size range within the spectrum of supplied load which requires the steepest gradient in transport equations of the form of (8). Even in mountainous terrain, much of the sediment discharged by a river is relatively fine; hence the concentration term is high for fine sediment, although the critical shear stress term is low. If sediment supply has a size distribution which is lognormally distributed about fine sand then a plot of solutions of (8) as a function of grain size shows two regions, each dominated by one of the two terms discussed above (Figure 1). In the fine size range there is a peak "required" gradient at a grain size somewhat coarser than the median grain size (usually in the sand size range for natural streams) due to the high supply rate, whereas the critical shear stress term produces a sloping line increasing indefinitely as size increases. If the limiting coarse grain size $d_c$ is small, then the peak gradient in the fine size range will determine the channel gradient, and the coarser grain sizes, although present and participating in downstream transport, will be diluted in their representation in the bed due to the small value of the areal density $\xi$. On the other hand, if $d_c$ is large, then the gradient will be determined by

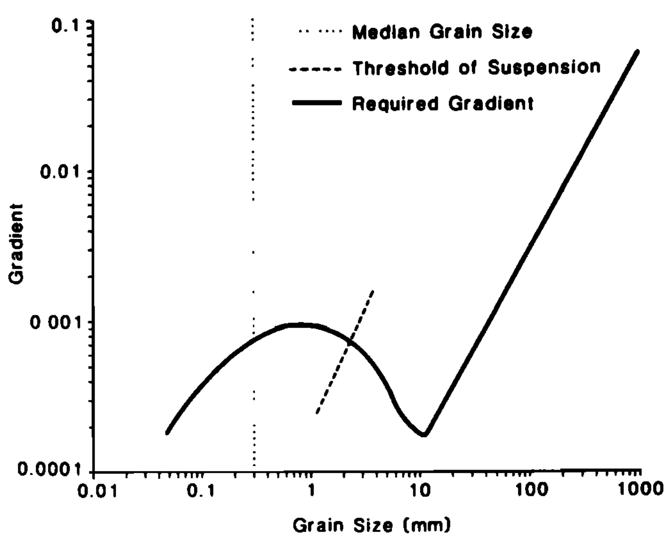


**Figure 1.** Theoretical curve of required gradient versus median grain size. Median grain size 0.3 mm and gradients are calculated using the Einstein-Brown transport formula [from *Howard* 1987, Figure 4.2].

coarse gravel or boulders near threshold, and sand-sized material will be carried as wash load and will be poorly represented on the bed.

**Distribution of alluvial channel types.** Fine-bed alluvial channels occur primarily in lowlands and are favored by low-relief, abundant supply of fine sediment, and absence of coarse detritus (either due to little production in headwater areas or due to abrasion or sorting out of coarse debris during transport). Deposits from fine-bed alluvial channels are those most commonly represented in fluvial depositional environments. The characteristic timescale of response of fine-bed alluvial channels to changes in sediment supply, discharge, base level, or tectonic deformation is the shortest of all the bed types [*Howard*, 1982].

Coarse-bed rivers are common in mountainous regions, where physical weathering processes produce coarse detritus. Such streams range from those carrying an abundant sediment load ("live bed" channels with dominance of the $\Phi/K_e$ term in (8)) to those with very low transport rates of bed sediment and gradients just steep enough to permit some transport at very high flow stages ("threshold" channels with dominance of $1/\Psi_c$ in (8)). Channels may alternate between live bed and threshold conditions if sediment supply rates are episodic.

Live bed gravel channels are similar to sand bed channels in that the gradient is affected by both sediment size and sediment supply rate. Live bed gravel may occur in mountainous, alpine, arid, and arctic areas where sediment yields relative to discharge are high and physical weathering predominates over chemical. Such channels generally convey a wide range of grain sizes on the bed, and most grain sizes are mobilized at about the same flow stage (the "equal mobility" concept [*Parker and Klingeman*, 1982]). The major difference from fine-bed channels is that both downstream sorting and abrasion play an important role, so that grain sizes diminish fairly rapidly downstream. In channels where the gravel only thinly mantles the bedrock and therefore provides little storage of sediment, the primary causes of downstream fining must be particle breakdown or downstream reduction in size of locally contributed sediment. However, even in such channels there may be appreciable temporary sorting effects if sediment supply from slopes is episodic [*Dietrich et al.*, 1993]. In depositional environments (alluvial piedmonts and fans), most modeling studies ascribe a leading role to sorting [*Parker*, 1991a, b; *Paola et al.*, 1992b; *van Niekerk et al.*, 1992].

Threshold gravel channels generally occur in areas where present or past physical weathering has supplied coarse gravel, but overall sediment yields are low, such as in the Appalachian Mountains. In such situations, reworking of the gravel by floods maintains gradients close to threshold conditions. But the supply rates and sizes of gravel and boulders often varies spatially in a complicated way, so that there is no unique pattern of downstream change in grain size or channel gradient [*Hack*, 1957; *Brush*, 1961]. Large changes in grain size and gradient can occur over short distances [*Ferguson and Ashworth*, 1991]. However, *Pizzuto* [1992] has shown that gravel stream gradients in large basins in the Appalachians can be closely estimated by a routing model combining a model of hydraulic geometry and transport with the assumption that gravel is produced in headwater basins and is abraded systematically during transport. In the context of regional response to long-term tectonics, the important issue is that such channels are relatively steep, thereby affecting overall relief, and the gradients are controlled by a minor but coarse component of the total sediment yield, the limiting grain size $d_c$ discussed previously. The slow rates of transport and comminution of the coarse detritus means that channel profile evolution is slow, despite the occasional influence of major floods. Much of the coarse debris in Appalachian Mountain streams may have been produced by periglacial physical weathering during glacial maxima [*Pizzuto*, 1992].

Channel gradient in threshold gravel streams may be a dependent or independent variable. For long-term equilibrium between supply of coarse debris from slopes and its comminution and selective transport within the fluvial system, gradient becomes a dependent variable. However, tectonic deformation or constructional landscape processes such as glaciation may impose or modify valley gradients, so that the resulting downstream distribution of grain sizes reflect sorting effects on the local alluvium. The rapid downstream fining observed by *Ferguson and Ashworth* [1991] is an example. Climatic changes can also alter the rates of production and removal of coarse debris in the fluvial system, so that gradients and grain sizes may be relict features.

**Occurrence of bedrock channels.** Channel incision into bedrock occurs when the supply of sediment to the channel cannot keep it continuously mantled with an alluvial cover, usually due either to steep gradients or to meager sediment supply. Thus bedrock channels are favored by one or more of the following factors: high relief, high uplift rates, local upwarping or faulting, resistant bedrock, low sediment yields, and possibly a dominance of debris flow transport. The headwater tributaries of many rivers draining mountainous areas are bedrock floored. Because of scouring and plucking that occurs during high flow stages, channels with a thin alluvial cover can slowly erode the underlying bedrock while maintaining an alluvial cover during low flow conditions [*Howard and Kerby*, 1983]; but the bedrock erosional capacity of alluvial channels is limited, so that if downstream erosion rates exceed this capacity, local gradients steepen and bedrock becomes exposed [*Merritts and Vincent*, 1989], often when channel gradients exceed about 3-10%. This may occur in particularly resistant rock, as a result of differential uplift along a river profile, or as a result of relative land-sea elevation changes (such as the Fall Line in the Appalachian Mid-Atlantic region [*Reed*, 1981; *Hack*, 1982]). The important characteristic of bedrock channels is their resistance to erosion; exposure of resistant bedrock isolates headwater areas from short-term effects of base level fluctuations. The timescale for upstream migration of base level control in bedrock channels is much longer than that of regrading of alluvial channels.

**Mixed alluvial-bedrock channels.** Channels in which bedrock exposures alternate with short alluvial sections are common [*Miller*, 1991; *Seidl and Dietrich*, 1992; *Wohl*, 1992a, 1993]. Such mixed channels arise from at least three scenarios. The first is where regional rates of stream downcutting are such that, on the average, the gradient required for bedrock incision is marginally steeper than for an equivalent fully alluvial channel. Under such conditions, an alluvial channel might require episodic exposure of bedrock in order to erode bedrock as well as lowering bed elevation through sediment flux divergence. Also, in such channels, bedrock exposures commonly occur due to small-scale areal variations in bedrock resistance. The second case occurs when changes in sediment load and discharge occasioned by climatic oscillations cause the channel to alternate between bedrock (during times of low sediment yield) and alluvial cover. Seasonal variations of this type between sand-bed alluvial and bedrock channels have been documented in badlands [*Howard and Kerby*, 1983]. In larger temperate-climate river systems bedrock exposure might be caused by alternating glaciopluvial and interglacial climates, with low sediment yields at present exposing bedrock. Catastrophic flooding with headwater debris avalanching might also discontinuously mantle a bedrock channel with immobile coarse debris. The third case occurs when sudden drop of baselevel causes dissection of a former alluvial channel system. Although knickpoint migration causes the greatest dissection, channel sections well upstream from the knickpoint experience some steepening and incision, as is discussed further below. This case may account for the sparse, incomplete alluvial cover along sections of bedrock channel between knickpoints along Elder Creek, northern California studied by *Seidl and Dietrich* [1992]. The extensive occurrence of mixed bedrock/alluvial channel streams of the Appalachian Mountains, United States [e.g., *Brush*, 1961], may be due either to dissection above the fall line knickpoint or to postglacial decrease in sediment loading.

## Effects of Bed Type on Long-Term Stream Evolution

The role of bed type in controlling fluvial evolution is illustrated below with topical discussions of channel gradients in the Grand Canyon, the role of bedrock knickpoints in evolution of stream profiles, and the behavior of waterfalls.

**The Grand Canyon, an example of gradient control by coarse debris.** Coarse-bed reaches at local sources of bouldery detritus can have appreciable influence on river gradients and thus on the connection between base level changes and uplift on headwater erosion rates. A premier example is the Colorado River in the Grand Canyon. The profile of the Colorado River is very irregular (Figure 2). Portions of the river, such as the now-submerged Glen Canyon section, are low-gradient, fine-bed alluvial. However, in the Grand Canyon the gradient is controlled primarily by rapids developed in coarse boulders contributed by debris flows debauching into the river from small tributaries draining the steep canyon walls [*Howard and Dolan*, 1981; *Kieffer*, 1985] (Figure 3). These rapids have gradients near the threshold of motion for the coarsest fraction of the debris flow sediment. Between the rapids are long sections with sandy bed and low gradient; these sand sections contribute little to the overall elevation drop through the canyon.

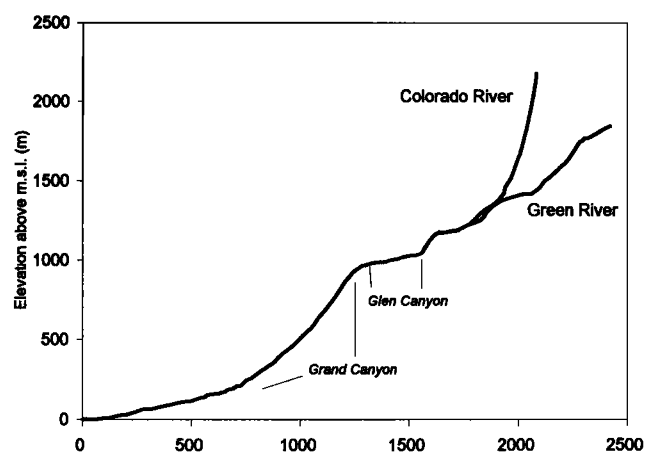


Figure 2. Longitudinal profile of the Colorado and Green Rivers, showing convexities and sections of variable gradient (based on work by the *U.S. Bureau of Reclamation* [1946, Figure 16]). The section labeled "Grand Canyon" includes the nominal Grand Canyon as well as Marble and Boulder Canyons.

Figure 3. A rapids of the Colorado River in the Grand Canyon. Coarse sediment delivered by debris flow from the tributary has created a fan, narrowed the river, and forced the Colorado River into a rapids. Repeated debris flows over a long time period has caused the Colorado River to preferentially erode the opposite bank. The tributary fan is partially mantled by thin sand terraces of the Colorado River (photo A. Howard).

The overall gradient through the Grand Canyon is determined by the balance between production and mobilization of coarse debris and its removal after abrasion and weathering within the rapids [*Howard and Dolan*, 1981; *Kieffer*, 1985; *Webb et al.*, 1989]. This balance is poorly characterized, but the delivery of coarse debris is certainly related to the physical characteristics of the exposed rocks as well as the relief and width of the canyon and thus indirectly to past rates of base level lowering. The balance of addition and removal have probably also been strongly influenced by climatic changes throughout the late Cenozoic.

Exposures of bedrock on the bed of the Colorado River in the Grand Canyon are very rare. Thickness of bed sediment near the Glen Canyon and Hoover dam sites averaged 20 m and locally exceeds 60 m [*Howard and Dolan*, 1981]. Scour holes below rapids commonly exceed 20 m in depth. These observations suggest that bedrock control is minimal and that the river has excess scouring capacity even in the metamorphic and igneous rock portions of the canyon. Were the relatively small quantities of coarse boulders not locally produced within the canyon, thinning of the alluvial cover would probably have accelerated bedrock erosion and lowering of the river profile, producing considerably deeper dissection of the Colorado Plateau and its bordering mountain ranges than has occurred. Thus the production of coarse debris in steeplands created by relative uplift has a negative feedback on erosion rates through its slow transport and comminution within the fluvial system and its role in maintaining steep channel gradients.

**Knickpoints.** We define a knickpoint to be a relatively steep gradient section of channel between lower-gradient sections, no matter whether it is produced by tectonic deformation, base level changes, or variable rock resistance. In alluvial streams, sediment transport acts rapidly to smooth perturbations in stream profiles by a combination of erosion of steep channel sections and redeposition downstream, including steep gradients introduced by faulting, base level change, or tectonic deformation [e.g., *Brush and Wolman*, 1960]. This mode of knickpoint decay has been called "inclination" or "rotation" by *Gardner* [1983].

When bedrock exposures occur along a river, lowering of base level elicits erosional response upstream only to the degree that the bedrock exposures can be eroded. If, as suggested below, erosion rate can be characterized as a power function with positive exponents on drainage area and gradient, then upstream propagation of base level changes is assured. In general, channel erosion into uniform bedrock following such a power function tends to smooth out irregularities and perturbations of the bed profile due, for example, to sudden changes in baselevel, so that knickpoints migrate upstream but gradually flatten, as observed experimentally by *Gardner* [1983] and termed "replacement". However, upstream migration of knickpoints with nearly constant drop and steepness can occur in two circumstances. The first is where gently dipping resistant bedrock is sandwiched between less resistant layers. *Miller* [1991] discusses natural occurrences of small, migrating knickpoints in horizontal sedimentary strata; *Holland and Pickup* [1976] performed flume experiments of knickpoint migration in layered sediments; and *Howard* [1971a, 1988] provides simulations of parallel knickpoint migration in layered rock. The second case occurs where steep, bedrock-floored channels alternate downstream with low gradient, generally alluvial channels. Bedrock-floored knickpoints occur in the profiles of many rivers. The fall-line steepening is a prominent feature of Piedmont streams in the Mid-Atlantic United States (Figure 4). Detailed terrace mapping [*Dunford-Jackson*, 1978; *Reed*, 1981] in Virginia shows that alluvial terraces generally extend downstream from the upper lip of knickpoints, with the terraces "running aground" onto the flatter channel sections between knickpoints (Figure 4). A similar situation occurs in Elder Creek, California [*Seidl and Dietrich*, 1992]. The simplest interpretation is that

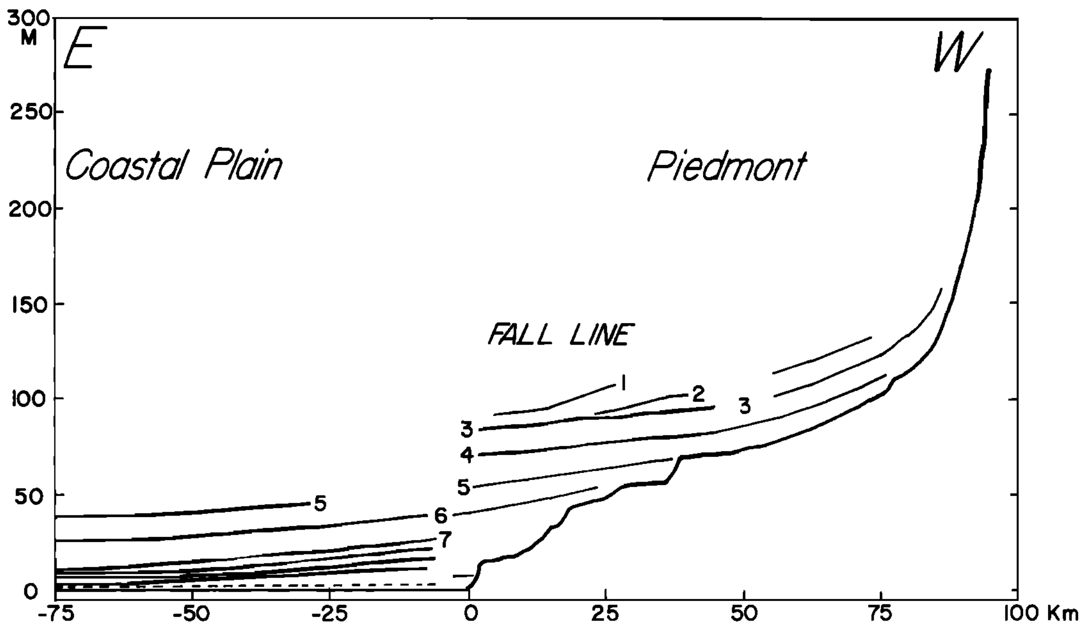


**Figure 4.** Profile and terraces of the Rappahannock River on the Virginia Coastal Plain and Piedmont. Profile extends upstream from below the Fall Zone near Fredericksburg, Virginia (Piedmont terraces from *Dunford-Jackson* [1978, Figure 4-0] and Coastal Plain terraces from *Pavich et. al.* [1989, Figure 11]). Based upon relative weathering and altitude, *Markewich et al.* [1987] suggest an age of about $5x10^5$ years for terrace 7 and $1x10^6$ years for terrace 5. Correlations for terraces 1-3 are uncertain near the fall line and may be faulted or tilted.

rapid downstream incision is transmitted upstream as discrete knickpoints, such that knickpoints farthest upstream reflect the earliest downcutting events. Headward migration and long-term maintenance of knickpoints in bedrock have also been demonstrated by *Young and McDougall* [1993], where, for example, a knickpoint about 250 m high has migrated about 15 km in 20 m.y. without diminishment of gradient and without obvious lithologic control.

Rapid, episodic drop of baselevel producing migrating knickpoints not only can occur as a result of relative land-ocean elevation changes but also occurs in tributary streams can also occur due to master stream incision due to climatic change, exposure of weak rocks, faulting, stream piracy, etc. [*Seidl and Dietrich*, 1992]. Knickpoints have also originated on portions of the Hawaiian island coastlines as a result of massive submarine landslides creating high scarps at the edge of the islands [*Seidl, et al.*, 1994].

Thus it appears that knickpoints in bedrock channels in homogeneous rocks can migrate long distances upstream in contrast to the experiments and conclusions of *Gardner* [1983] that knickpoints decline by replacement. In a later section we introduce a simulation model that illustrates such knickpoint migration.

**Waterfalls.** The classic model of development and migration of waterfalls illustrated in nearly every elementary geology textbook is exemplified by the Niagara Falls, where plunge pool erosion of underlying shale purportedly undermines the superjacent limestone [e.g., *Gilbert*, 1907]. As pointed out by *Horton* [1945], shear stress exerted on channel beds by flowing water reaches a maximum at inclinations of 45° and diminishes to zero as vertical free-fall conditions are approached. Retreat of waterfalls requires other processes, with plunge pool undermining being the most widely recognized. However, the general applicability of this model is questionable, even in the case of Niagara Falls [*Tinkler*, 1993]. Weathering, stress relief fracturing of the caprock, and its undermining due to gravitational creep of the subjacent shale may play a role in addition to hydraulic erosion. In arid landscapes with permeable caprocks overlying impermeable shales, groundwater sapping is locally important in canyon development [*Laity and Malin*, 1985; *Howard and Kochel*, 1988].

Backcutting via plunge pool undermining may have limited applicability for waterfalls in massive rock. Waterfalls with basal plunge pools abound on the basaltic Hawaiian Islands (Figure 5). However, backcutting and overlying rock collapse concentrated at plunge pools are not commonly observed. *Kochel and Piper* [1986] and *Baker et al.* [1990] attribute canyon backcutting at waterfalls to sapping by basal groundwater emerging at the foot of the waterfalls. However, well-developed alcoves, secondary porosity, or obviously weathered rocks are rare.

Hawaiian waterfalls are commonly stepped, with several plunge pools interrupting the cascading falls (Figure 5). It seems likely that downcutting at the plunge pools due to the momentum of falling water and debris is more prevalent than backwasting. In this interpretation, the waterfalls expand downward through time with relatively little backwasting; in fact, many waterfalls are but little incised into the surrounding slopes (Figure 5), suggesting that rates of headward retreat are not much greater than erosion rates on adjacent slopes. The vertical limit to such downcutting would be the elevation at which transport of debris away from the plunge pool is inhibited through development of an alluvial stream graded to a downstream base level. Occurrences of steep bedrock sections between coarse-bed alluvial sections are common in the Hawaiian Islands [*Seidl, et al.*, 1994]. Also under such conditions, waterfalls, rather than representing locations of rapid backwasting, may instead be sites of inhibited backcutting, above which streams are relatively isolated from base level control. Backwasting rates at such waterfalls may be as much related to rates of rock weathering, including stress relief fracturing, as to hydraulic processes. However, when channel segments above and below a waterfall are boulder covered, erosion rates on these segments are much diminished, and headward retreat of the waterfall, even though slow, may dominate channel incision [*Seidl et al.*, 1994].

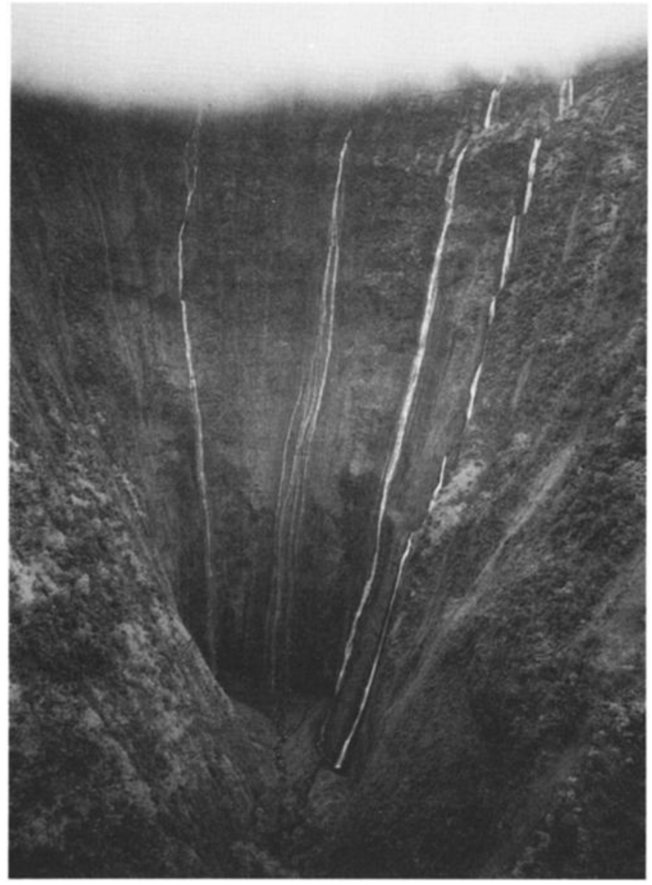

**Figure 5.** Waterfalls and plunge pools at a canyon head in the Kohala region of the island of Hawaii. Note the minimal evidence for bedrock undermining at the plunge pools (photo A. Howard).

## Quantitative Modeling of Channel Evolution

The discussion now turns to process description of bedrock erosion and alluvial sediment transport within the context of large-scale modeling.

### Erosion in Bedrock Channels

In streams with bedrock beds the critical concern is the rate of bed scour. Erosion may occur by several mechanisms, including plucking, abrasion by sediment, solution, and weathering. The relative importance of these processes depends upon rock type, channel hydraulics, water chemistry, sediment type and load, and climate. Thus there is no universal law of bed erosion, and due to the general slowness of bed erosion in resistant rocks, few process observations have been made.

As a first-order approximation, the functional relationship in (2) might be approximated by a power law:

$$\frac{\partial y_b}{\partial t} = -K_r\, q^a\left[1 + K_1 q^s d^c\right] S^k, \tag{10}$$

where the bracketed term reflects the possibility that some bed erosion may occur even in clear water discharge due to plucking and weathering. The constants $K_r$ and $K_1$, and possibly the exponents, would vary among rock types. In areas of uniform lithology, climate, and relief $q$, $q_s$, and $d$ should vary systematically with drainage area $A$, so that further simplification may be appropriate:

$$\frac{\partial y_b}{\partial t} = -K_2 A^m S^n, \tag{11}$$

where the exponents can be expected to be positive.

*Howard and Kerby* [1983] and *Howard* [1994] suggest that bedrock erosion in some rock types may be proportional to a dominant bed shear stress $\tau$:

$$\frac{\partial y_b}{\partial t} = -K_t\, \tau\,. \tag{12}$$

where $K_t$ is bedrock erodibility. They also assume that most erosion occurs as a result of high (flood) discharges and that erosion by any given flood is small compared to overall basin relief. They therefore assume that a characteristic, or dominant discharge can be defined that represents the average effect of the natural sequence of flows. Furthermore, the erodibility $K_t$ in (12) is assumed to be adjusted for the flow duration of the dominant discharge. The power law equations of hydraulic geometry introduced above ((5)-(7)) are assumed to be valid, together with expressions relating dominant discharge $Q$ and channel width $W$ to drainage area:

$$Q = K_a A^e, \tag{13}$$

$$W = K_w Q^b = K_w K_a^{\,b} A^{be}. \tag{14}$$

Combining (12) with (5), (6), (7), (13) and (14) gives

$$\frac{\partial y_b}{\partial t} = -K_t K_z A^{0.6e(1-b)} S^{0.7}, \tag{15}$$

where

$$K_z = \rho_f\, g \left\{\frac{N_m K_a^{\,1-b}}{K_p K_w K_n}\right\}^{3/5}. \tag{16}$$

Thus for this model, $n=0.7$ and $m\approx 0.25$ in (11) for typical exponent values $b\approx 0.5$ and $e\approx 0.8$ [e.g., *Knighton* 1987]. *Howard and Kerby* [1983] measured erosion rates in bedrock channels in shale badlands and found that spatial variations of erosion rates are well approximated by (15) with estimated values of $n=0.7$ and $m=0.45$, values which are reasonably close to the model prediction.

*Seidl and Dietrich* [1992] look at gradient relationships at stream junctions in bedrock channels to estimate a value for the ratio $m/n$. They assume that erosion rates of main streams and their tributaries should be nearly equal if the junctions are accordant and channel profiles are smooth. If the subscript 1 refers to the main stream and 2 to the tributary, then equality of erosion rates implies the following relationship:

$$\frac{m}{n} = \log(S_2/S_1)/\log(A_2/A_1)\,. \tag{17}$$

They find an estimated value for the ratio close to unity, which is consonant with a stream power rate law for bed erosion if discharge is directly proportional to drainage area:

$$\frac{\partial y_b}{\partial t} = -K_p\, \rho_f\, A\, S. \tag{18}$$

However, the *Seidl and Dietrich* [1992] analysis cannot distinguish the absolute value of $m$ and $n$, only their ratio, so that values of, say, $m=0.5$ and $n=0.5$ are also consonant with their data. This deficiency is addressed in their study of fluvial dissection of Hawaiian volcanos, where areal variations in amount of dissection correlate reasonably with the area-slope product [*Seidl, et al.*, 1994]. *Young and McDougall* [1993] and *Wohl* [1992b, 1993] also suggest that bedrock stream erosion in southeastern Australia may occur in proportion to stream power.

*Foley* [1980] has developed a model for circumstances where erosion by bedload is dominant. He uses theoretical treatments by *Bitter* [1963a,b], *Neilson and Gilchrist* [1968] and *Finney* [1960] to parameterize erosion rate by particles due to cutting and deformation (surface distortion) abrasion, providing procedures for estimating parameters from measurements of Moh's hardness. His analysis gives abrasion rates as complicated functions of particle mass $M$, relative velocity $v_r$, angle of impact $\theta$, and impactor and surface properties. A simpler empirical formulation reported by *Head and Harr* [1970] and *Engel* [1976, pp. 149-157] relates single-particle wear erosion $E$ for brittle materials to $v_r$ and $\theta$:

$$E \propto v_r^{\,d}\, \theta^f, \tag{19}$$

where the exponents are empirically evaluated. *Foley* [1980] shows that erosion by bedload should be proportional to the sediment load $q_s$ times the single-particle erosion rate divided by the characteristic saltation pathlength $\lambda$. Combining this with (19) yields the following:

$$\frac{\partial y_b}{\partial t} \propto -\frac{q_s v_r^{\,d}\, \theta^f}{\lambda}. \tag{20}$$

In bedrock channels $q_s$ is less than the equilibrium rate given by sediment transport relationships for equivalent discharge, gradient, and bedload grain size. Regression analysis of data from simulations of bedload saltation reported by *Wiberg and Smith* [1985] and *Wiberg* [1987] suggest the following dependencies for $v_r$, $\lambda$, and $\theta$ at high transport stages ($v_*/v_{*c} > 1.7$), where $v_*$ is shear velocity and $v_{*c}$ is the critical shear velocity for movement of grains of size $d$:

$$v_r \propto v_*^{\,1.6}\, d^{-0.3}, \tag{21}$$

$$\theta \propto v_*^{\,-0.5}\, d^{\,0.3}, \tag{22}$$

$$\lambda \propto v_*^{\,2}. \tag{23}$$

Although these simulations are for bedload grain motion above alluvial beds, bedload grain trajectories above a bedrock bed should show similar dependencies upon $v_*$ and $d$. Use of (21)-(23) together with equations of hydraulic geometry (5)-(7) permit (20) to be recast into the form of (10):

$$\frac{\partial y_b}{\partial t} \propto -q_s\, q^{0.5d-0.15f-0.6}\, S^{0.55d-0.2f-0.7} \cdot d^{0.3f-0.3d}. \tag{24}$$

Empirical correlations [*Head and Harr*, 1970] for brittle materials indicate values of about 3.0 for the exponent $d$ and 2.7 for $f$, suggesting a negligible dependency of erosion rate on grain size.

Although the relationship follows the form of (10), appropriate values for the exponents are not as well constrained as the analysis would suggest. Studies of wear usually concern impact velocities in the range of 20 to 300 m/s [*Engel*, 1976], well outside the probable upper limit of about 5 m/s in streams. The above relationship may approximately characterize cutting and surface deformation due to bedload impact on a flat bed, but other processes may also be involved, such as weathering, particle fragmentation, and scour by suspended sediment or bed material in vortices, resulting in potholes and longitudinal grooves [*Wohl*, 1992b, 1993].

Thus three different models of bedrock channel erosion (*Howard and Kerby's* [1983] shear stress model; *Seidl and Dietrich's* [1992] stream power model; and the above sediment scour model) result in equations (i.e., (15), (18), and (24)) that can be expressed in the form of (10) or (11), but with different exponents and constants of proportionality. At present, our knowledge of spatial variations in bedrock channel erosion rates and of inherent erodibility of bedrock is insufficient to permit adequate evaluation of the relative merits of these models.

Bedrock streams have no simple downstream hydraulic geometry because gradient is a semi-independent variable. However, if bedrock is homogeneous and the rate of base level lowering has been constant for a long time, then erosion rates should be uniform throughout the basin and (11) implies

$$S \propto A^{m/n}. \tag{25}$$

In some areas, such as the U.S. Pacific coastal ranges and steep hollows of the Appalachians, erosion by debris avalanches may dominate through removing accumulated sediment and weathered bedrock from channels [*Dietrich and Dunne*, 1978; *Dietrich et al.*, 1982; *Pierson*, 1980; *Benda*, 1990; *Wohl and Pearthree*, 1991]. Due to the high sediment concentrations, debris flows have fundamentally different mechanics of transport and erosion than normal river flows, and they will require a separate analysis whose long-term spatial patterns of erosion may not be described very well by (11). *Seidl and Dietrich* [1992] suggest that debris flows in mountainous regions are the primary agent of bed scour in headwater channels with gradients greater than 0.2.

In some cases detachment in bedrock channels may be limited by the rate of bed weathering. *Howard* [1994] discusses how weathering and shear detachment may interact in bedrock channels in shale badlands, and *Howard* [1990] models the interaction of weathering and detachment by debris avalanches in forming bedrock chutes on mountainous slopes.

The previous discussion of knickpoint retreat indicates that knickpoints formed by rapid incision in bedrock channels can migrate upstream for long distances under suitable circumstances. This scenario is tested in a simulation model of stream profile evolution. Drainage area and discharge are assumed to increase as the square of distance downstream. Streams are assumed initially to be bedrock with a concave profile in equilibrium with a slow, constant rate of base level lowering (negative relative times in Figure 6a), so that all parts of the profile are eroding at an equal rate (time zero in Figures 6a-6e). After time zero, base level is assumed to be characterized by long periods of stability interrupted by brief episodes of uplift (Figure 6a). Figures 6b-6e show stream profiles developed in response to the base level lowering scenario, with profiles keyed to the times shown in Figure 6a. Bedrock erosion is assumed to be governed by (11); Figures 6b and 6d assume unity $m$ and $n$ for erosion rates $E$ proportional to stream power [*Seidl and Dietrich*, 1992], and the 0.3 and 0.7 values correspond to erosion in proportion to shear stress [*Howard and Kerby*, 1983]. The heavy lines are alluvial channel sections (discussed below); light lines are bedrock channels.

A power law relationship for bedrock erosion implies that gradients will gradually diminish if base level is fixed and that the lowest gradients will occur at the downstream end. However, as gradients decline, a minimum gradient will be reached where the gradient will only be just sufficient to transport sediment supplied from upstream, and that section of the stream will be converted to fine- or coarse-bed alluvial, depending upon sediment supply characteristics. This conversion will occur first at the downstream end and migrate upstream, gradually replacing the bedrock channel (time 1 in Figure 6).

The minimum alluvial channel gradient $S_m$ can be evaluated from (8). Hydraulic geometry relationships (13)-(14) can be used to express $S_m$ as a function of drainage area $A$:

$$S_m = K_m A^u. \tag{26}$$

In the simulations $u$ equals -0.25, consistent with a sand bed channel at high sediment transport rates with areally uniform and temporally constant sediment yield [*Howard*, 1980]. If the alluvial channels are coarse bed with gradients close to the threshold of motion the values of $K_m$ and $u$ will differ, with $u \approx -0.4$ for uniform grain size [*Howard*, 1980; *Knighton*, 1987], and $K_m$ will vary nearly linearly with bed grain size (8). In channels with sediment transport dominated by debris flows, there may be little area dependence upon transport capacity, so that $u \approx 0$ [*Seidl and Dietrich*, 1992]. So long as base level remains constant, the alluvial section of the channel is assumed not to lower but does gradually extend headward in response to continued erosion of the bedrock channel (e.g., times 1, 4, and 5 in Figure 6). In a more realistic simulation, continued erosion of headwater areas would cause some decline in supplied load and grain size, requiring some regrading of the alluvial channel section.

Immediately after a rapid base level drop, the farthest downstream section of the channel is steepened to values well above $S_m$ (e.g., time 2), so that this section is reconverted into a bedrock-floored knickpoint, which gradually

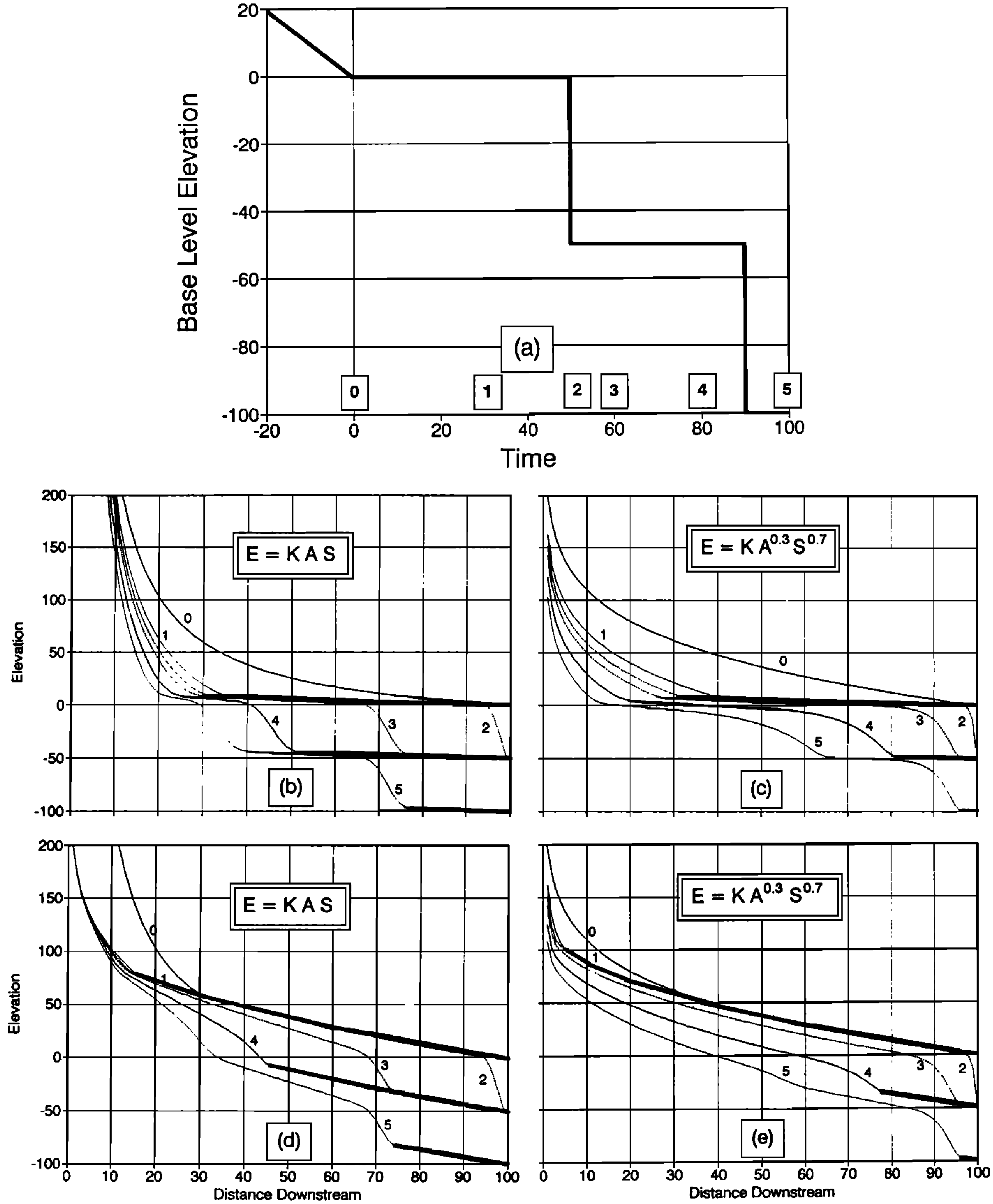


**Figure 6.** Simulations of knickpoint evolution. Time and distance scales are in arbitrary units. Drainage area is assumed to be proportional to the square of distance downstream, with unit drainage area at unit distance. (a) Temporal change in downstream base level (material coordinates are used in this simulation); (b) successive stream profiles produced for the erosional history in Figure 6a with bedrock erosion proportional to stream power ($m$=1, $n$=1, and $K_2$=0.0004 in (11); $K_m$=1 in (26)). Heavy lines are alluvial channel segments that form during periods of base level stability; (c) conditions similar to Figure 6b except bedrock erosion proportional to shear stress and $K_2$=0.096; (d) conditions similar to Figure 6b except for steeper assumed alluvial channel gradients ($K_m$=6.5); note that at time zero there is a downstream transition from bedrock to alluvial channel; (e) conditions similar to Figure 6c except for steeper assumed alluvial channel gradient.

erodes headward, replacing the upstream alluvial channel section (times 3 and 4).

Former alluvial channel sections which lie upstream from a bedrock knickpoint also gradually steepen in advance of the arrival of the knickpoint. This gradual steepening is an effect of the base level lowering which is smaller in magnitude but more rapidly transmitted upstream than the knickpoint. This steepening can occur in the model because during each time increment a potential bedrock erosion rate is calculated, even in alluvial channel sections. That erosion is permitted to occur unless the gradient would be reduced below the critical gradient $S_m$ as a result of the erosion. Because the gradient is steepened, the alluvial channel sections upstream from a knickpoint are rapidly reconverted to bedrock, although the gradients are only slightly steeper than an equivalent alluvial channel. In natural streams, one might find patchy alluvial cover or short alternating sections of alluvial and bedrock channels in such circumstances. *Gardner* [1983] noted a similar steepening upstream from the knickpoints in his experiments but attributed it to drawdown effects.

The combination of upstream migration of knickpoints with slight steepening and lowering of the former upstream alluvial sections would typically result in creation of alluvial terraces from remnants of the former alluvial floodplain. Such terraces should occur high above the river below the knickpoint but be only slightly dissected above the knickpoint. Such terraces occur in the Rappahannock River in Virginia (Figure 4) and have been interpreted to have arisen from knickpoint migration of the type discussed here [*Dunford-Jackson*, 1978].

The shape, height, and gradient of the knickpoint, and its temporal perseverance during upstream migration, depend upon the value of the exponents $m$ and $n$, the magnitude of base level drop, and the steepness of the alluvial channel sections. For high values of $m$ and $n$ (erosion proportional to stream power), knickpoints remain steep as they migrate upstream and former alluvial sections are little eroded until they are engulfed by the migrating knickpoint because of the strong dependency of erosion rates upon gradient (Figure 6b). On the other hand, for the case of fractional powers of $m$ and $n$, knickpoints are convexly rounded, they maintain a generally constant gradient but decreasing height during upstream migration, and the former alluvial sections undergo appreciable erosion during subsequent dissection although their gradient is only modestly increased until the knickpoint migrates through (Figure 6c). If the gradient of the alluvial channels that form during times of base level stability are steeper than the previous simulations, then erosion of the former alluvial sections after base level lowering is more pronounced and knickpoints lower and gradually loose individuality (Figures 6d and 6e). The relevant criterion is the ratio of the alluvial channel gradient to that of the knickpoint. The knickpoint gradient is a function of the exponents $m$ and $n$, the magnitude of the base level drop, and the distance of knickpoint migration.

Although the simulations assume a downstream increase in discharge, development and migration of knickpoints also occur when discharge is constant or decreasing downstream, although the pattern of upstream migration is different.

### Sediment Transport in Alluvial Channels

Modeling of bed elevation changes in alluvial channels using the conservation equation (1) requires estimation of the downstream trend in transport rate, which in turn depends upon sediment supply rate and grain size distribution, channel width, and discharge. Treatment of downstream changes in transport rate also must also account for abrasion and sorting. Complicating the issue for long term drainage basin evolution is the necessity for developing transport relationships that integrate the effects of the natural temporal spectrum of discharges and sediment supply.

Equilibrium transport relationships for narrow grain size ranges of supplied sediment under constant discharge are reasonably well developed, although there are a plethora of empirical transport laws (see summaries by *Vanoni* [1975], *Chang*, [1988], and *Gomez and Church* [1989]). Most of these can be formulated into relationships similar to (4), although some are valid for only a limited range of grain sizes (e.g., gravel or sand) or for high or low transport rates. Some are for bedload transport only, whereas some include suspended load transport of bed material at high flow stages in sand bed streams (total load formulas). In channels with a narrow grain size range of bed material, hysteretic effects in transport rates as a function of bed shear are minor. However, stage-related changes in form drag due to bedform development can introduce temporal variations and time lags in the proportion of total perimeter shear that is available for transporting sediment. Nonetheless, for long-term, large-scale evaluation of sediment transport rates, the use of an appropriate transport relationship together with an empirical resistance relationship generally permits evaluation of transport rates with fair accuracy if a representative, or dominant discharge is used which is an average of the spectrum of natural discharges weighted by their transport capacity, but only so long as the grain size range of supplied sediment is small.

Transient modeling of stream profile evolution in response to changes in hydraulic regime or base level is possible [*Howard*, 1982; *Snow and Slingerland*, 1987, 1990; *Willgoose et al.*, 1991a,b; *Bonneau and Snow*, 1992]. Transient responses of sand bed channels to differential uplift have been documented [*Burnett and Schumm*, 1983; *Ouchi*, 1985]. Sand or sand-silt bed channels are most suitable for modeling using the approach discussed above. In such channels the range of grain sizes on the bed is usually small, and gravel, although present, is transported in generally negligible quantities. In addition, downstream changes in grain size due to comminution or sorting are appreciable only over distances greater than several hundred kilometers, although more rapid downstream fining sometimes occurs [*Pickup*, 1984]. Furthermore, because of the low gradients of sand bed channels their contribution to overall relief may be small, so that in large-scale modeling, errors in estimating gradients may be inconsequential. However, two complications arise in large-scale modeling. Because of the wide range of grain sizes fed into headwater areas, a model must be capable of predicting when and where sand bed channels will occur. Furthermore, changes in hydraulic regime or tectonic warping can cause transitions to other channel types. Diminishment of sediment load (e.g., by upstream reservoirs) can lead to development of coarse-bed armoring. Similar transitions to coarse-bed conditions might accompany tectonic steepening of river profiles. A transition to bedrock channels may accompany rapid downcutting and removal of the alluvial bed (Figure 6).

In channels carrying a wide range of sediment sizes the simple transport relationships of (3) and (4) are not satisfactory. In recent years a number of transport relationships for mixed grain sizes have been proposed [e.g., *Shih and Komar*, 1990; *Parker*, 1990; *Bridge and Bennett*, 1992; *van Niekirk et al.*, 1992]. Some of these models are computationally very demanding and may not be suitable for the type of large-scale, long-term modeling discussed here. None of these models predicts the transitions between sand bed and gravel bed channels discussed above; such abrupt transitions may be due to change in transport efficiency associated with the difference between gravel-on-gravel and gravel-on-sand conditions (Y. Kodama, personal communication, 1993).

Effects of downstream abrasion upon grain sizes have generally been treated empirically, with mean grain size decreasing as either an exponential or power function of travel distance (see reviews by *Kodama* [1992], *Pizzuto* [1992], and *Mikos* [1993]). *Parker* [1991a,b] has developed a more mechanistic approach. Models of sediment transport, sorting, and abrasion in such streams are reasonably well advanced [e.g., *Parker*, 1990, 1991a,b]). In situ abrasion within streams [*Schumm and Stevens*, 1973] and during temporary storage in floodplains [*Bradley*, 1970] may help to account for very rapid downstream grain size diminishment in coarse-bed streams [e.g., *Pizzuto*, 1992; *Kodama*, 1992].

Downstream fining as a result of sorting should only occur in areas in which sediment is actively depositing. *Paola et al.* [1992a] use the simplest approach, modeling sorting as a successive depletion of grains in transport from coarsest to finest. The models of *Parker* [1991a,b] and *van Niekerk et al.* [1992] offer more detailed modeling.

## Discussion: A Suggested Modeling Approach

For coupling of erosional processes with tectonic and climatic forcing on large spatial scales and over long time spans, a critical concern is prediction of spatial and temporal rates of erosion and deposition. If erosion of the landscape were everywhere in balance with a long-term constant rate of uplift, then correlation studies of erosion rates as a function of relief [e.g., *Ahnert*, 1970, 1984; *Ruxton and McDougall*, 1967; *Pinet and Souriau*, 1987; *Milliman and Syvitski*, 1992], climate [*Langbein and Schumm*, 1958; *Wilson*, 1973; *Ohmori*, 1983; *Schmidt*, 1985; *Pinet and Souriau*, 1987; *Milliman and Syvitski*, 1992] and uplift rates [*Schumm*, 1963; *Adams*, 1985; *Yoshikawa*, 1985] would suffice to couple tectonics and erosion. *Hack* [1960, 1975] makes an argument for such an equilibrium in the Appalachians, *Suppe* [1981] and *Dahlen and Suppe* [1988] for Taiwan, and *Adams* [1985] for the New Zealand Alps. However, serious discrepancies between long-term uplift or deformation rates and denudation exist in many tectonic regions (e.g., the Tibetan Plateau and the Altiplano of South America [*Dahlen and Suppe*, 1988; *Isacks*, 1992] and the fall line knickpoints of the Appalachians). On shorter timescales and smaller spatial scales, river profiles are commonly directly affected by tectonic deformation [e.g., *Burnett and Schumm*, 1983; *Ouchi*, 1985; *Gregory and Schumm*, 1987; *Merritts and Vincent*, 1989].

Thus mechanistic, predictive models of relief development and associated weathering, erosion and deposition with parameterization appropriate for regional or continental spatial scales and $>10^5$ year temporal scales must be developed to adequately couple tectonics and geomorphology. In a large-scale erosional model, the focus should be on the evolution of the river longitudinal profile and erosion, transport, and deposition of sediment. Numerical, coupled slope-channel models of landform evolution have been developed in the last several years [e.g., Ahnert, 1976; *Koons*, 1989; *Willgoose et al.*, 1991a,b; *Chase*, 1992; *Lifton and Chase*, 1992; *Howard*, 1994]. However, it is impractical (and unnecessary) to extend this type of model to regional or continental scales. Adequate representation of slope morphology would require orders of magnitude more memory and computational resources than are presently available. On the other hand, the strong feedback between channel profile evolution and slope processes requires that such interactions not be ignored. The approach suggested here is to explicitly model profile evolution of high-order channels while implicitly treating the interaction between channel evolution and sub-grid-scale response (flood hydrology and the delivery rates and grain size distribution of sediment from slopes and low-order channels). This takes an opposite approach to that used by *Koons* [1989], which modeled slope erosion using a diffusion equation but simply specified the profile of high-order channels.

A large-scale fluvial model might be based on a matrix of cells, with each cell representing a high-order channel plus surrounding sub-grid-scale contributing area, such as the network models of *Howard* [1971b, 1991]. Typical cell dimensions might range from 1 to 10 km square. The important issues would be characterization of sub-grid-scale erosion and sediment contribution, routing of fluvial sediment downstream, rate of channel bed erosion or deposition, tectonic deformation, temporal and spatial changes between channel types, flow directions, and initial and boundary conditions. Some of these issues have been discussed above, but the remaining issues are beyond the scope of this paper. However, the treatment of sub-grid-scale processes is particularly crucial, so that we suggest one possible approach.

Sediment yields from local slopes and channels might be modeled as a convolution function of present and past rates of incision of the high-order channel flowing through each grid cell:

$$q_h(t) = \frac{\rho_s\,\delta}{\lambda+1}\sum_{i=0}^{\infty}\frac{\partial y_b}{\partial t}(t-i)\left[\frac{\lambda}{\lambda+1}\right]^i, \tag{27}$$

where $q_h$ is the sediment influx (per unit length) to the channel from slope erosion at time $t$, $\delta$ is the dimension of a unit cell in the simulation, $\lambda$ is a characteristic relaxation timescale ($\lambda \geq 0$) measured in iterations, $\partial y_b/\partial t$ is the local channel erosion rate at time $t$-$i$ in the past, and the summation goes over all iteration time steps $i$. If the channel erosion rate is temporally constant or if $\lambda = 0$, then

$$q_h = \rho_s\,\delta\,\frac{\partial y_b}{\partial t}. \tag{28}$$

The weighting by $\delta$ comes from considering the cell area to be $\delta^2$, the length of the channel through the cell to be $\delta$, and the width of the channel to be small compared to the cell dimensions. Equation (27) can be expressed as an equivalent difference formula which is easier to compute:

$$q_h(t) = \frac{\rho_s\,\delta}{\lambda+1}\frac{\partial y_b}{\partial t}(t) + q_h(t-1)\left[\frac{\lambda}{\lambda+1}\right]. \tag{29}$$

The use of large scale simulation models that explicitly treat slope erosion [e.g., *Ahnert*, 1976; *Willgoose*, 1991a,b; *Howard*, 1994) can help to scale λ as a function of parameters governing mass wasting and fluvial erosion. Figure 7 shows an example of temporal variation in $q_h$ as a function of changes in $\partial y_b/\partial t$ using (29).

The grain size distribution of sediment supplied from local slopes is also important in determining bedload transport rates. A reasonable assumption would be a lognormal distribution with logarithmic mean and standard deviations $\mu$ and $\sigma$, respectively. Both $\mu$ and $\sigma$ would be functions of bedrock characteristics, climate, and present and past channel erosion rates (expressed through a convolution function as above), under the assumption that the steeper relief associated with more rapid erosion would produce coarser and possibly more variable debris.

The primary issue in application to natural landscapes is providing reasonable estimates of λ, $\mu$, and $\sigma$. The characteristic relaxation time λ scales the time required for changes in stream erosion rates to be transmitted upslope and upstream within sub-grid-scale tributaries as perturbations of channel gradient, slope steepness, and drainage density. In the limiting case of $\lambda=0$, slope and low-order channel response to change in erosion rate is instantaneous, and the entire cell erodes at the same rate. Two broad categories of slopes have been identified: regolith-mantled (transport-limited) slopes whose rate of erosion depends upon the capability of erosional processes to remove the regolith, and bedrock (weathering-limited) slopes whose erosion rate depends upon the rate of weathering [*Culling*, 1960; *Carson and Kirkby*, 1972]. In low-relief landscapes with thick regolith, low drainage density, and a dominance by creepike processes, λ may be fairly long. In some mountainous areas, efficient frost weathering produces abundant coarse debris and slopes are close to threshold of stability [*Carson*, 1971; *Carson and Petley*, 1970]. In such cases, slope steepness varies little with erosion rate, and channel downcutting provokes an immediate response in mass wasting, so that λ would be close to zero. High-relief areas with

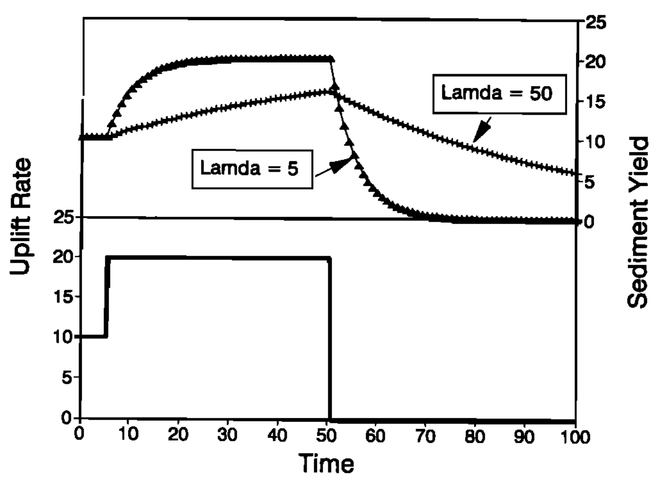


**Figure 7.** Sediment yield for two different values of λ as a function of time and uplift rate using (29). Time and uplift rate scales are arbitrary with $\rho_s=1$ and $\delta=1$. Initial sediment yield is assumed to be in equilibrium with an uplift rate of 10.

bedrock slopes would respond very slowly (very long λ) to erosion of master streams. *Anderson and Humphrey* [1990] suggest weathering and stream downcutting and transport are essentially decoupled in such circumstances (indefinite λ). However, most physical weathering processes, such as frost action and development of sheeting fracturing, work from the surface inwards and diminish in intensity with depth. On steeper slopes mass wasting requires less weathering to remove the partially weathered bedrock than on gentler slopes, so that there is a positive feedback between slope steepness and erosion rates. Furthermore, some physical weathering processes, such as fracturing due to gravitational stresses, are directly related to slope steepness. Thus there is an indirect positive coupling between relief generation by stream incision and erosion on bedrock slopes. The contrast in timescales between regolith-mantled slopes (low to moderate λ) and bedrock slopes (long λ) is somewhat analogous to the difference in response times between alluvial and bedrock channels.

Evaluation of $\sigma$ and $\mu$ (or other suitable descriptors of grain sizes supplied from slope erosion) require sampling of sediment transport in headwater basins. Because of difficulties of accurate measurement of the full spectrum of sizes in transport, the most reliable method is bulk analysis of sediment deposited in headwater reservoirs or lakes [e.g., *Smith et al.*, 1960]. However, such data for mountainous areas are rare, and measurement in downstream reservoirs is confounded by downstream fining and sediment storage.

Glaciated areas will require a separate parameterization of the relationship between uplift rates and sub-grid-scale erosion rates.

A number of the other issues that must be addressed in large-scale erosion models can be only briefly mentioned. Sediment must be routed through the fluvial system, with appropriate attention to prediction of bed type, sorting and abrasion using the quantitative approaches summarized earlier. Local bed type in alluvial channels depends upon which term is dominant in (8), and transitions between bedrock and alluvial channels could be treated as in the profile simulations in Figure 6. In addition, boundary and initial conditions must be specified, including tectonic deformation.

## Conclusions

The emphasis in this paper is on our inadequate understanding of long-term evolution of fluvial channels and of the controlling processes. Several types of channels occur in nature, with bedrock, fine-bed alluvial, and threshold coarse-bed alluvial being the end members. Each type requires a different approach to modeling and prediction. Mechanisms and rates of erosion in bedrock channels are poorly characterized, but an approach relating erosion rate to a power function of drainage area and channel gradient may be sufficient for many situations. Fine-bed alluvial channels are the best understood, with the critical issue being characterization of sediment yield from slopes and the interactions between channel entrenchment/aggradation and rates of slope erosion. Gravel channels often have gradients near the threshold of motion, and the important issue is determining delivery rates and size distributions of gravel from slopes and the roles of sorting and abrasion in downstream transport of this debris.

The most critical uncertainties in prediction of long-term evolution of fluvial systems are (1) determination of what type of channel will occur in a given topographic-geologic-hydraulic-climatologic setting, (2) parameterization of slope-channel interactions (including size distribution and amount of sediment shed to channels), (3) quantitative characterization of erosion rates in bedrock channels, and (4) the role of debris production, sorting, and comminution in evolution of gravel bed channels.

Not discussed in this paper, but of considerable importance in long-term channel evolution are possible changes in channel pattern (meandering, braided or straight) and drainage network pattern through divide migration or stream capture. In addition, climate and climatic change influences often outweigh baselevel effects in mountainous regions (e.g., altiplanation, glaciation, high-elevation deserts).

Our recommendations for future study addressing regional scale landform evolution are to proceed on several fronts. The first is development and testing of simulation models of the type proposed above in order to assess the nature of channel type interactions and the types of simplifications that can be made in such modeling. At the same time, regional studies of channel geomorphology are needed, particularly in high relief areas, with an emphasis on channel type and bed sediment size, and processes of transport and erosion. Characterization of sediment supply rates and grain sizes from headwater slopes and channels is also essential, particularly in mountainous areas. Finally, application of absolute age dating techniques to estimation of erosion rates is necessary for model calibration.

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