Natural oscillations in coupled geomorphic systems: An alternative origin for cyclic sedimentation

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ABSTRACT

Internally oscillating cycles of cutting and filling occur naturally in many coupled geomorphic systems. As a demonstration, we have modeled stream entrenchment and backfilling where alluvial systems couple to uplifting mountain blocks. This simple model tracks bedrock stream erosion and transport of alluvium to base level. Internal oscillations arise from coupling of a kinematic wave model for mountain erosion to a diffusion model for the alluvial basin. Decaying periodic cutting and filling occur on time scales of 10^5 yr; the largest magnitudes are found at the coupling point. Model results suggest that cyclic sedimentation may result from the tintinnabulation of single perturbations applied to natural systems and need not record cyclic changes in climate, tectonics, or base level.

INTRODUCTION

The nature of the stratigraphic record is one of episodic, often cyclic, sedimentation, where aggradation events are punctuated by times of nondeposition or erosion. Cyclicity occurs over a variety of space and time scales, from deposition of individual beds to the development of large-scale stratigraphic cycles. The interpretation of cyclic sedimentation is problematic. Most interpretations hinge on changes in external forcing, such as climate, tectonics, or base level, or autocyclic changes. An alternative mechanism discussed here is the internal oscillation inherent within linked geomorphic systems that become preserved in the stratigraphic record.

Geomorphic processes are often modeled with simple mathematical analogs that, on their own, do not display complex behavior (e.g., Ahnert, 1987). However, real-world landscapes are rarely produced by only one process. Even simple landscapes are composed of linked erosional, transport, and depositional systems. Irrespective of the complexity of the coupling, it is the existence of feedbacks between the processes that leads to internally governed oscillations. Coupling of processes leads to complex behavior even in simple systems, especially if signal propagation is fundamentally different between the connected transport systems (Gaponev-Grekhov and Rabinovich, 1992). The degree of complexity can range from driven oscillations all the way to chaos.

We investigate a simple two-part geomorphic system with different processes operating in separate spatial domains—a river that erodes an uplifting terrain and subsequently transports the products to a depositional sink, such as a lake or ocean. Our focus is on the intermediate time scale (10^4–10^6 yr), which is sufficiently long to average many potential difficulties, such as detailed sediment transport. Even this simple coupled system produces complex responses in the form of oscillations at more than one frequency. These natural oscillations are expressed as cycles of sedimentation and erosion that vary in space and time; i.e., the types of stratigraphic cycles widely seen in the rock record.

OSCILLATIONS IN ALLUVIAL FANS

As an example of internal oscillations, we examine repeated cutting and filling in alluvial systems, which is perhaps best developed at the proximal parts of alluvial fans. Alluvial fans and their juxtaposed mountain-front source area represent a simple coupled transport and depositional system. Episodes of entrenchment occur where alluvial fans merge along the mountain front and where the alluvial streams of the fan meet the steep incised bedrock of the mountain block. These episodes of erosion are superimposed on the net aggradation of the fan. These incision events have been interpreted as the result of (1) changes in climate with resultant changes in peak discharge, (2) change in character of the sediment load, (3) tectonic uplift along the mountain front and the subsequent return of streams to an equilibrium profile, and (4) base-level changes (e.g., Bull, 1977). Other studies of alluvial fans suggest that cut-and-fill events may occur spontaneously because of poorly understood intrinsic factors within the fan itself (e.g., Hooke, 1967; Schumm, 1977).

By developing a process model of alluvial systems, we can explore the origins of internal oscillations in natural systems. Alluvial-fan systems are well defined both in physical extent and in their boundary conditions, and they possess an easily understood theoretical equilibrium or steady-state profile. Such steady-state topography exists throughout a coupled mountain-basin system if the mountains are eroding at the same rate as they are uplifted, and the alluvial system is graded to the slope required to transport all sediment through the system to base level. Our model uses this steady-state profile as the initial condition. We investigate deviations from this equilibrium profile that result from perturbing variables such as uplift or climate.

MODEL

The essential elements of the model are a continuously rising mountain block driven by tectonic uplift coupled to a lowland alluvial system (Fig. 1). Bedrock-controlled rivers flowing through the mountain block behave differently from rivers within the alluvial basin. The mountain region is primarily erosional, and the amount and distribution of erosion is controlled by an incising river. The lowland alluvial system may be depositional, erosional, or variable along its length depending upon local conditions, but behaves primarily as a transport system carrying erosional debris from the mountains to the sea. Rivers in the bedrock mountain block and in the alluvial basin are coupled at the head of the alluvial fan.

Our model couples a simple description of an eroding mountain river system to a simple form of an alluvial lowland river (Fig. 1) in such a manner as to preserve time-varying behavior. Deposition and erosion occur in the system following the prescribed set of rules below. The model boundary conditions are base level at the

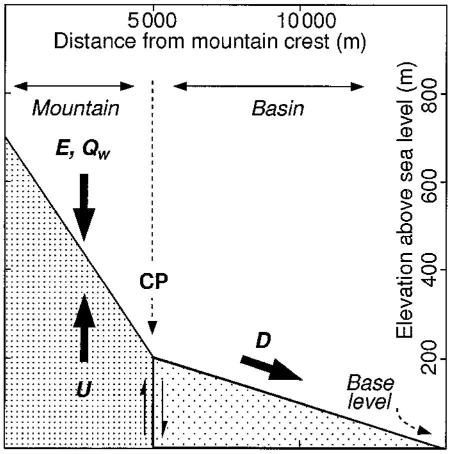

Figure 1. Steady-state river profile for coupled mountain block and basin pair used by model. Tectonic uplift (*U*) affects mountain block, which is coupled to alluvial basin (stippled) by coupling point (CP), here defined by fault. Rainfall (water flux, (Q_w) and erosion (*E*) affect mountain block streams, whereas water flux (Q_w) and diffusivity (*D*) affect streams in alluvial basin. River profile is set by parameters used in model (see text). Figures 2 and 3 are plotted as perturbations from this steady-state profile.

lowest point, tectonic uplift rate (U) of the mountain block, and climate. Climate is defined as rainfall rate and is introduced into the model by making the water flux in the river system directly proportional to rainfall. Note that, for simplicity, the coupling point (CP, Fig. 1) and the location of base level are considered fixed in horizontal space but are free to move vertically. The alluvial fan is not allowed to grow headward into the mountains nor is the shoreline allowed to prograde seaward.

The alluvial river system between the mountains and base level is modeled by using the diffusion equation. This equation is based on mass conservation in a sediment-transporting system and has been shown by several workers to be both theoretically and experimentally a valid first-order approximation for the time-averaged behavior of alluvial rivers (e.g., Begin et al., 1981; Paola et al., 1992). The simplest form of the diffusion equation applicable to natural river systems is

$$\frac{\partial z}{\partial t} = D\, Q_w \frac{\partial^2 z}{\partial x^2}, \qquad (1)$$

where the diffusion coefficient is split into a constant diffusivity, D, which multiplies the time-averaged water flux, Q_w. The vertical elevation above base level is z, the horizontal distance relative to the coupling point is x, and time is t. The coefficient D contains many aspects of the physics of river transport (Paola et al., 1992), including channel pattern, channel width, and several other parameters that are assumed to remain constant over the duration of the model. Water flux is expressed in nondimensional terms and assigned an initial value of 1; thus D retains the usual values and dimensions of a diffusion coefficient, and Q_w may be varied from 1 to indicate deviations from normal water flow. The diffusion coefficient applies to time-averaged, full-width transport and numerically reflects the value chosen for the cross-stream basin width.

The river system in the mountains is considered to be erosional. Erosion of bedrock by rivers is still being investigated, but most formulations invoke a relation between bedrock erosion rate and the stream power (e.g., Seidl and Dietrich, 1992). In this model, channel width is held constant to yield our erosion equation for the upland bedrock channels:

$$\frac{\partial z}{\partial t} = U - E\, Q_w \frac{\partial z}{\partial x}, \qquad (2)$$

where the term containing E is directly proportional to stream power; E is an erosion coefficient that, like D, contains aspects of the problem that are assumed to remain constant, such as the bedrock erosivity and river width. Equation 2 is a kinematic-wave equation with wave velocity $E\, Q_w$, which describes the erosion of bedrock channels on a long time scale. We assume that the flanking valley walls in the mountains erode at the same rate as the valley floor.

The volume of detritus delivered to the fan head is calculated by integrating the erosion rate (equation 2) over the time step and over the area of the mountain drainage. Because the mountain river system and the lowland river system are linked at the coupling point, the model explicitly requires continuity of elevation and sediment transport across this point. Note that this does not imply continuity of slope.

Equations 1 and 2 are implemented as a finite difference numerical scheme. Base level ($z[0, t]$), which acts as a sediment sink and is either fixed or moves vertically as a prescribed function of time, is the dominant boundary condition. The specific input parameters $U(t)$, $E(t)$, $D(t)$, $Q_w(t)$, and $z(0, t)$, are, in general, functions of both x and t. However, we only allow parameters to vary over time.

This model of a coupled erosional and depositional system is based upon, but perhaps oversimplifies, nature. However, it is not important that the model captures all of the complexity of natural systems, but that it shows that even the simplest coupled systems in nature can develop complex responses.

Model Setup

We use the above model to investigate the behavior of a small mountain catchment and attendant alluvial system. We consider a mountain catchment with 5 km stream length and unit width (in this case 1 km) that connects at the coupling point to an alluvial basin that extends 10 km from the mountain front to base level (Fig. 1). The basin contains an alluvial fan and its downstream fluvial continuation. The horizontal model length is thus 15 km. There is implicit integration in the cross-valley or basin-width direction, reflected in the values chosen for E and D, which are based on a model width of 1 km. This choice has no qualitative affect on the model results. However, the width does control some of the derived numbers, such as the volume of sediment in transport, and therefore, the actual frequency of oscillation.

Model parameters start at: $U = 1$ mm/yr, $Q_w = 1$ in nondimensional units (i.e., a unit-flood event), $E = 10$ mm/yr, and $D = 250$ m^2/yr. These parameters allow a steady-state solution when uplift is just balanced by erosion and transport. The derived steady-state value for the slope of the bedrock channel is 5.7°; the implied total yearly sediment output is 5000 m^3 of rock. The steady-state slope of the alluvial river is $\sim 2 \times 10^{-2}$ (1.15°). The elevation of base level is 0; the coupling point is at 200 m elevation, and the highest point of the mountain rivers is 700 m. The steady-state river long profile for these parameters is shown in Figure 1. In the model, only modest changes in these variables are allowed because order of magnitude changes yield nonrealistic landscapes. This constraint on the parameter values allows us to express model results in dimensional terms.

Model Result

With the controlling parameters at their starting values and topography at steady-state values, the model is run with perturbations in a single parameter. Essentially we give the model one or more small nudges and observe the time evolution of the topography as represented by the change in river profile over the length of the model.

Two typical system profiles, along with the envelope that bounds the profiles over time (Fig. 2), show the effect on the model of a periodic 100 m change in sea level every 50 ka. We plot deviations away from the steady-state profile of Figure 1, because the changes in profile are small compared to the relief of the entire profile. The effect of

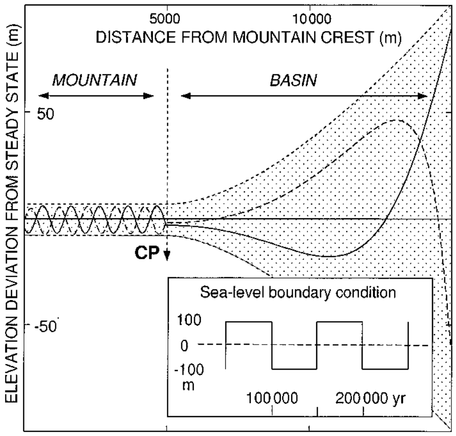

Figure 2. Curves showing deviations away from steady-state profile of Figure 1 during highstand and lowstand of sea level. Sea-level change (inset) is oscillatory and stepwise with magnitude of 100 m and 100 ka period. Solid curve shows particular change in river profile when sea level has risen to 100 m above datum, and dashed curve shows profile when sea level has dropped to 100 m below datum. Both curves show profile changes after many sea-level oscillations. Shaded region encloses total range of profiles produced by record of sea level. CP is coupling point.

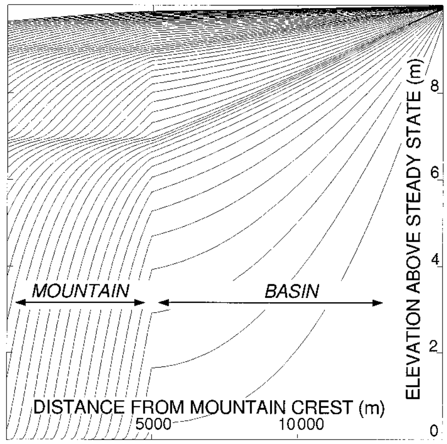

Figure 3. Change in datum profile following single 10 m stepwise increase in sea level. Successive profiles show changes in steady-state profile (Fig. 1) and are shown at 35 ka intervals; earliest profile is at bottom and latest interval is at top. Total time for complete change in profile is 1.96 m.y.

base-level change dies out upstream through the length of the alluvial system, and is very small in the mountain block. This decay upstream of the effect of base-level change was noted by Leopold and Bull (1979), among others. However, the effects of previous oscillations of base level are best seen in the mountain block, where the stepped sea-level oscillation has been attenuated and smoothed into a lower amplitude sinusoid, traveling up the river system as a wave train. The upstream decay of the effect of base-level change is governed by the diffusion coefficient and the frequency of oscillation in the alluvial part, but oscillations do not decay in the mountain block.

Part of a cycle of oscillation of the river profile is seen preserved in the alluvial basin (Fig. 2), where the deviations above and below datum may be interpreted as deposition and erosion. However, steady tectonic uplift in the mountain block results in kinematic waves migrating up the bedrock river valley that are primarily erosive with no deposition. The envelope of the disturbance caused by repeated base-level oscillations is not fixed. Doubling the frequency of base-level change causes a reduction of the magnitude of the disturbance upstream by a factor of $\sqrt{2}$, whereas doubling the diffusion coefficient doubles the upstream magnitude of disturbance. The base-level disturbance propagates upstream in the alluvial system as a rapidly decaying wave moving at a velocity given by $\sqrt{4\pi f D Q_w}$ (where f is the frequency of oscillations), which in this case is ~0.2 m/yr. Once any disturbance reaches the mountain block the disturbance continues to propagate upstream more slowly, at a velocity of EQ_w or, in this case, 0.01 m/yr.

The result of a single stepwise base-level rise of 10 m (Fig. 3) over time causes the entire system to become elevated by 10 m. Note that this does not form bedrock in the mountains but merely slows erosion, allowing the mountains to elevate by tectonic uplift. However, the final state is approached, not by a steady change, but by a decaying series of aggradational steps that take place over ~2 m.y. The stratigraphic pattern of the deposits appears to be cyclic, or at least a decaying series of similar depositional sequences. The profile in the mountains records decreased erosion, not aggradation, and the lower part of the alluvial system is dominated by the single depositional sequence of the base-level rise; thus, the alluvial-fan head is the locus of greatest cyclic deposition. Aggradation at the fan head is caused by the decrease in river slope, which is imposed by the base-level rise. However, fan-head aggradation locally forces a decrease in the slope of the adjacent part of the mountain river system. As this wave of decreased slope migrates up the mountain block, there is a reduction in erosion rate of the mountain bedrock, leading to a decrease in sediment discharge from this source. The decreased sediment supply, in turn, reduces the aggradation rate at the fan head. Ultimately the wave of decreased slope reaches the mountain crest, and the sediment supply again increases as the mountain river slope returns to its previous, steeper, profile, leading to another round of fan-head aggradation and a repeat of the cycle. Diffusion in the alluvial system dampens the process over time, causing the cycle size to decay over several oscillations. The total response of the system to a single base-level increase is a cyclic series of depositional events, wherein the cycle time is controlled by the duration of the kinematic wave sweeping into the mountains (0.5 m.y). The coupling point is the location of the largest oscillating deviations from the steady state and is the origin of both the waves that sweep into the mountains and the waves of erosion and deposition that travel back down the alluvial system after the initial wave associated with base-level rise propagates up the entire system.

The response of the coupled system to periodic perturbations of its controlling parameters can be very complex. This is illustrated in Figure 4A, which shows the elevation of the coupling point over time. The uplift rate has been perturbed every 50 ka, resulting in an oscillation of aggradation-degradation events on the 50 ka time scale superimposed on a larger magnitude oscillation of 10^6 yr duration. The most notable result is that the strongest response of the system is the secondary 10^6 yr events, not the primary 50 ka cycles. Indeed, the geologic record would probably only reveal the larger, longer depositional cycle that begins 0.5 m.y. after the start of the repeated uplifts. Figure 4A is only one realization of the results of cyclic changes imposed upon the model. In this particular case, the driving cycle is commensurate with the inherent time constant of the system, which leads to a strong damping of the 50 ka driving cycle. In other realizations, the driving cycle can be enhanced by the internal time constant, or there can be other complex interactions. An example of the complexity that can be achieved even in this simple system is shown in Figure 4B, which shows the response of the coupling point to the rainfall rate (Q_w) varying by 5% every 75 ka.

Typically, changes of <10% in the controlling variables create aggradation-degradation cycles on the scale of metres. Thus, the system is sensitive to small changes in extrinsic forcing factors and suggests that even exceedingly small changes in climate or tectonics may lead to significant cut and fill events in the stratigraphic record. In contrast, the responses to changes in base level are typically far less sensitive, and the sensitivity decreases with increasing frequency of change. This frequency dependence is created by the fact that the model is driven through the alluvial river system only in the

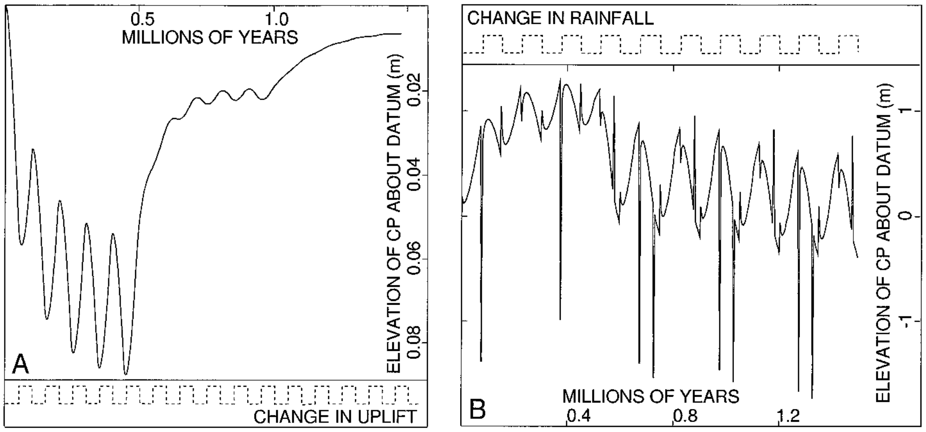

Figure 4. A: Elevation of coupling point (CP) is shown for 1.5 m.y. following start of varying uplift rate in mountains by 5% every 50 ka. B: Elevation of CP is shown for 1.5 m.y. following start of varying rainfall rate 5% every 75 ka.

case of base-level change forcing. With all the other driving functions, the coupling point acts as the locus or origin of the changes in the model. Even in the case of base-level change, the coupling point becomes the virtual locus after the initial change has propagated up from the lower end of the alluvial system.

DISCUSSION AND CONCLUSIONS

The most fundamental and robust result of the modeling is that coupled processes in nature may produce complex, although here not chaotic, behavior. These cyclic changes are neither truly autocyclic nor allocyclic, but instead result from the coupling of linked transport systems, each of which has a different mode of signal propagation. We have expressed a simple coupled transport system of rivers linking erosional source areas in the mountains with deposits in an adjacent alluvial basin. Over a wide variety of conditions, single perturbations that take place anywhere in the coupled system, for example, changes in rain fall, base level, sediment flux, erosion rates, river type or uplift, produce dampened, long-term internal oscillations. The oscillations are repeated cutting and filling events that may be preserved in the stratigraphy. It is significant that model results suggest that even minor changes of $<10\%$ in any of these parameters, except base level, can yield repeated cutting and filling events of up to several metres.

Regardless of where the initial perturbation takes place, the resultant oscillations are greatest at the coupling point. As an example, a short period drop in base level will produce a signal that first migrates up the alluvial river system via a decaying diffusional wave that, over time, will generate a longer period signal traveling back down stream. This secondary signal will oscillate between cutting and filling that will dampen over time. These results are consistent with the entrenchment found at the heads of alluvial fans (Schumm, 1977; Bull, 1977). These secondary events will always migrate away from the coupling point. In the case of base-level change, this direction is contrapuntal to the migration direction of the original perturbation. These secondary effects might be misinterpreted as caused by an unrelated event occurring in the upper reaches of the transport system, such as a tectonic or climatic change.

In the model, oscillations always dampen over time, they never amplify. The time scale for decay of oscillation is not related to the duration of the original perturbation, but instead is controlled by the size of the coupled systems, the erosion rates in the mountains, and the diffusivity of the river systems. In the model runs we explored relatively small coupled systems, and the resulting duration of each oscillation was on the order of a million years. The model results are valid for systems of different size, erosivity, and diffusivity, but in general, the larger the coupled systems, the longer the decay constant. Because most Earth systems are not steady over such long time periods, it may be impossible to find evidence for a single perturbation with its decaying oscillations, before a new perturbation takes place. Thus, the stratigraphic record will record the tintinnabulation of various perturbations superimposed on each other over time, thereby convolving millions of years of past climate, tectonics, and base level. In fact, it is possible that the behavior of the river system does not reflect recent changes in the system, but instead records events that took place in the distant past.

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REFERENCES CITED

Ahnert, F., 1987, Approaches to dynamic equilibrium in theoretical simulations of slope development: Earth Surfaces Processes and Landforms, v. 12, p. 3–15.

Begin, Z. B., Meyer, D. F., and Schumm, S. S., 1981, Development of longitudinal profiles of alluvial channels in response to base-level lowering: Earth Surface Processes and Landforms, v. 6, p. 49–68.

Bull, W. B., 1977, The alluvial-fan environment: Progressive Physical Geography, v. 1, p. 222–270.

Gaponov-Grekhov, A. V., and Rabinovich, M. I., 1992, Non-linearities in action: Berlin, Springer-Verlag, 191 p.

Hooke, R. L., 1967, Processes on arid-region alluvial fans: Journal of Geology, v. 75, p. 438–460.

Leopold, L. B., and Bull, W. B., 1979, Base level, aggradation, and grade: American Philosophical Society Proceedings, v. 123, p. 168–202.

Paola, C., Heller, P. L., and Angevine, C. L., 1992, The large-scale dynamics of grain-size variation in alluvial basins, 1: Theory: Basin Research, v. 4, p. 73–90.

Schumm, S. A., 1977, The fluvial system: New York, Wiley, 338 p.

Seidl, M. A., and Dietrich, W. E., 1992, The problem of channel erosion into bedrock, *in* Schmidt, K. H., and de Ploey, J., eds., Functional geomorphology: Catena, Supplement 23, p. 59–74.

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