Relationships among probability distributions of stream discharges in floods, climate, bed load transport, and river incision

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[1] Analyses of mean daily discharges and annual peak discharges for streams in 14 of the United States and spanning a wide range of climates show that frequency of occurrence relationships for the large-discharge tails of both follow power laws. The number $N(Q)$ of days on which the discharge exceeds Q, or the number of years in which the peak discharge exceeds Q, is related to Q by $N(Q) \propto Q^{-\alpha}$. Values of the exponent α ($1 < \alpha < 6$) decrease in magnitude with increasing aridity so that the ratio of the frequency of occurrence of very large discharges to that of smaller discharges is higher in arid than in humid environments of the United States. To examine the effect of climate change on bed load transport and river incision, we obtain a curve-fit relating mean annual discharge per unit area of drainage basin (an effective precipitation rate $\overline{P}$) to α: $\alpha - 1 \propto \overline{P}^n$, where $n \approx 1.6$. Using this relationship, we confirm that rivers in arid regions should incise less rapidly as climate becomes yet more arid. (If no water flows, the ‘‘river’’ transports no sediment.) Whether aridification of an initially humid environment leads to increased or decreased incision rates, however, depends on the minimum (threshold) discharge capable not only of transporting bed load but also sufficient to scour alluvium from the riverbed and then erode the bedrock. The curve fit relating $\overline{P}$ to α implies that for aridification to accelerate incision, floods that recur only once or twice per millennium (or less frequently) must carry out most of the incision. An overestimate of n could permit smaller, more frequent floods to incise, but it appears that in only special circumstances will aridification accelerate steam incision.

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1. Introduction

[2] Numerous geomorphic observations, at least from temperate regions, suggest that rivers incise valleys only during the largest, and hence rarest, floods [e.g., *Baker*, 1977; *Baker and Pickup*, 1987; *Eaton et al.*, 2003; *Gintz et al.*, 1996; *Graf*, 1979; *Howard*, 1998; *Kochel*, 1988; *Wohl*, 1992; *Wolman and Gerson*, 1978]. For most of the time, rivers transport little sediment. Although floods that recur approximately annually transport most of the suspended sediment [e.g., *Wolman and Miller*, 1960] and in some regions much of the bed load [e.g., *Pickup and Warner*, 1976; *Torizzo and Pitlick*, 2004], these floods seem to affect the landscape little compared with rarer, larger floods that not only transport bed load, but also expose the bedrock floor to attack [e.g., *Magilligan*, 1992; *Pickup and Warner*, 1976]. Thus, insofar as the logic given here applies to river systems in temperate climates, the rate at which rivers incise should depend on the recurrence intervals of their largest discharges.

[3] If rivers are to attack bedrock, they must remove the nearly ubiquitous sedimentary cover on streambeds, which requires that a downstream gradient in bed load transport be maintained to thin the cover [e.g., *Hancock and Anderson*, 2002]. All theories of bed load transport [e.g., *Bagnold*, 1977; *Carson and Kirkby*, 1972; *Dade and Friend*, 1998; *Parker et al.*, 2003; *Seminara et al.*, 2002] and abundant field and laboratory measurements [e.g., *Bagnold*, 1980; *Metivier and Meunier*, 2003; *Meunier and Métivier*, 2006; *Rickenmann*, 1997, 2001] include a threshold below which no bed load is transported. Although the threshold is most commonly cast as a local critical shear stress, there will be a corresponding threshold for discharge. Treatments that include stochastic forcing of varying discharge and thresholds for bed load transport show both to be important in estimates of erosion [e.g., *Lague et al.*, 2005; *Molnar*, 2001; *Snyder et al.*, 2003; *Tucker*, 2004; *Tucker and Bras*, 2000]. Clearly, as a climate becomes more arid and discharge decreases, rivers lose the capability to transport sediment. At the opposite extreme, however, if the threshold for

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removing a sedimentary cover and actually eroding the underlying bedrock is sufficiently high, only rare floods will be able to erode. Thus whether a climate change leads to faster or slower river incision depends upon both the magnitude of the threshold for such incision to occur and the frequency with which it is exceeded.

[4] The preceding logic fails where bedrock is widely exposed and where abrasion, rather than plucking of blocks from the bed, occurs rapidly. Although the rate of abrasion is likely to depend strongly on the speed of water flow [e.g., *Hancock et al.*, 1998; *Sklar and Dietrich*, 2004; *Whipple et al.*, 2000], the threshold for abrasion is likely to be low, whereas for plucking of bedrock and transport of bed load it will be larger and dependent on the size of cobbles or boulders that must be moved. Although we pursue the logic that stream incision occurs only in rare floods when a large a threshold is exceeded, we also recognize that in what is perhaps the best study of bedrock incision, *Hartshorn et al.* [2002] showed that incision of the LiWu valley floor in Taiwan occurs during several floods in a year. Their study examined not only a riverbed where abrasion plays an important role, but also one in a climate different from those of rivers that we studied. Moreover, in an analysis similar to ours, *Snyder et al.* [2003] inferred that erosive events occur relatively frequently, more than once per year. Elsewhere, however, *Pickup and Warner* [1976] showed that although bed load is moved often (nearly annually) along the Cumberland River in Australia, scouring of the alluvial cover overlying bedrock is much rarer. Similarly, *Magilligan* [1992] argued that floods that recur only once or twice per 100 years, and perhaps less frequently, move the largest boulders along the Galena River in Wisconsin. Thus our study cannot be generalized to all regions and climates.

[5] Estimating erosion over geologic times requires a quantification of how floods of different sizes are distributed. Hydrologists have a long tradition of describing flood distributions with probability distributions that include exponential tails at the infrequent, high-magnitude end, in part because they work well for much of the observed magnitude range of floods. *Benson* [1968], in particular, tested six such distributions and recommended that the log-Pearson Type III distribution be used in further analysis. In contrast, *Turcotte and Greene* [1993], *Malamud and Turcotte* [2006], and *Turcotte* [1994] suggested that the distribution of the largest floods, and hence largest discharges, was best fit by a power law distribution:

$$N(Q) \propto Q^{-\alpha} \qquad (1)$$

Here $N(Q)$ is the number of floods with discharge greater than Q, and α is an empirical constant. (Note that neither (1) nor (2) below is a probability density function.) The 1993 flood on the Mississippi River dramatizes the difference between a power law relationship and more traditional statistical distributions; *Malamud et al.* [1996] showed that the former suggests a 100-year recurrence interval, but the latter implies 1000 years between comparable floods. Obviously, the difference between as many as 10 or only one such event per 1000 years matters both for estimating the geomorphic work that can be accomplished and for assessing hazards on human timescales.

[6] The power law relationship in (1) can be treated as a part of an incomplete gamma function [*Davy and Crave*, 2000], which requires one additional parameter to describe the entire distribution. *Lague et al.* [2005] argued that the probability density function that leads to the cumulative distribution describable with an incomplete gamma function fits observed recurrence of discharge on a number of rivers in Taiwan. We exploit the incomplete gamma function below, but we limit analysis of data to forms like (1), because of our focus on the large magnitude end of the distribution.

[7] *Turcotte and Greene* [1993] considered annual peak discharge on 10 rivers for which *Benson* [1968] had compiled measurements. Their results suggest that large floods in arid regions occur more frequently per smaller flood (i.e., α is smaller) than in more temperate environments, a tendency that others have inferred from independent analyses [e.g., *Baker*, 1977; *Baker and Penteado-Obellana*, 1977; *Kochel*, 1988; *Pitlick*, 1994; *Schick*, 1988]. If this dependence of variability (parameterized by α or differently) on climate applied in general, then a shift toward a more arid climate might allow rivers to perform more geomorphic work either by transporting sediment more efficiently or by eroding their bedrock floors more rapidly. Whether such a climate change would lead to greater or less sediment transport and erosion, however, depends both on how transport and erosion depend on discharge and on the magnitude of the threshold above which discharge must be for a river to transport bed load and to incise [e.g., *Lague et al.*, 2005; *Molnar*, 2001; *Tucker*, 2004; *Tucker and Bras*, 2000].

[8] Because the shear stress that a river applies to its bed does not increase linearly with discharge, and because increased discharge implies a wider stream, a shift toward more large magnitude floods need not require more rapid incision than for instance a constant discharge [*Lague et al.*, 2005; *Tucker*, 2004; *Tucker and Bras*, 2000]. Thus deciding whether a climate change toward greater aridity will accelerate bed load transport or bedrock incision depends to some extent on what rules one assumes to relate discharge to bed load transport and incision. If, however, the shift toward greater aridity increases the frequency of discharges in excess of the threshold for bed load transport and incision, then regardless of what one assumes for an erosion rule, an arid climate should erode more rapidly than a humid one. We pursue this perspective and ask what magnitude of threshold discharge must exist for a more arid climate to erode more rapidly than a humid one and how often must that threshold be exceeded.

[9] Because Turcotte and Greene considered only 10 rivers, and only annual peak discharges, their inferred relationship with climate should be considered tentative. To test this relationship further, we analyzed flood statistics using average daily discharges from gauges on 440 rivers in climates that cover a wide range of annual precipitation. Our goals include (1) testing whether equation (1) accurately describes the large-discharge tail of the cumulative distribution, (2) quantifying, insofar as it is valid, Turcotte and Greene's relationship between aridity and flood variability (Figure 1), and (3) evaluating both how large the threshold for bed load transport and incision must be for a shift to a

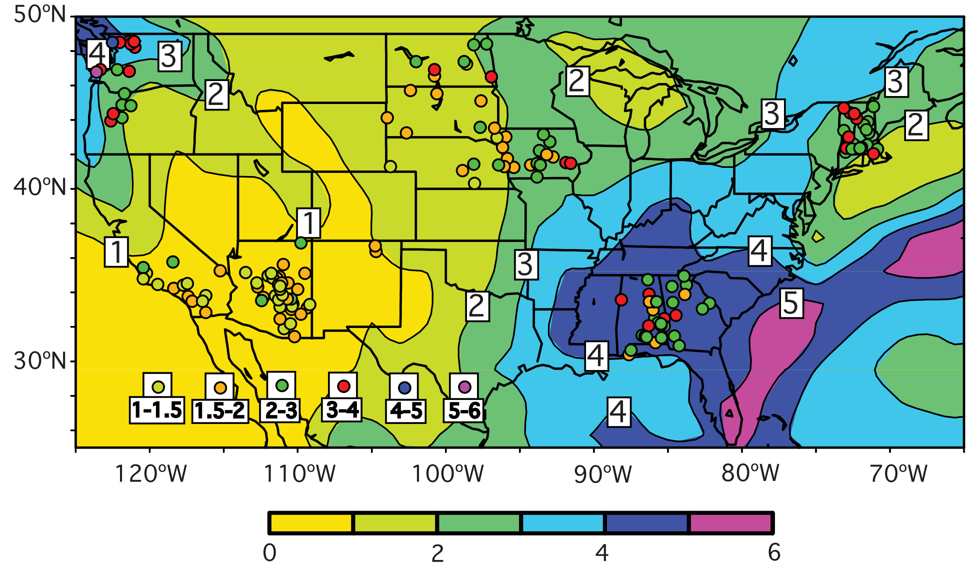

Figure 1. Map of the contiguous United States showing locations of stream gauges that we studied and values of α in equation (1) that we estimated from mean daily discharge measurements, superimposed on contours of mean daily rainfall. To estimate mean daily rainfall, we used the NCEP reanalysis data [*Kalnay et al.*, 1996] available from the NOAA-CIRES Climate Diagnostics Center, Boulder, Colorado, from its Web site at http://www.cdc.noaa.gov/. Note that small values of α correlate with regions of low precipitation.

more arid climate to increase erosion rates, and how often such discharges are likely to be exceeded.

2. Frequency of Occurrence of Discharge and Climate

2.1. Stream Discharge Data and Analysis

[10] We obtained streamflow data from the National Water Information System online hydrological database (NWISWeb, http://waterdata.usgs.gov/nwis/rt) maintained by the United States Geological Survey. We analyzed both mean daily discharges for the period of record at each site, and the maximum instantaneous peak discharge in each year for which there are measurements (annual peak discharge). Because annual peak discharges can last for short times, which may depend on sampling rates at the gauges, their values typically exceed those for the largest mean daily discharge. In this sense annual peak discharge may be more important for erosion than daily discharge. On the other hand, if during the period of record the two largest discharges at a gauge occurred in the same year, the use of annual peak discharge data would overlook the smaller of the two. Daily discharge measurements provide a statistically more reliable description of recurrence than annual peak discharge measurements, by including 365 times as many observations. Consideration of both annual peak and daily discharge time series allows us to assess how 24-hour averages affect estimates based on annual peak discharge data and overcomes the poorly resolved, large flood events in annual peak discharges.

[11] We analyzed data from 440 stream gauges in 14 of the United States that are sufficiently dispersed geographically to include a wide range of climates (Figure 1 and Table 1). Although tropical climates are not represented here, and glaciated regions are sparse (in Washington only), the regions studied include a wide range of mean annual precipitation (Figure 1), as well as different seasonal precipitation, including monsoon climates with summer rain (Arizona and New Mexico), "Mediterranean climates" with winter rain (California), southeastern states with hurricanes that drop exceptionally heavy rains, and northern states affected by spring runoff from snowmelt that affects flooding there. These regions, however, do not sample well the intense concentrated precipitation that characterizes summer monsoons in India and the Himalaya or typhoons in Taiwan. Thus, although we consider the results obtained here to have more than mere parochial application to the USA, they cannot be generalized to the entire earth.

[12] We first tested whether magnitude-frequency distributions obey either (1) or an exponential relationship:

$$N(Q) = N_0 \exp\left(-\frac{Q}{Q_0}\right) \tag{2}$$

where N_0 and Q_0 are empirical quantities. Second, we determined the extent to which the distributions of annual peak discharge correlate with those for large daily discharges. We restricted analysis to gauges for which the period of record exceeds 20 years. The longest record is 46,809 days or 128.24 years (gauge ID = IA MISS 05420500). The average period of record is 19,413 days or 53.18 years. Where possible, we eliminated records affected by dams, diversions, and human modifications of groundwater. At some gauges such effects contaminate only part of a long record and are well documented. For example, annual peak discharges are available since 1940 and daily discharges since 1939 for USGS Gauge 01169000 on the North River at Shattuckville, Massachusetts, but USGS records report that a nearby mill affected flow prior to

Table 1. List of States for Which Daily Discharge and Peak Annual Discharge Were Studied

State	Number of Gauges
Alabama	26
Arizona	54
California	17
Georgia	15
Iowa	54
Massachusetts	20
Minnesota	21
North Dakota	23
Nebraska	14
New Hampshire	14
New Mexico	33
Oregon	43
South Dakota	42
Vermont	32
Washington	32
Total	440

1950. Accordingly, we removed data from prior to 1950. Because such documentation is not available for many online gauges, however, some data that contribute to our conclusions may be contaminated by flow that does not reflect concurrent or recent precipitation.

[13] For each gauge, we found the maximum daily discharge, Q_M. We then sorted daily discharges into 100 bins: $(i - 1)Q_M/100 \leq Q < iQ_M/100$, for i = 1,100. By definition, $N(Q)$ decreases with increasing Q. Commonly that decrease is steep where Q is large, and N relatively small, but less steep for smaller Q (Figure 2); neither a simple power law relationship (1) nor an exponential relationship (2) fits the full distribution of discharges. For peak annual discharge, we used 20 bins.

[14] As our focus is on the distribution of large discharges that matter geomorphically, we considered the large-Q range only. For all records, the cumulative distribution of the few largest daily discharges can be matched by both power laws (1) and exponential functions (2), but because incision may occur in floods that recur over intervals longer than the stream gauge record, we rely on the gauges with the more precisely estimated distributions. For more than 35% of the gauges tested, the power law relationship, equation (1), describes the frequency-magnitude distribution for mean daily discharges greater than 20% of the maximum daily discharge with uncertainties in α less than 17% of the estimated values. At most gauges, the power law relationship includes daily discharges that recur more commonly than once per year. In only rare cases (3% of all considered) does $N(Q)$ match an exponential relationship (2) as well as or better than a power law relationship (1). Ignoring these 3%, the remaining 62% of the cases can also be fit by equation (1), but only for values of mean daily discharge that are greater than ~60% of its maximum at the gauge, or for which uncertainties in estimated values of α are greater than 17% of the estimated values. We presume that the durations of the records are too short to resolve well these distributions, and we focus the remaining discussion on the 35% of the streams (159 of the 440 gauges) with least uncertain values of α.

[15] Others have reported that for the large discharge tail in the cumulative distribution, a power law relationship as in (1) works better than other distributions. Casting recurrence in terms of intervals of time between discharges equal to or greater than some magnitude, *Malamud and Turcotte* [2006] examined six very long records (>100 years) from gauges in the USA, and hence with many data, and found power laws to work better than log-Gumbel or log-Pearson type 3 distributions. Moreover, *Malamud and Turcotte* [2006] included paleoflood estimates and showed that predicted recurrence intervals, of thousands of years, are within a factor of 3 of those determined geologically. *Lague et al.* [2005] also found power laws to fit the large-discharge tail better than other distributions, including the exponential distribution in (2). They showed that an exponential distribution predicts fewer large discharges per smaller one over the range described well by a power law distribution.

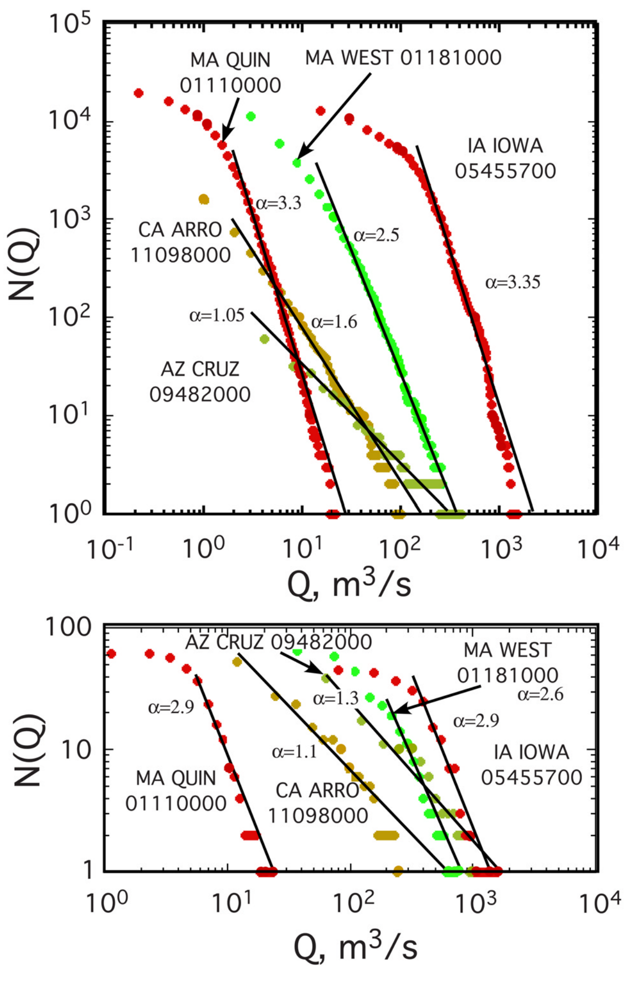

Figure 2. Examples of plots of $N(Q)$ versus Q for both (top) daily discharge data and (bottom) annual peak discharge, showing cases where a power law relationship fits well. Results from gauges are color coded as in Figure 1. Note the smaller values of α for rivers in the more arid climates. Values of α for the different gauges are AZ CRUZ 09482000, 1.04 ± 0.03 (top) and 1.30 ± 0.12 (bottom); CA ARRO 11098000, 1.59 ± 0.03 (top) and 1.14 ± 0.07 (bottom); IA IOWA 05455700, 3.35 ± 0.08 (top) and 2.92 ± 0.29 (bottom); MA QUIN 01110000, 3.33 ± 0.04 (top) and 2.90 ± 0.17 (bottom); and MA_WEST_01181000, 2.47 ± 0.04 (top) and 2.56 ± 0.20 (bottom).

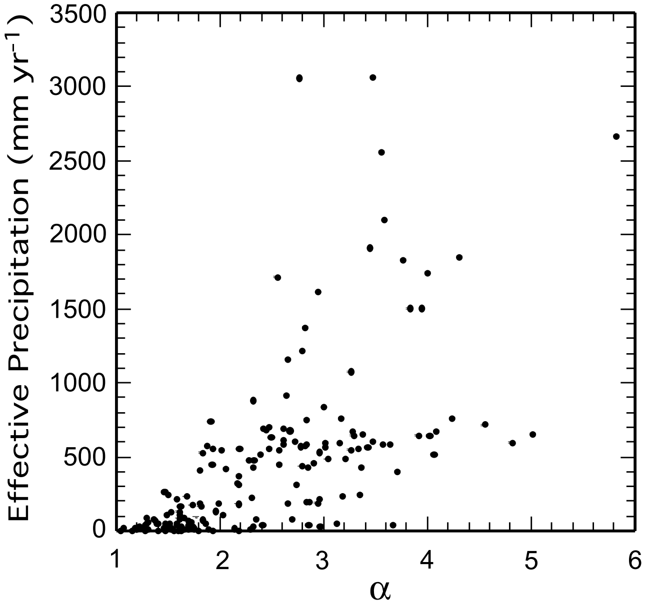

Figure 3. Plot of measured values of effective precipitation versus estimated values of α from the 155 gauges that we selected. Note that larger values of α correlate with higher effective precipitation rates, consistent with α correlating positively, if crudely, with aridity.

[16] The values of α that we estimated range from just above 1 to almost 6, corresponding to radically different ratios of large to very large discharges on these rivers. Values of α inferred from annual peak discharge time series and daily discharge time series differ, on average, by only 15%. This agreement suggests that the recurrence of large daily discharges can be extrapolated further, at least to the peak discharges that common last only a few hours but that do much geomorphic work. In four cases, values of α were unusually disparate (differing as much as a factor of two from one another); we ignore those four gauges in the remainder of the study. Last, to quantify differences in climate, we used the daily discharge data to evaluate the relationship of total (effective) precipitation and discharge variability (α) at the 155 gauges for which equation (1) fits the frequency-magnitude distribution over the highest 80% of the discharges, and for which estimates of α are uncertain by less than 17%.

2.2. Effective Mean Annual Precipitation

[17] Although stream gauge data are readily available throughout the United States, comparable coincident precipitation records are not. We therefore use an effective precipitation at each stream gauge as a proxy for the average total precipitation in the catchment. To determine an effective annual precipitation, we divided the average annual discharge for the period of record by the total drainage area above each gauge, which NWISWeb provides for each gauge. Clearly, effective precipitation ignores evapotranspiration and temporal variations in subsurface flow.

[18] Where data exist, we also calculated average precipitation for the past 100 years using the National Oceanic and Atmospheric Administration Cooperative Institute for Research in the Environmental Sciences (NOAA-CIRES) Climate Diagnostic Center online database (http://www.cdc.noaa.gov/USclimate/USclimdivs.html). For a few gauges (<10), the average precipitation was higher than the effective precipitation by an order of magnitude. These gauges, however, are not included among the 155 gauges that we considered and therefore do not contaminate the data that we use. We presume that they represent diversions of water into or out of the rivers above the gauges, effectively decoupling the gauged flow from the climate within the basin.

2.3. Effective Precipitation and Climate

[19] To relate climatic differences to discharge variability, we sought a relationship between the power law exponent, α, which quantifies discharge variability, and effective mean annual precipitation $\overline{P}$ using the results for the 155 gauges with precise values of α. Despite considerable scatter, α increases with effective precipitation (Figure 3). Thus, in general, large floods occur more frequently per small flood in arid climates than in more humid or temperate climates, as *Turcotte and Greene*'s [1993] analysis of *Benson*'s [1968] annual peak discharge data suggested.

[20] At first glance at least, the scatter in Figure 3 does not encourage the quantification of a relationship between α and $\overline{P}$, and lacking a theory to explain any trend, we merely sought empirical curve fits to these data. In particular, we replotted the data on a log-log plot, and following a suggestion by D. Lague (personal communication, 2005), we subdivided the data into bins according to the areas of the drainage basins. He argued that the variability of discharge is likely to be greater in small than in large basins, because heavy precipitation might occur over small basins, but not over large ones. Such logic derives support from a compilation of peak discharges by *O'Connor and Costa* [2004], who showed that although peak discharges increase with drainage basin area, the relationship is not linear. They showed that the largest peak discharges vary with drainage basin area to the 0.53 power, or approximately as the square root of area, and hence less rapidly that mean annual discharge varies with drainage basin area. Consistent with Lague's expectation, fits of $\log_{10}(\overline{P})$ versus $\log_{10}(\alpha - 1)$ (Figure 4) yield straight lines that are roughly parallel, at least for drainage basin areas greater than $\sim$100 km^2, but with less variability (greater α) for larger basins.

[21] Although taking into account the different drainage basin areas does reduce some of the scatter in a relationship between $\overline{P}$ and $\alpha - 1$, enough scatter remains that it would be folly to assume that we can predict $\overline{P}$ accurately from estimates of $\alpha - 1$. Moreover, the likelihood that an accurate relationship can be obtained seems remote given some other likely reasons for the scatter. As *O'Connor and Costa* [2004] emphasized in their discussion of peak discharges, many nonclimatic factors affect temporal variability of discharge, such as orographic precipitation, especially if enhanced by convective storms, and in turn if such storms are "anchored" to particular topographic features. Moreover, as they noted, soil permeability, snowmelt, and topographic features that route runoff will affect flooding. Readers might be surprised that in the United States flooding offers the greatest risk to south central Texas near the Balcones Escarpment, not a major orographic feature.

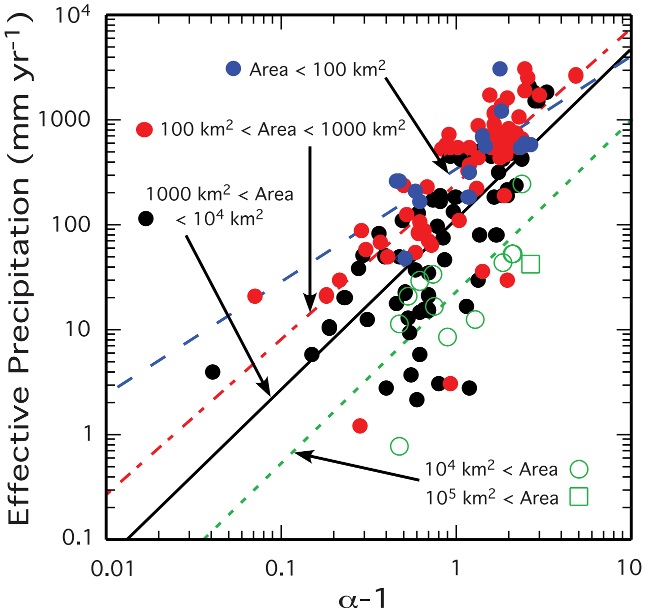

Figure 4. Plot of the data in Figure 3, but as $\log_{10}(\alpha - 1)$ versus $\log_{10}$ (effective precipitation). Values of effective precipitation are binned into groups with different drainage basin areas, and straight lines show plots of the form $\overline{P} = B(\alpha - 1)^n$. For area < 100 km^2, $B = 343$ mm yr^{-1}, $n = 1.08$, and $R = 0.39$; for 100 km^2 < area < 1000 km^2, $B = 242$ mm yr^{-1}, $n = 1.47$, and $R = 0.73$; for 1000 km^2 < area < 10^4 km^2, $B = 101$ mm yr^{-1}, $n = 1.62$, and $R = 0.85$; and for 10^4 km^2 < area < 10^5 km^2, $B = 22.5$ mm yr^{-1}, $n = 1.63$, and $R = 0.76$.

(Some readers will be disappointed that Crawford, Texas does not lie in this region of high flood risk.)

[22] Despite the caveats given above, let us use a relationship of the following form to explore how a climate change manifested by a change in α might affect erosion:

$$\overline{P} = B(\alpha - 1)^n \tag{3}$$

Because α depends on the drainage basin area, so also must B, which is measured in mm yr^{-1}, for $\overline{P}$ should not depend on area. The fits in Figure 4 suggest that $n \approx 1.6$, but we consider $1 \leq n \leq 2$ below.

3. Possible Impact of Climate Change on River Incision

[23] We seek an understanding of how climate change, manifested by changes in $\overline{P}$ and α, might affect river incision, assuming that a threshold discharge for incision exists. By using (3) to link $\overline{P}$ and α, we assume that the variability deduced from different regions also applies to temporal variations in climate at a specific location, an assumption that we cannot test easily. Moreover, with no theoretical basis for (3), the inferences that follow must be considered tentative and at best suggestive, not rigorously founded.

[24] With (3) we can, in principle and at least crudely, relate mean annual discharge, equal to the product of the effective precipitation $\overline{P}$ with the drainage basin area, to the variability quantified by α. Because the cumulative distribution given by (1) does not describe the relative frequency of occurrence of small discharges, we need a distribution with one additional parameter.

3.1. Climate and Frequency of Occurrence of Large Discharge Events

[25] The probability density function proposed by *Davy and Crave* [2000] not only includes a power law distribution for large discharge events, but also a second parameter, the mean daily discharge $\overline{Q}$, which obviously is proportional to the mean annual discharge that we have used to characterize the degree of aridity:

$$p(Q)dQ = \frac{\left[\overline{Q}(\alpha - 1)\right]^\alpha}{\Gamma(\alpha)} Q^{-1-\alpha} \exp\left(-(\alpha - 1)\frac{\overline{Q}}{Q}\right) dQ \tag{4}$$

In (4), $\Gamma(\alpha)$ is the gamma function of α, and we have rewritten *Davy and Crave*'s [2000] expression in terms of the parameter α that we use. Clearly, for $Q \gg \overline{Q}$, the exponential factor approaches 1, and $p(Q) \propto Q^{-1-\alpha}$. Thus the cumulative distribution will be proportional to $Q^{-\alpha}$, as in (1). As *Lague et al.* [2005] show, the cumulative distribution is given by

$$N'\left(Q \geq Q_*\right) = \int_{Q_*}^{\infty} p(Q)dQ = \Gamma\left(\frac{(\alpha - 1)\overline{Q}}{Q_*}, \alpha\right) \tag{5}$$

where $N'(Q \geq Q_*)$ is the probability per year that the daily discharge Q will exceed Q_*, and

$$\Gamma(x, a) = \frac{1}{\Gamma(a)} \int_0^x y^{a-1} e^{-y} dy \tag{6}$$

is the incomplete gamma function. If $Q_* = 0$, $N' = 1$, and N' decreases as Q_* increases. If we multiply N' by 365 days, we obtain the number of days per year for which we expect the daily discharge Q to equal or exceed Q_*. We may use (5) to estimate the number of days per year that the daily discharge is likely to exceed the mean discharge daily $\overline{Q}$ by substituting $\overline{Q} = Q_*$ into (5) and multiplying by 365 days: $365 \cdot \Gamma(\alpha - 1, \alpha)$. The number of days decreases as α decreases (Figure 5). With this insertion, (5) quantifies the fact that as the ratio of large discharges to small ones increases (decreased α), a larger fraction of the mean daily discharge is carried on fewer days, those with the rare large discharges, than when the ratio of large to small discharges is smaller (increased α). The mean discharge daily in (5) is given by $\overline{Q} = \overline{P}A/365$, where A is the drainage basin area, and division by 365 converts mean annual discharge into mean daily discharge. When (3) is used, this becomes

$$\overline{Q} = AB\frac{(\alpha - 1)^n}{365} \tag{7}$$

[26] We seek a cumulative distribution of discharge as function of discharge, so that we can compare such distributions for different climates with each other. Because N' in (5) is a cumulative probability for daily discharge, by multiplying it by the appropriate value of $\overline{P}$ and by the

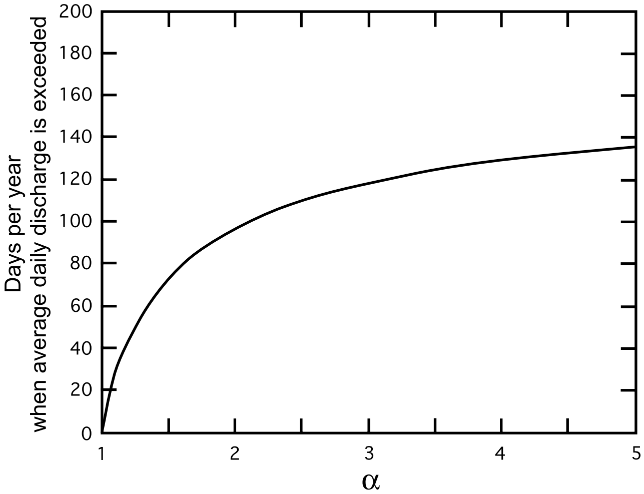

Figure 5. Plot of the frequency of occurrence of discharges that equal or exceed mean daily discharge as a function of α. Note that when α is larger than ~ 2 and variability is relatively small, the mean daily discharge is exceeded more than 100 times per year. As α approaches 1, however, an increasing fraction of the mean annual discharge is carried by rare large floods, and the mean daily discharge is exceeded rarely.

drainage basin area, A, we obtain a predicted cumulative distribution for daily discharge.

$$D(Q \geq Q*) = \overline{P}AN'(Q \geq Q*) = \overline{P}A\Gamma\left(\frac{(\alpha-1)\overline{Q}}{Q*}, \alpha\right) \quad (8)$$

where $D(Q \geq Q*)$ is the total expected discharge per year due only to those floods with $Q \geq Q*$. Inserting (3) into (8) expresses the predicted cumulative distribution for daily discharge solely in terms of α:

$$D(Q \geq Q*) = AB(\alpha-1)^n\Gamma\left(\frac{AB(\alpha-1)^{n+1}}{365Q*}, \alpha\right) \quad (9)$$

[27] Our focus is on climate change, and its effect for specific basins, not a comparison among basins with different areas. Neither the drainage basin area, A, will change if climate changes, nor should B in (3). Thus it makes sense to render (9) dimensionless by scaling discharges by $AB/365$:

$$D'\left(Q' \geq Q'_*\right) = \frac{(\alpha-1)^n}{365}\Gamma\left(\frac{(\alpha-1)^{n+1}}{Q'_*}, \alpha\right) \quad (10)$$

[28] Plots of $365D'(Q' \geq Q'*)$ given by (10) versus $Q'*$ for different values of α show not only how the slope of the large-discharge portion of the distribution varies with α, but also the degree to which the mean annual discharge, $365D'(Q' \geq 0)$, depends on α (Figure 6). The combination of these two effects means that most discharges occur more frequently in humid than in arid climates; for any pair of values of α, however, corresponding to two different climates, there is a (large) discharge above which higher discharges are more common in the arid climate.

[29] Before considering either the effect of errors in n, defined in (3) and used in (10), or how erosion is affected, note that if $\alpha = 2$, (10) is independent of n. As it should, the mean annual precipitation, proportional to $(\alpha - 1)^n$ in (3), increases monotonically with α, but as $\alpha \rightarrow 1$, that dependence becomes very marked (Figure 6). Because a shift to a more arid climate should result in a shift to smaller α, the impact of climate change should be greater in arid than in humid regions. Similarly, the mean daily discharge given by (7) and the position of the elbow separating the low-discharge and high-discharge ends of $D'(Q' \geq Q'*)$ in (10) depend on $(\alpha - 1)$ raised to the powers n and $n + 1$, respectively, and both decrease rapidly as $\alpha \rightarrow 1$ (Figure 6). Thus, insofar as both the power law cumulative distribution in (1) and the relationship between variability and mean annual precipitation as expressed by (3) are reasonable, we should expect behavior for $\alpha < 2$ to be different from that with $\alpha > 2$.

3.2. Effect of Climate Change on Bed Load Transport and River Incision

[30] We explicitly assume that bed load is not transported by small discharges, and that rivers cannot incise bedrock unless some threshold shear stress is exceeded. Without specifying how, we also assume that a threshold shear stress

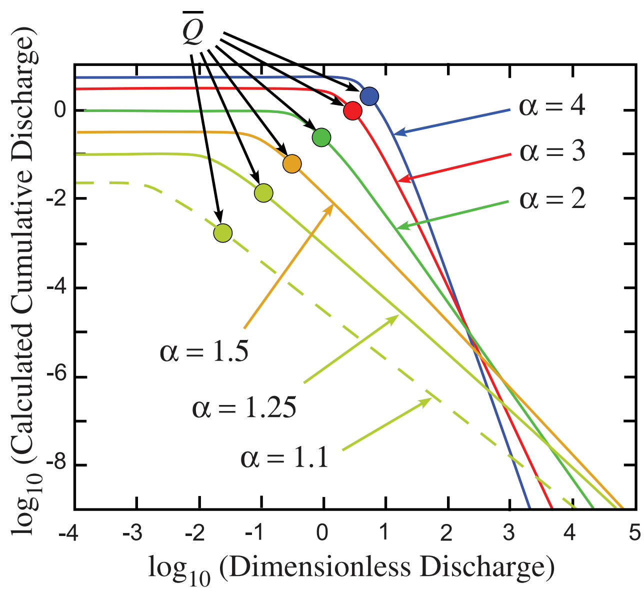

Figure 6. Plot of theoretical values of $\log_{10}$ $[365D'(Q' \geq Q'*)]$ versus $\log_{10}$ $Q'*$ based on equation (10) for several values of α. The circles for each curve show the corresponding mean annual discharge. This plot shows the relative likelihood of a particular (dimensionless) discharge occurring as a function of α for basins with the same areal extent. For large α, characteristic of a moist climate, more annual discharge occurs, and relatively small discharges are more common than in dry climates (small α). For very large discharges, however, the largest discharges could occur more commonly, if still rarely, in arid regions, because the small values of α compensate for the smaller mean annual discharge. To make this plot, we used $n = 1.6$ in (3).

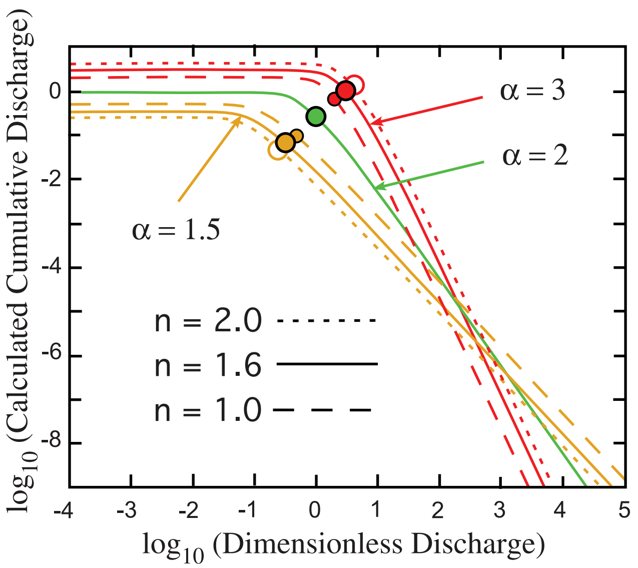

Figure 7. Effect of errors in n, defined by (3) and used in (10), on theoretical values of $\log_{10}$ [$365D'(Q' \geq Q'*)$] versus $\log_{10} Q'*$, for three values of α. As shown in (10), if α = 2, $D'(Q' \geq Q'*)$ is independent of n. Larger values of n enhance climatic differences, manifested by different α.

can be expressed in terms of a threshold discharge. For sufficiently large discharges, a shift to a more arid climate, manifested by a shift to smaller α, will result in more events with the very largest discharges, a fact illustrated by the crossing of the cumulative distributions in Figure 6 at large discharge. Suppose that the threshold discharge for bed load transport and bed load incision were so large that only discharges greater than this crossover discharge moved enough bed load to expose the bed to incision by other processes, or to incise the streambed by bed load motion. Then, regardless of how bed load transport and incision depended on discharge, provided that both increase monotonically with discharge, a climate shift toward greater aridity would imply increased incision.

[31] If a climate change from one characterized by α > 2 to another with α = 2 resulted in faster river incision, we might ask: What would this require for the magnitude of the threshold discharge and the frequency of occurrence of discharges that exceed it? Consider the cumulative distribution of discharges for α = 2 in Figure 6; the green dot defines the (dimensionless) mean daily discharge. From Figure 5, we know that for α = 2, the mean daily discharge is exceeded roughly 100 days per year. For α = 3 or 4, the cumulative distributions (Figure 6) intersect that for α = 2 where $\log_{10}$ (dimensionless discharge) ≈ 2.3, for which discharges exceed the mean daily discharge for α = 2 by ~200(=$10^{2.3}$) times. From the relative frequency of occurrence shown by the vertical axis in Figure 6, such large discharges recur approximately 50,000 times less often than do those equal to or exceeding the mean daily discharge. Thus, if climate became more arid, with an decrease from α = 3 or 4 to α = 2, and this shift made rivers more erosive, only floods that recur less frequently than approximately once in every 500 years (100 events per year ÷ 50,000) would contribute significantly to that erosion. Thus, if aridification increases river incision, the important flood events must occur rarely on human timescales, and perhaps only the 1000-year floods are important.

[32] Consider a very arid climate characterized by α ≈ 1.5 that becomes yet more arid, so that α decreases. Again, let us assume that this climate change leads to more rapid river incision and ask what that hypothesis implies for the frequency of occurrence of discharges capable of carrying out incision. For α ≈ 1.5 the mean daily discharge (brown dot in Figure 6) would be exceeded only ~70 times per year. For a shift to α ≈ 1.25, for instance, by the same logic as in the preceding paragraph, the discharges that would recur comparably often would be approximately 10^5 times larger than the mean daily discharge (Figure 6), and such discharges should recur 10^8 times less often that the mean daily discharge, or with a recurrence interval of a million years. Thus we must conclude that in the limit of arid climates, those already with low α, increasing aridity should lead to less rapid incision.

[33] The results presented above depend on the value of n in (3). For larger values of n, increased aridity requires yet larger threshold discharges, which therefore must occur less frequently that they would in less arid climates (Figure 7). Conversely, if n = 1, for increased aridity associated with a shift from α > 2 to α = 2 to lead to faster incision, the threshold discharge need be only ~30 times larger than the mean daily discharge, and it might recur only ~1000 times more often than the mean daily discharge for α = 2, or roughly every 10 years. Such erosive events are still much rarer than those in Taiwan [*Hartshorn et al.*, 2002] or along the Coast Ranges of California [*Snyder et al.*, 2003], but maybe not for some other regions [e.g., [e.g., *Magilligan*, 1992; *Pickup and Warner*, 1976]. Thus, given the absence of a theoretical basis for (3) and the uncertainty in n, we cannot exclude the possibility that aridification in some regions leads to more rapid incision. Even with n = 1, however, a shift from an arid climate to a yet more arid one, described by a shift from α ≈ 1.5 to α < 1.5, is not likely to accelerate incision.

4. Discussion and Conclusions

[34] The frequency-magnitude distribution of the largest daily discharges obeys a power law distribution of the form in (1). This distribution appears to be robust for 155 of the 440 gauges that we studied, for which the lowest discharge value we considered was smaller than 20% of the maximum measured discharge at the gauge. Uncertainties in estimated values of α for these gauges are all less than 17%. Equation (1) can be applied to most of the other gauges, but either uncertainties in α are larger than 17% of their inferred values, or the range of validity of (1) is limited to a fraction of the largest daily discharge that is greater than 20%. Moreover, for most of the 155 gauges, inferred values of α derived from daily discharges agree within 15% of those for annual peak discharges, for which data are much fewer and uncertainties are therefore much larger. Our results therefore corroborate *Turcotte and Greene*'s [1993] suggestion that a power law distribution describes the frequency-magnitude distribution of discharge for the large floods on a given river.

[35] The values of the exponent α depend on climate, as *Turcotte and Greene* [1993] also found (Figure 1). As the effective precipitation, the ratio of annual mean discharge at a gauge divided by drainage basin area, increases, α also increases (Figure 3). Conversely, the frequency of occurrence of the largest discharges compared with frequency of occurrence of more typical discharges is higher in arid regions than in more humid regions.

[36] It follows that climate change can alter the rate of bed load transport and of stream incision, but whether a shift toward more arid climates leads to faster or slower river incision depends on the degree of climate change, its impact on river discharge, and the threshold discharge that must be exceeded for incision to occur. If incision occurs in only rare large floods, then a shift to a more arid climate might make increased aridity more erosive than a humid climate. Thus the threshold that discharge must exceed to incise its bed determines the simplest criterion for incision.

[37] We posed the question: If a climate change toward greater aridity were to lead to more rapid stream incision, what would the recurrence interval be for those discharge events that incised the streambed? Using a cumulative distribution for discharge (10) suggested by *Davy and Crave* [2000] and *Lague et al.* [2005], we estimate the magnitude of such a threshold discharge, and from it the expected recurrence interval for such discharges.

[38] In very arid climates, a shift toward greater aridity leads to a decrease in the incision rate, because too little water is available to incise the streambed. (Obviously, in the limit of no flow at all, the incision rate drops to zero.) This result corroborates findings of *Lague et al.* [2005]. In a relatively humid environment, a shift toward a more arid climate might accelerate stream incision, but for this to be so, the threshold discharge for incision must be so large that floods capable of incising river beds occur rarely. For a value of $n = 1.6$ in (3), that recurrence interval is 500 years, but if a smaller value of n were more appropriate, the minimum recurrence interval could be as short as 10 years. Thus, contrary to what one of us wrote earlier [*Molnar*, 2001], the aridification of most landscapes does not seem likely to lead to faster river incision, except perhaps in regions where only very rare floods excavate the riverbed [e.g., *Magilligan*, 1992].

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