

# The Natural Channel of Brandywine Creek Pennsylvania

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GEOLOGICAL SURVEY PROFESSIONAL PAPER 271



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*By* M. GORDON WOLMAN

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UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1955

**UNITED STATES DEPARTMENT OF THE INTERIOR**

**Douglas McKay, *Secretary***

**GEOLOGICAL SURVEY**

**W. E. Wrather, *Director***

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For sale by the Superintendent of Documents, U. S. Government Printing Office  
Washington 25, D. C. - Price 55 cents (paper cover)

## PREFACE

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This report was prepared in the Water Resources Division of the United States Geological Survey as part of the investigations of water resource problems conducted by the Technical Coordination Branch, R. W. Davenport, chief. The author is greatly indebted to Luna B. Leopold for his help in the field and for his many discussions of the manuscript with the author. The advice and suggestions of W. B. Langbein have also been invaluable. In addition to many colleagues in the Geological Survey to whom the author is indebted for aid and counsel, the cooperation of R. W. Struble of the Brandywine Valley Association is appreciated. The critical comments of Profs. C. E. Stearns, M. P. Billings, J. H. Mackin, and H. A. Thomas, Jr., who kindly read the manuscript, have also been extremely helpful.

M. G. W.

## SYMBOLS

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<p><i>a</i> coefficient representing width at unit discharge</p> <p><i>A</i> area of cross section of flowing water</p> <p><i>b</i> exponent in width-discharge relation</p> <p><i>c</i> coefficient representing depth at unit discharge</p> <p><i>C</i> Chezy coefficient in <math>v = C\sqrt{R}s</math></p> <p><i>d</i> mean depth</p> <p><i>f</i> exponent in depth-discharge relation</p> <p><i>g</i> acceleration due to gravity</p> <p><i>j</i> exponent in suspended load-discharge relation</p> <p><i>k</i> coefficient representing velocity at unit discharge</p> <p><i>L</i> suspended-sediment load, weight per unit of time</p> <p><i>m</i> exponent in velocity-discharge relation</p> <p><i>n</i> Manning roughness factor</p>	<p><i>n'</i> a roughness parameter</p> <p><i>p</i> coefficient representing load at unit discharge</p> <p><i>Q</i> water discharge in cubic feet per second (cfs)</p> <p><i>r</i> coefficient representing roughness parameter at unit discharge</p> <p><i>R</i> hydraulic radius, approximately equal to mean depth in wide channels</p> <p><i>s</i> slope of water surface in open channels</p> <p><i>t</i> coefficient representing slope at unit discharge</p> <p><i>v</i> velocity</p> <p><i>w</i> width</p> <p><i>y</i> exponent in roughness-discharge relation</p> <p><i>z</i> exponent in slope-discharge relation</p>
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# THE NATURAL CHANNEL OF BRANDYWINE CREEK, PENNSYLVANIA

By M. GORDON WOLMAN

## ABSTRACT

This study of the channel of Brandywine Creek, Pennsylvania, consists of three parts. The first is an analysis of the changes which take place in the width, depth, velocity, slope of the water surface, suspended load, and roughness factor with changing discharge below the bankfull stage at each of several widely separated cross sections of the channel. Expressed as functions of the discharge, it is found that the variables behave systematically. In every section studied, as the discharge increases, the velocity increases to about the 0.6 power, depth to the 0.4, and load to the 2.0 power of the discharge. The roughness decreases to the 0.2 power of the discharge. The relative magnitudes and the direction of these variations are similar to those which have been observed in other rivers in the United States, primarily in the West. Some modifications of the hypotheses applicable to the western rivers are probably required because on Brandywine Creek the difference between the materials on the bed and in the banks is considerably greater than it is on most of the western rivers studied.

In the second part of the paper the progressive changes of the same variables in the downstream direction with increasing discharge at a given frequency are described. Despite the disorderly appearance of the stream, it is found that the variables display a progressive, orderly change in the downstream direction when traced from the headwater tributaries through the trunk stream of Brandywine Creek. At a given frequency of flow, width increases with discharge to about the 0.5 power. Depth increases downstream somewhat less rapidly, while the slope and roughness both decrease in the downstream direction. Despite a decrease in the size of the material on the bed, both the mean velocity and the mean bed velocity increase downstream. The rates of change of these variables are in close accord with the changes observed on rivers flowing in alluvium and in stable irrigation canals. These relationships hold for all flows up to the bankfull stage. Analysis of the streamflow records indicates that the annual maximum discharge equals or exceeds the bankfull stage roughly once every 2 years.

The regularity in the behavior of the variables with changing discharges both at-a-station and in the downstream direction and the similar rates of change of the variables on Brandywine Creek and in stable irrigation canals suggest the existence of a quasi-equilibrium in the channel of the creek.

Part three of this study is concerned with this concept of equilibrium in streams. By analogy with canals and with several rivers in diverse regions of the United States it may be concluded that this quasi-equilibrium is closely related to the discharge, and to the concentration of the suspended load. The shape and longitudinal profile of the channel are determined by these two independent factors which operate within the limits set by the local geology. The latter determines the initial size, shape, and resistance of the material provided to the channel.

The existence of a quasi-equilibrium among the variables studied suggests that most reaches on Brandywine Creek are at grade. This is true if the term "grade," when applied to natural rivers, is synonymous with quasi-equilibrium. The adjustability of the variables in the channel rather than the stability of any particular shape or longitudinal profile of the channel is emphasized when the word "equilibrium" is applied to Brandywine Creek.

## HISTORY AND PERSPECTIVE

Although the laws and literature of ancient Babylonia and Egypt contain references to the natural characteristics of river channels, no systematic development of a general principle can be discerned before the 19th century. From the early 19th century on, however, the history of speculation on the natural characteristics of rivers illustrates the gradual and systematic refinement of a broad generalization. In 1802 Playfair observed that the complex drainage network, the proportionality of the rivers to their valleys, and the "nice adjustment" between the slopes of the main stream and its tributaries made it "infinitely improbable" that each of the valleys was not the work of the stream that flowed in it. Since the acceptance of Playfair's deduction, elaboration of the generalization has proceeded in two ways. First, the concept of the cycle of erosion was introduced in part to describe successive stages in the erosional history of river valleys. Second, qualitative and, more recently, quantitative inquiries have been made into the processes operating through these successive stages. This investigation of the Brandywine Creek in Pennsylvania falls into the second category.

In the first approach William Morris Davis (1899) postulated a cycle of erosion in which he described a simplified sequence of stages in the life history of a region. Because of the normally slow rate at which processes of land sculpture proceed, the actual sequence of events in a cycle was inferred by comparing observations of different rivers and of the same river at different reaches. Thus, Davis's system is a qualitative description which may adequately describe a sequence of landforms, but it does not necessarily embody the physical processes that operate in the development of these forms.

In the second approach, fluvia processes, because of

their fundamental importance in the cycle of erosion, have received special emphasis. In considering the behavior of rivers, geologists and engineers have been largely concerned with the relationship of a number of variables which enter into river processes. Gilbert (1877) presented an excellent qualitative picture of the mechanics of natural streams in his classic discussion of land sculpture based on his observations on the relation among competence, capacity, declivity, velocity and quantity of water.

The idea of equilibrium was introduced in studies of natural river channels as early as the 16th century (Daussé, 1872, p. 340). Later morphologic studies by Baulig (1925) and Mackin (1948) and many others have been primarily concerned with channel equilibrium. The qualitative analyses of Baulig and Mackin on the interaction of several variables which determine the size, shape, and profile of a natural channel are an application and extension of the kinds of principles enunciated earlier by French and Italian engineers (Daussé, 1872, p. 340; Baulig, 1925, reprinted 1950, p. 43, 50).

In the last part of the present study the author has attempted to relate this idea of equilibrium to the observed behavior of a number of variables measured on Brandywine Creek, Pennsylvania.

To further our understanding of river morphology, qualitative analyses such as those of Baulig and Mackin must be supplemented by quantitative studies for several reasons. First, only a quantitative study shows the magnitude of the variables involved. Second, if their relative magnitudes are known, the choice of variables is simpler, as is the recognition of their importance in the process being considered. Third, the simultaneous action of a number of factors is extremely difficult to treat qualitatively. Fourth, quantitative methods permit hypotheses to be tested or verified either by repetition of the study or by controlled experiments.

Engineering problems of water supply and river control require measurements of streamflow and sediment content and have led to a large accumulation of quantitative data. Such data can be useful in studying fluvial processes and the sculpture of the land as Leopold and Maddock (1953) demonstrated in their work on the hydraulic geometry of river channels. Using data obtained from streamflow measurements, including measurements of the width, depth, velocity, discharge, and sediment load of the stream, they were able to describe quantitatively some features of the simultaneous adjustments of these variables by which an equilibrium may be maintained in natural rivers. This study provides the immediate background for the present work.

This study of Brandywine Creek is an attempt to describe the way in which a particular natural stream channel changes in profile and in cross section from its headwaters to its mouth. Measurements of the width, depth, velocity, slope, suspended load, and size of bed material of the stream furnish the basis for a quantitative description of these changes. These measurements were specifically designed to test the applicability of the methods developed by Leopold and Maddock to a stream considerably different from the rivers upon which the techniques were developed. The data on Brandywine Creek, in addition, provide means for an independent test of the validity of some of the generalizations on natural channels. Because the stream is small enough to be studied in detail, the author has been able to study in particular the extent to which variations within individual reaches of the channel affect the more general relationships described by Leopold and Maddock.

Previous studies utilizing standard stream-gaging records have lacked measurements of slope of the river, which is not ordinarily measured at gaging stations. In understanding fluvial processes, however, this information may be important. On Brandywine Creek measurements were made of the slope as well as the depth and velocity of the water. From these known quantities a hydraulic roughness of the channel was computed. Geologic factors such as grain size and hardness which interact with the hydraulic variables to produce a particular natural channel behave essentially as friction or roughness parameters. The correlation of the computed roughness with the geological characteristics of the terrain in this study has been limited to measurements of the grain size of material on the bed of the stream. Unfortunately, in regard to other geological variables such as load, this investigation is severely limited by the paucity of measurements of suspended and bed loads. This deficiency is recognized by the author but practical problems made additional measurements of the load impossible.

The study is divided into three parts. The first is a description of the changes which take place in a given reach of the channel as the volume of the flow changes. The second is a discussion of the way in which the hydraulic and geologic variables change in the downstream direction at a given flow. The last is an attempt to relate these progressive downstream changes to the concept of equilibrium in a natural river. Although the measurements in parts one and two may have some intrinsic value of their own, their primary significance here is as a means of illustrating the condition of equilibrium. In part one the study of the behavior of the variables at a given station is based primarily upon data from seven reaches on the Brandywine Creek that

were studied in detail. The changes which take place in the downstream direction are based upon comparisons of nine major stations as well as upon a great many more cross sections at which measurements were made. Such a study of the entire river requires extensive coverage of the drainage basin. At the same time, however, because it is desirable to obtain data on the way in which width, depth, velocity, slope and roughness change with increasing discharge at a given station, the choice of the specific locations for study and hence the extent of the coverage are partially governed by the necessity of having accurate measurements of the discharge.

The author is well aware that in a quantitative study of fluvial processes too much emphasis can be placed upon mathematical relationships to the exclusion of an understanding of their physical meaning. Avoidance of this danger is made easier by the fact that the data on Brandywine Creek rarely permit such an elaborate mathematical treatment.

#### THE AREA OF STUDY: BRANDYWINE CREEK GEOLOGIC AND GEOGRAPHIC SETTING

To fulfill the aims of this study the river chosen for investigation should meet three principal specifications. First, it must traverse a region of heterogeneous rocks in order to preclude the possibility that any regularity in its behavior could simply be explained as a result of especially uniform and thus unrepresentative conditions. Second, it must be a single representative river rather than individual reaches on a number of streams. Third, if one is to test the general applicability of the relationships derived by Leopold and Maddock, the river must be in a region different in climate and physiography from the regions represented by the rivers studied by them. These considerations suggested the selection of a river typical of the Middle Atlantic region of the eastern United States. Although there are no simple fixed characteristics of such streams, they are customarily envisioned as streams which flow southeast across the Appalachian Valley and Ridge province and across the Piedmont. In their lower reaches most of them cross the Fall Line, or Fall Zone, where the basement rocks have been exposed by erosion of the sediments of the Atlantic Coastal Plain. Many of the characteristic rivers of this region flow diagonally across or at right angles to the alternating bands of resistant and weak rocks which strike northeast. They do not flow on homogeneous material.

Brandywine Creek in southeastern Pennsylvania fulfills these basic requirements. This creek has a drainage area of 314 square miles and an overall length of about 45 miles from its headwaters to its

mouth below Wilmington, Del. (fig. 1). The drainage basin lies almost entirely within the Honeybrook, Phoenixville, Coatesville, and West Chester quadrangles in Pennsylvania.

At its highest elevation, approximately 900 feet above sea level on the southeast slope of Welsh Mountain, Brandywine Creek is perennially fed by springs. Most of the major tributaries throughout the drainage basin can be traced to their sources in springs that reach the surface at the apices of shallow, amphitheaterlike bowls on the uplands. Flowing through an unglaciated area from Welsh Mountain to its mouth near the junction of the Christina and Delaware Rivers, the Brandywine crosses a number of different types of rock (fig. 2). After traversing a rolling upland of metamorphosed sediments and intrusive rocks, the stream successively crosses the North Valley Hills, the Chester Valley, and the South Valley Hills. The first of these is a northeast-striking anticline with a resistant quartzite core; the valley itself is composed of Ordovician limestone and dolomites. On the southern flank of the valley there is a belt of phyllite. The Brandywine then traverses an area underlain by schist. Above Chadds Ford the course of the stream through the schist is interrupted by a belt of gneiss, and about 4 miles above the city of Wilmington the stream encounters the Fall Zone in an area underlain by gabbro. The Coastal Plain sediments are not reached until the river nears its mouth. The passage of the Brandywine through Wilmington is obstructed by numerous low dams and the very lowest portion at Wilmington Harbor is subject to tidal influence. This study is concerned only with that part of Brandywine Creek which lies above the Fall Line.

Figures 3-9 show the characteristic vegetation of the region bordering the river channel. Although trees are found adjacent to the channel, most of the flood plain of Brandywine Creek is in permanent grass pasture. The drainage basin as a whole is farmland, about a third of which is covered by woodland. The average annual rainfall, which is distributed rather evenly throughout the year, is about 47 inches (U. S. Dept. of Agriculture, 1941, p. 1087).

In addition to the theoretical requirements which Brandywine Creek satisfies, as do many other streams in the eastern United States, it also fulfills a number of important practical requirements: 1. It has a relatively small drainage area. 2. The creek itself is small and shallow enough at most stages for measurements of flow to be made by wading. 3. A permanent recording stream gage has been in operation since 1932 at Chadds Ford. This relatively long and continuous record of discharge is required in comparing flows of

THE NATURAL CHANNEL OF BRANDYWINE CREEK, PENNSYLVANIA

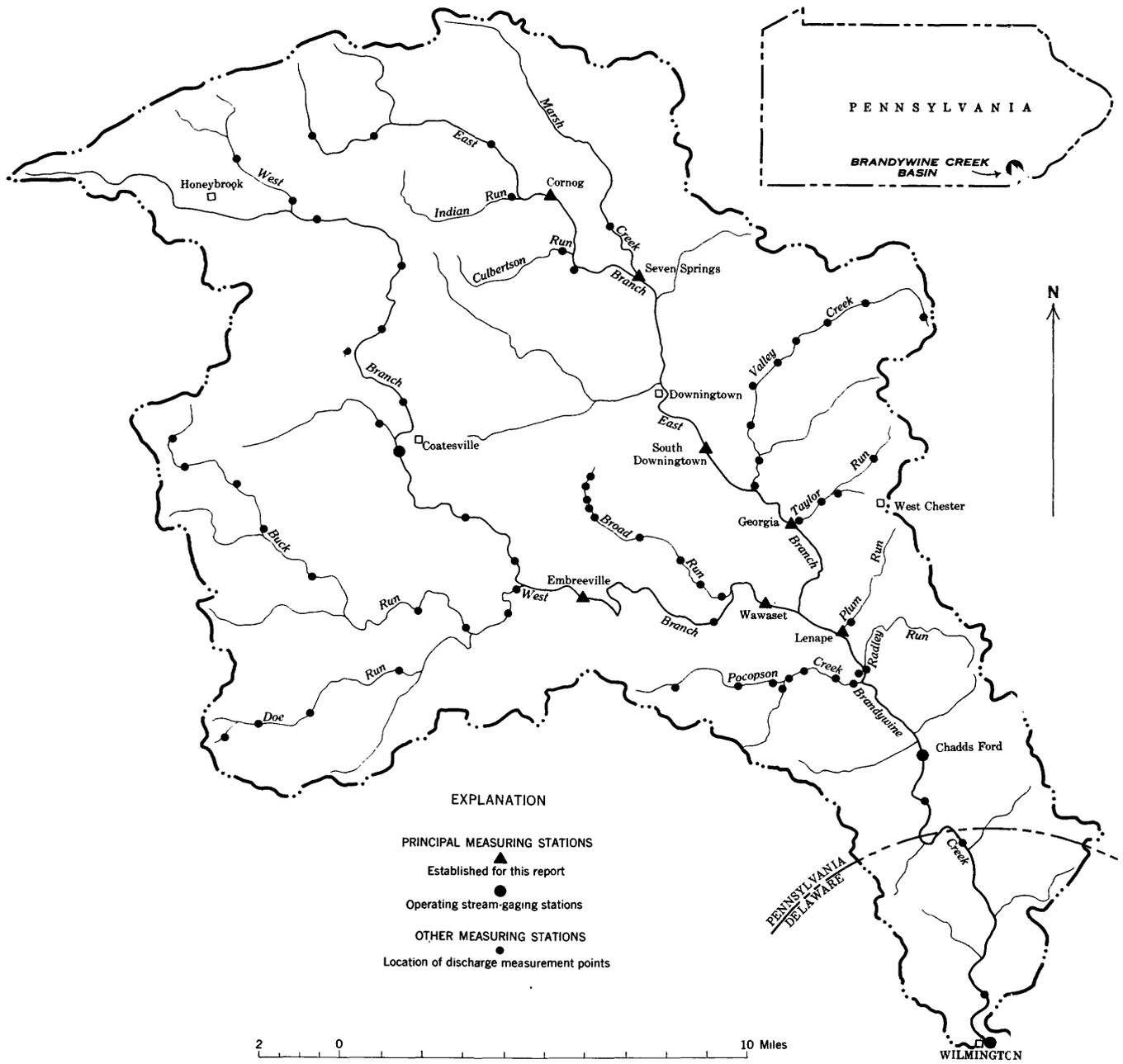


FIGURE 1.—Map of the drainage basin of Brandywine Creek, Pennsylvania, showing major tributaries, measuring stations established for this study, gaging stations of the U. S. Geological Survey, and locations of discharge measurements on tributaries. Inset map shows location of Brandywine Creek in southeastern Pennsylvania.

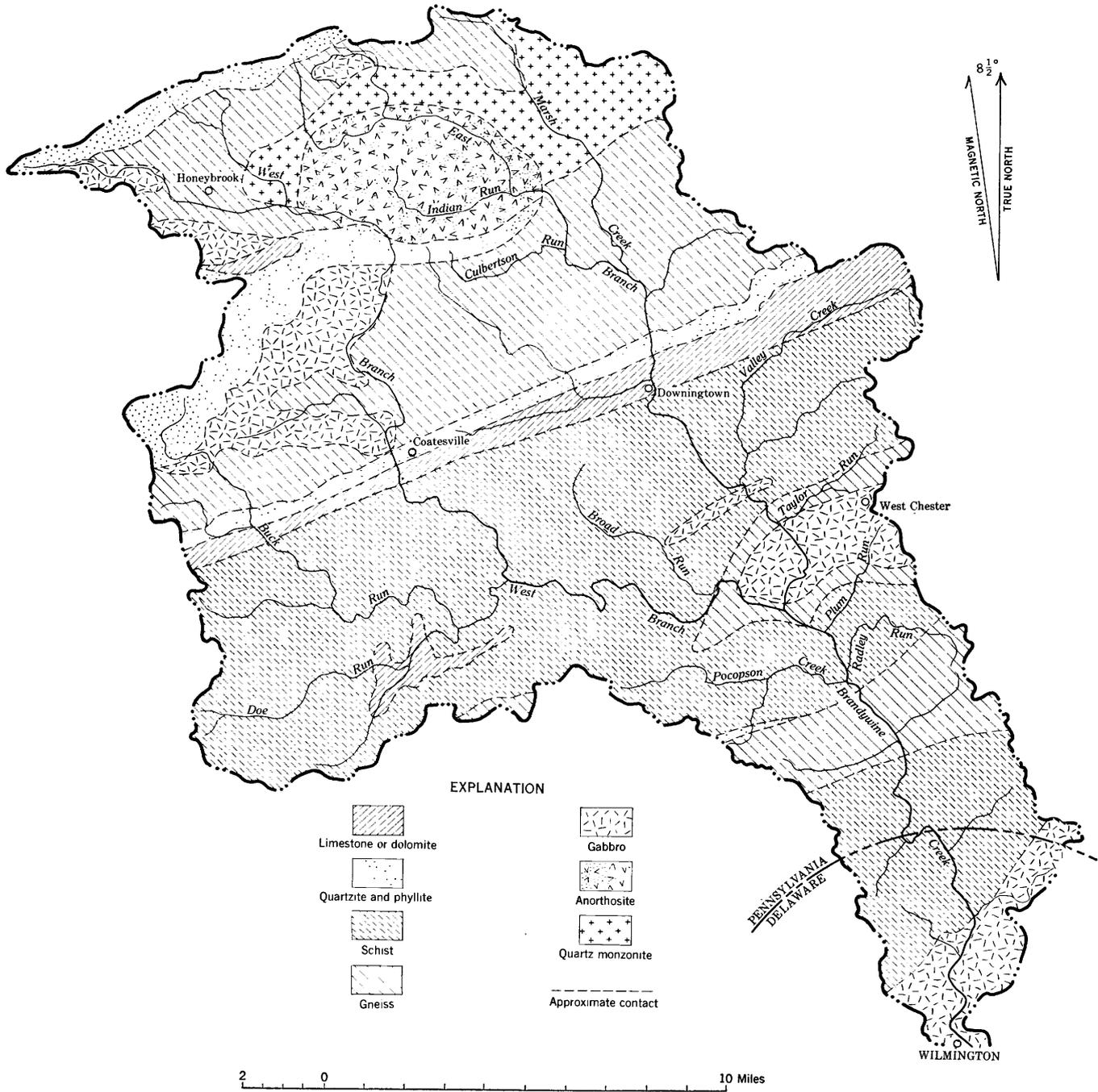


FIGURE 2.—Generalized geologic map of drainage basin of Brandywine Creek, Pennsylvania, illustrating the diverse lithology (after Stose and Ljungstedt, 1931).

different durations or frequencies. 4. There is a sediment sampling station on Brandywine Creek at Wilmington, Del. Such sampling stations are rare in Eastern United States. 5. The area is readily traversed by automobile, a fact which makes it possible to obtain measurements of flow in periods of high water.

#### CHOICE OF VARIABLES

In an investigation of the river channel both the hydraulic and the geologic factors that determine the shape and longitudinal profile of the channel must be considered. A study of the geological aspects alone, such as the distribution of hard and soft rocks, or the grain size, shape, and sorting of bed material in the channel is insufficient. Although such studies describe the physical features of the channel, they shed very little light on the mechanism by which those features are produced. Similarly, a purely theoretical study of the hydraulics of an open channel cannot take into account the variety of modifications of any hypothesis which the geological setting requires. In the present work an attempt to bridge this gap was made by measuring some of the hydraulic variables in the field.

As one might suppose, it would be impossible to consider every variable. However, a theoretical minimum number of variables must be considered. The methods of the present investigation and the reasons that specific data were collected are best explained in terms of the requirements of the hydraulic study and the practical problems involved in measuring the necessary variables in the field.

This study deals with the flow of water in open channels. Such flow can be described in simple form by the familiar Chezy formula,

$$v = C\sqrt{Rs} \quad (1)$$

where  $v$  equals the mean velocity;  $R$  the hydraulic radius, defined as the area of the wetted cross section divided by the wetted perimeter;  $s$ , the slope of the energy gradient, in steady uniform flow equal to the slope of the water surface; and  $C$ , a coefficient of resistance. Because the coefficient  $C$  is not a constant, in the numerical evaluation of the three major variables  $C$  must also be evaluated. The Manning formula is one among many developed to overcome this difficulty (Ganguillet and Kutter, 1893; Houk, 1918; Schnackenburg, 1951). In this formula

$$v = \frac{1.486}{n} R^{2/3} s^{1/2} \quad (2)$$

where  $n$  is a measure of the boundary roughness of the bed and banks of the channel. From these basic equations it is clear that at least four major variables

must be measured or computed to provide the bare essentials in any analysis of the flow of water in a given reach of the channel.

Standard stream-gaging measurements (Corbett and others, 1945) include data on the width, area of cross section, depth, and velocity of the flow. The discharge is computed from the measurements of the area and velocity. The mean depth as used here is the quotient of the cross-sectional area of the flowing water divided by the top width of the water surface. Mean velocity is equal to the discharge divided by the area of the cross section. For relatively wide sections the mean depth is approximately equal to the hydraulic radius. On Brandywine Creek, more than 160 discharge measurements have been made. These include measurements of the fundamental parameters of width, depth, and velocity at approximately 50 different locations along the creek.

The measurement of another principal variable, the slope of the water surface, is much more difficult, however, and is not a customary stream-gaging procedure. In order to measure this variable on Brandywine Creek, seven stations (fig. 1) were established on the main stream. At these sections not only the width, depth, and velocity were measured at different discharges, but also the slope of water surface.

#### REQUISITES OF A MEASURING REACH

The discharge at any given point is a function of the contributing drainage area. Comparison of the measured variables in the downstream direction with respect to discharge therefore requires a range of discharges and hence a choice of measuring reaches representing a range of sizes of contributing drainage areas. The seven major measuring reaches chosen for detailed study (fig. 1) have contributing drainage areas ranging from 25 square miles at Cornog to 259 square miles at Lenape (app. A). Their distribution spans a relatively large part of the total length of the stream.

Within a selected region the choice of the specific measuring reach, or length of channel, is more difficult. The necessity for accurate discharge measurements presents many criteria which must be met or compromised in the selection of each reach. On Brandywine Creek, these criteria include: 1. A natural cross section of the channel which could be waded under normal conditions of flow. 2. A straight reach of the channel which would permit a reasonably accurate determination of the slope of the water surface. 3. The presence of a bridge at or very near the natural cross section from which discharge measurements could be made at times of high water when wading was impossible. 4. A reach in which the velocity was great enough



FIGURE 3.—Measuring reach at Cornog. *A*, Upstream view; flow shown is equaled or exceeded about 30 percent of the time. *B*, Downstream view; flow shown is equaled or exceeded less than 1 percent of the time. Note large boulders not submerged until relatively high flow is attained.



FIGURE 4.—Measuring reach at Seven Springs looking downstream from bridge. *A*, Low flow that is equaled or exceeded about 17 percent of the time and that is similar to flow used in comparison of downstream changes in figure 25. *B*, High flow that is equaled or exceeded less than 1 percent of the time.



FIGURE 5.—Measuring reach at South Downtown. View upstream at low flow from lower end of reach. At right is characteristic flood plain.



FIGURE 6.—Measuring reach at Embreeville at discharge equaled or exceeded about 20 percent of the time. Downstream view.



FIGURE 7.—Measuring reach at Georgia looking downstream from bridge. Shows approximate range measured at the principal stations. *A*, Low flow; *B*, high flow.



FIGURE 8.—Measuring reach at Wawaset looking downstream from bridge at flow equaled or exceeded 15 percent of the time.

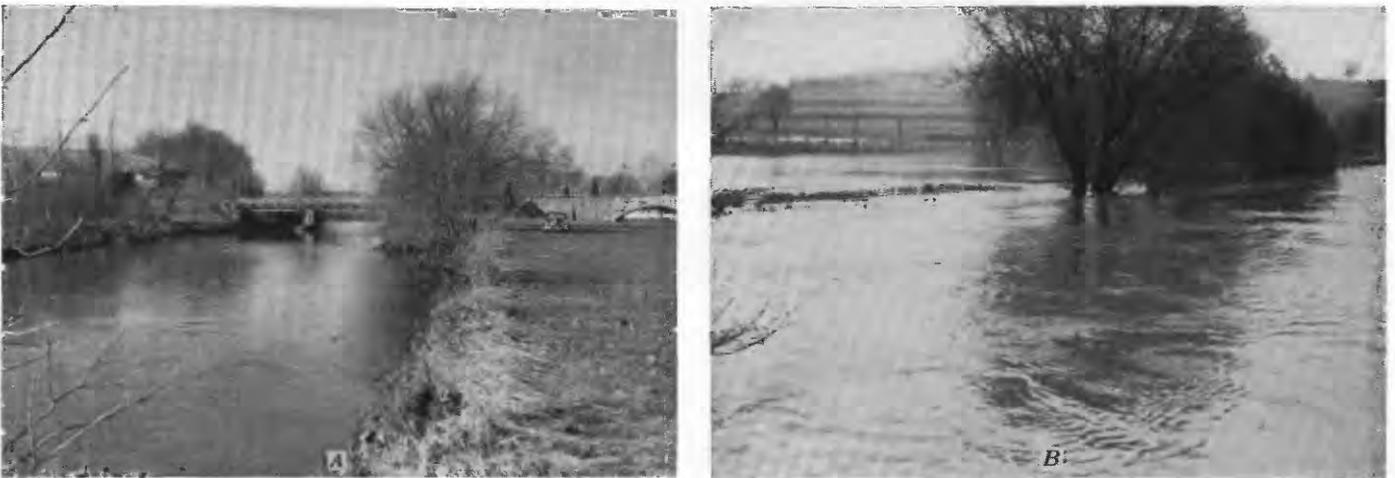


FIGURE 9.—Measuring reach at Lenape. *A*, View downstream at low flow when mean depth is about 1 foot deeper than lowest depth shown in figure 13. *B*, View downstream from bridge at high flow. Tree surrounded by water is near car in *A*. Note broad flood plain.

to permit an accurate determination of the discharge by means of the standard current meter. 5. A reach which was relatively accessible by automobile in rainy weather to allow the stream gager to visit as many stations as possible during a single rainstorm. 6. A reach at which a reasonable correlation between discharge and elevation of the water surface could be obtained.

It was impossible to satisfy all of these requirements at each of the gaging locations. Because only natural cross sections are required for this study, sensitive controls are lacking at many reaches. At Cornog, for example, the stage-discharge relation is poor or only fair by gaging-station standards. The gaging section at Georgia was chosen because of its drainage area, accessibility, and proximity to the forks of the river. The stage-discharge relation at Georgia is excellent because of the presence below the gage of a gravel riffle which acts as a control. Measurements of low discharges at this natural section however, were hampered by the low velocity of flow in the pool. Also, conditions in the pool may be unrepresentative of the reach as a whole. Photographs (figs. 3-9) of the river channel at the principal measuring sections show how diverse the individual stations are. How much of this diversity is related to the upstream or downstream position of the gaging locations along the longitudinal profile, and how much is simply a function of the local variations at the specific section at which the measurements were taken are discussed later under "Variability of the cross section in a short reach." The original spacing, on the basis of drainage areas, determined the position of the gaging sections in relation to the longitudinal profile. The specific criteria previously listed fixed the exact location of the measuring reach within the prescribed segment of the profile. The distinction between the degree of variation of the channel within a local area and the total variation in the downstream direction is an important aspect of the present investigation.

Despite the criteria which had to be met in their selection, the principal measuring stations should not be considered atypical of the Brandywine as a whole. Possibly somewhat straighter than most reaches, they are otherwise indistinguishable from adjacent parts of the stream.

#### MEASUREMENTS OF THE STREAM

The measurement of the discharge and of the slope of the water surface at each of the principal measuring sections consisted of the following steps: 1. The staff gage nearest to the measuring section was read. 2. A measurement of the discharge was obtained by use of a

current meter to measure the velocity of flow. Measurements were generally made by wading and were always made at precisely the same cross section at each station. 3. The height of the water surface at each end of the reach was determined from the gages at the upstream and at the downstream ends of the reach. 4. At high flows when wading was impossible, measurements of the discharge were made from bridges nearby (figs. 7-9). At these times the elevation of the water surface at the regular wading section was marked and the area of this cross section measured after the water had receded. The slope of the water surface was computed by dividing the difference in elevation between upstream and downstream gages by the measured distance between the two gages. In addition to these standard observations, measurements of the peak stage at each of the seven stations were obtained from peak-stage recorders.

By using the measurements of width, mean depth, mean velocity, and slope of the water surface, a roughness parameter  $n'$  was computed from a modified Manning equation. The quantity  $n'$  computed here is not equivalent to Manning's  $n$ . This distinction is discussed later. For each discharge measurement, then, four variables have been measured and one computed. The results of these measurements are listed in appendix A.

Accurate measurements of the water surface slope are extremely difficult to make (Houk, 1918, p. 78). First, the difference in elevation may be very small, and second, the water surface slope often changes within very short distances along the thalweg. Although no high order of precision is claimed for the measurements of slope listed in appendix A, they are reasonable and consistent and provide a satisfactory description of the range and relative magnitudes of the slopes concerned in this study. The values of  $n'$  have additional importance because there are few rivers on which measurements have been obtained that permit comparisons to be made of the changes taking place in any roughness parameter in the downstream direction.

On occasions when current-meter measurements could not be made, weighted floats were used to measure the velocity of the flowing water. Correction factors for the velocity obtained when the floats were used were later derived by comparing velocities of the floats with the mean velocity determined by use of the current meter. Measurements in which floats were used are given a different symbol in figures 10-13.

In addition to the seven measuring sections, established for this study, two river gaging stations have been operated by the U. S. Geological Survey on Brandywine Creek at Coatesville and at Chadds Ford. The recording gage at Chadds Ford gives a continuous

record of the height of the water surface. At Coatesville, from 1943 to 1951, when the station was discontinued, the height of the water on a graduated staff gage was read periodically by an observer. At both stations rating curves have been established and cover a range of river stages.

On Brandywine Creek, analyses have also been made of the bed and bank materials. Some measurements of the suspended-sediment load have also been made which, when supplemented by information obtained from other rivers, may be used in the analysis.

#### HYDRAULIC AND GEOLOGIC CONDITIONS WITHIN SELECTED REACHES

Before the behavior of the hydraulic parameters at each of the principal stations is described, one example will be presented to show the characteristic at-a-station changes of all of the variables. Because the behavior of the variables with changing discharge at Cornog is typical of their behavior at the other stations, it will be used as the illustrative example.

The hydraulic variables are treated in accordance with the methods developed by Leopold and Maddock (1952). At each station the width, depth, velocity, slope, and roughness parameter are plotted against discharge on log-log paper. Each variable is thus expressed as a function of discharge. When expressed in this way the data at a given station are remarkably consistent.

#### THE REACH AT CORNOG: AN EXAMPLE

Figure 10 shows the changes in the variables which take place at Cornog as the discharge increases. On log-log paper, the slope of the straight lines drawn through the plotted points is the value of the exponent in the equation expressing the relationship between the two variables. Thus  $b$ ,  $f$ ,  $m$ ,  $z$ , and  $y$  (fig. 10) are the exponents in the following equations:

$$w = aQ^b \quad (3)$$

$$d = cQ^f \quad (4)$$

$$v = kQ^m \quad (5)$$

$$s = tQ^z \quad (6)$$

$$n' = rQ^y \quad (7)$$

In these equations  $Q$  is the discharge in cubic feet per second;  $w$  is the width, in feet;  $d$ , the depth, in feet;  $v$ , the velocity, in feet per second;  $s$ , the slope of the water surface; and  $n'$ , a roughness factor;  $a$ ,  $c$ ,  $k$ ,  $t$ , and  $r$  are constants. At Cornog, values measured from the graphs of figure 10 are  $b = .04$ ,  $f = .40$ ,  $m = .52$ ,  $z = 0.0$ , and  $y = -.23$ . At the point where  $Q = 1$ , the width, depth, velocity, slope, and roughness equal the constants  $a$ ,  $c$ ,  $k$ ,  $t$ , and  $r$  respectively. In the cross section at

Cornog, the relationship of each variable to discharge is given by these equations:

$$w = 37Q^{.04}$$

$$d = 17Q^{.40}$$

$$v = 16Q^{.52}$$

$$s = .0032, \text{ that is, slope remains very nearly constant}$$

$$n' = .15Q^{-.23}$$

$$\text{and } b + f + m = 0.96$$

It has been shown (Leopold and Maddock, 1953, p. 8) that because

$$Q = Av = wdv$$

$$\text{and } w = aQ^b \quad (3)$$

$$d = cQ^f \quad (4)$$

$$v = kQ^m \quad (5)$$

$$\text{then } Q = aQ^b \times cQ^f \times kQ^m$$

$$\text{whence } b + f + m = 1$$

The experimental result at Cornog is in close accord with the theoretical expression.

The slope of the water surface in the measuring reach remains practically constant as the discharge increases. Similarly, the width changes only very slightly as the discharge increases at Cornog. Both the depth and the velocity, however, increase rapidly with increasing discharge at a given cross section, while at the same time the roughness decreases. As the discharge at Cornog rose from 5 to 830 cfs, an increase of 166 times, the depth increased approximately 7-fold and the velocity increased 30-fold. At the same time the roughness factor was reduced by only half. These figures indicate that these hydraulic variables change little compared with the discharge and the load, both of which are extremely variable at any given section, as will be shown.

The relative magnitude and the direction in which the changes in the variables at Cornog take place are characteristic of the major cross sections studied on Brandywine Creek (figs. 10-13). Very little scatter appears on any of these graphs. In contrast, the curves drawn for Coatesville (fig. 14), and Chadds Ford (fig. 15), based on stream measurements made in the vicinity of the gaging stations, show wide scatter. This scatter is characteristic of most of the curves drawn by Leopold and Maddock (1953) from similar data and has raised some doubt as to the validity of the relationships expressed by their curves. However, these relationships appear to be justified if the methods of measuring discharge are taken into account.

At the author's seven measuring sections, all discharge measurements were related to a single cross section. At Chadds Ford, Coatesville, and other gaging stations, discharge measurements may be made at

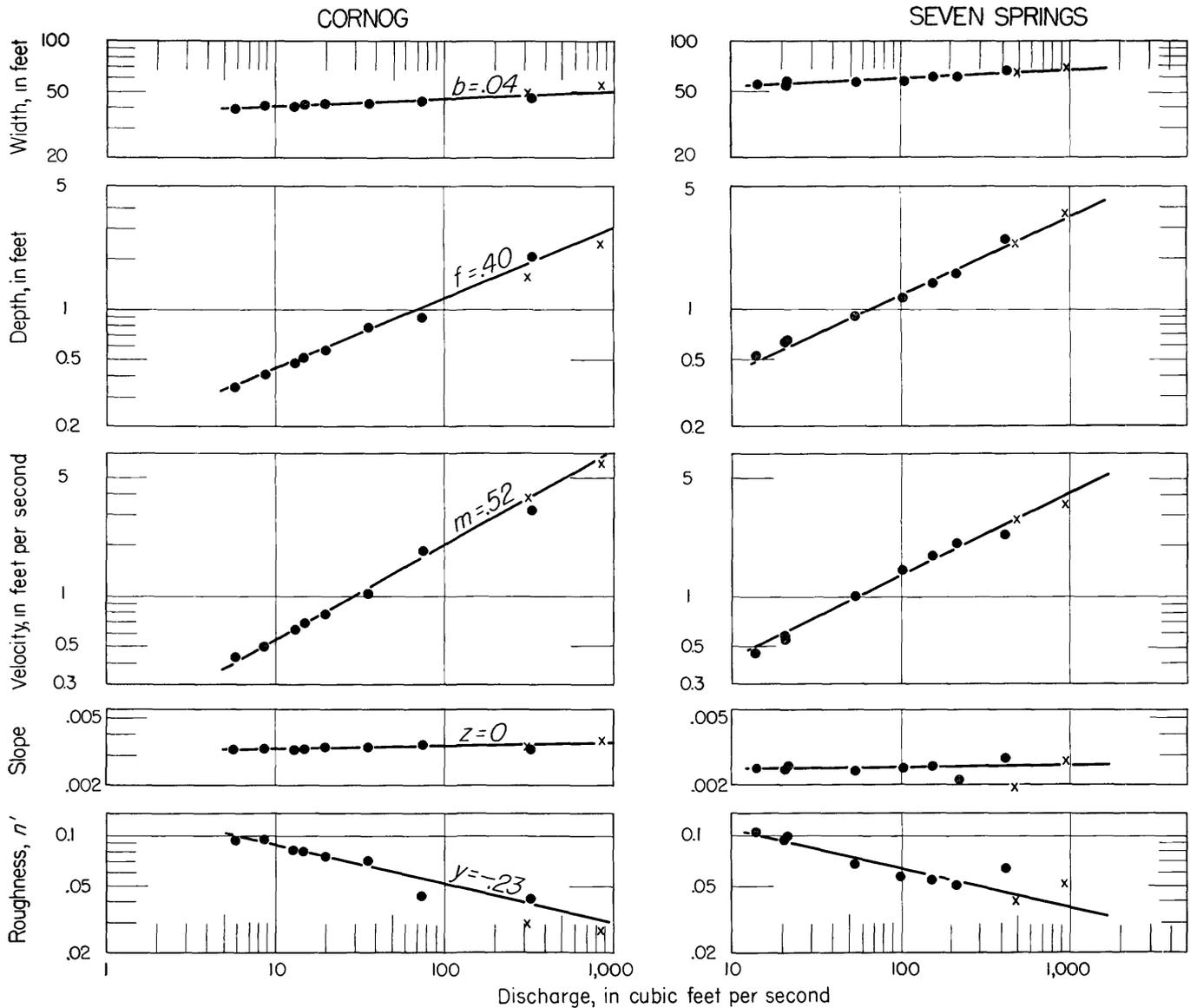


FIGURE 10.—Relation of width, mean depth, mean velocity, water surface slope, roughness parameter  $n'$  to discharge at Cornog and Seven Springs. Slopes of curves designated  $b, f, m, z,$  and  $y$  give values of exponents in equations relating variable to discharge. Symbol  $\bullet$  denotes velocity measurements made by current meter. Symbol  $X$  indicates velocity measured by floats.

various points in the vicinity of the station. A discharge measurement made at low flow in a narrow cross section has a velocity that may far exceed that of a much higher flow measured at a wider cross section. A study of the discharge records and of cross sections of the channel at Chadds Ford and Coatesville shows no significant difference between these and the seven other sections studied in detail in this investigation. The scatter on at-a-station curves is virtually eliminated when all of the measurements are made at the same cross section. This explanation of the cause of scatter is not new, but it is useful to anyone interested in interpreting the data in this or other studies that make use

of regular discharge measurements taken at gaging stations.

**WIDTH, DEPTH, AND VELOCITY**

The curves relating width to discharge (figs. 10-13) indicate that at most locations on Brandywine Creek the width remains remarkably constant as the discharge increases. With the exception of Chadds Ford and Coatesville the value  $b$  (table 1) is less than .1 at each of the sections. These low values of  $b$  reflect the rectangularity of the cross sections of the Brandywine (fig. 16).

The mean  $b$  value obtained by Leopold and Maddock

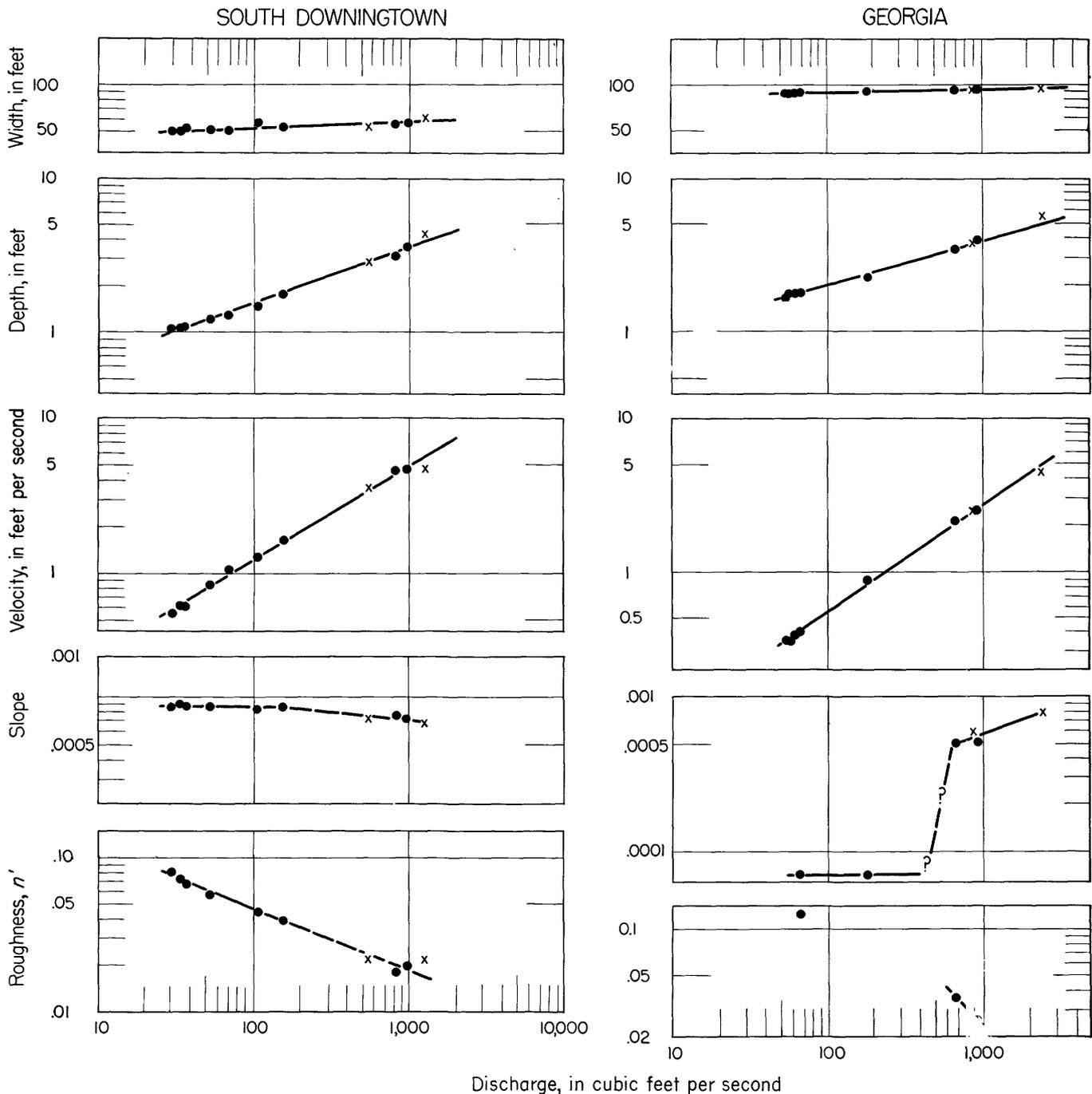


FIGURE 11.—Relation of width, mean depth, mean velocity, water surface slope, and roughness parameter  $n'$  to discharge at South Downingtown and Georgia. Symbol ● denotes velocity measured by current meter. Symbol X denotes velocity measured by floats.

(1952, p. 9) was .26. There is a strong suggestion that the higher  $b$  values result from the fact that the measurements are not all taken at the same cross section. It is also true, however, that the western streams toward which the Leopold and Maddock data are heavily weighted, often have less rectangular or more ellipsoidal channels. Some regional factors therefore may influence the rate of change of width with discharge.

The rates of change of depth and of velocity with increasing discharge at the principal measuring sections are shown in figures 10–13 and in table 1, where  $f$  is the rate of change of depth and  $m$  the rate of change of velocity. The curves indicate that both of these variables increase at all of the cross sections. In addition, the points on the graphs show that the two variables are closely related. Although at Georgia

the velocity increases rapidly at the expense of depth, at all of the other stations the rates of change of depth and velocity are similar to those described at Cornog. With the exception of Lenape where the rates of change of the two are equal, the  $f$  and  $m$  values (table 1) show that the rate of increase of velocity slightly exceeds the rate of increase of depth at the stations on Brandywine Creek.

At each station the range of discharges represent stages from very near the bed to within a foot of the bankfull stage. There is a suggestion in some of the data (figs. 10-13) that the depth-discharge and velocity-discharge curves may actually plot as curved rather than straight lines on log-log paper. Such a relation-

ship of the at-a-station curves is not uncommon. At present, however, the Brandywine data are not sufficient to justify drawing a curve through the points, and a straight line is used because it simplifies the general analysis.

The mean  $f$  value of .41 at the seven measuring stations established on Brandywine Creek for this study is practically the same as the mean of .40 found by Leopold and Maddock (1953, p. 9). The mean  $m$  of .55 from the Brandywine data, however, is considerably greater than .34 value found in their general study. These differences may or may not be greater than can be accounted for by the scatter alone.

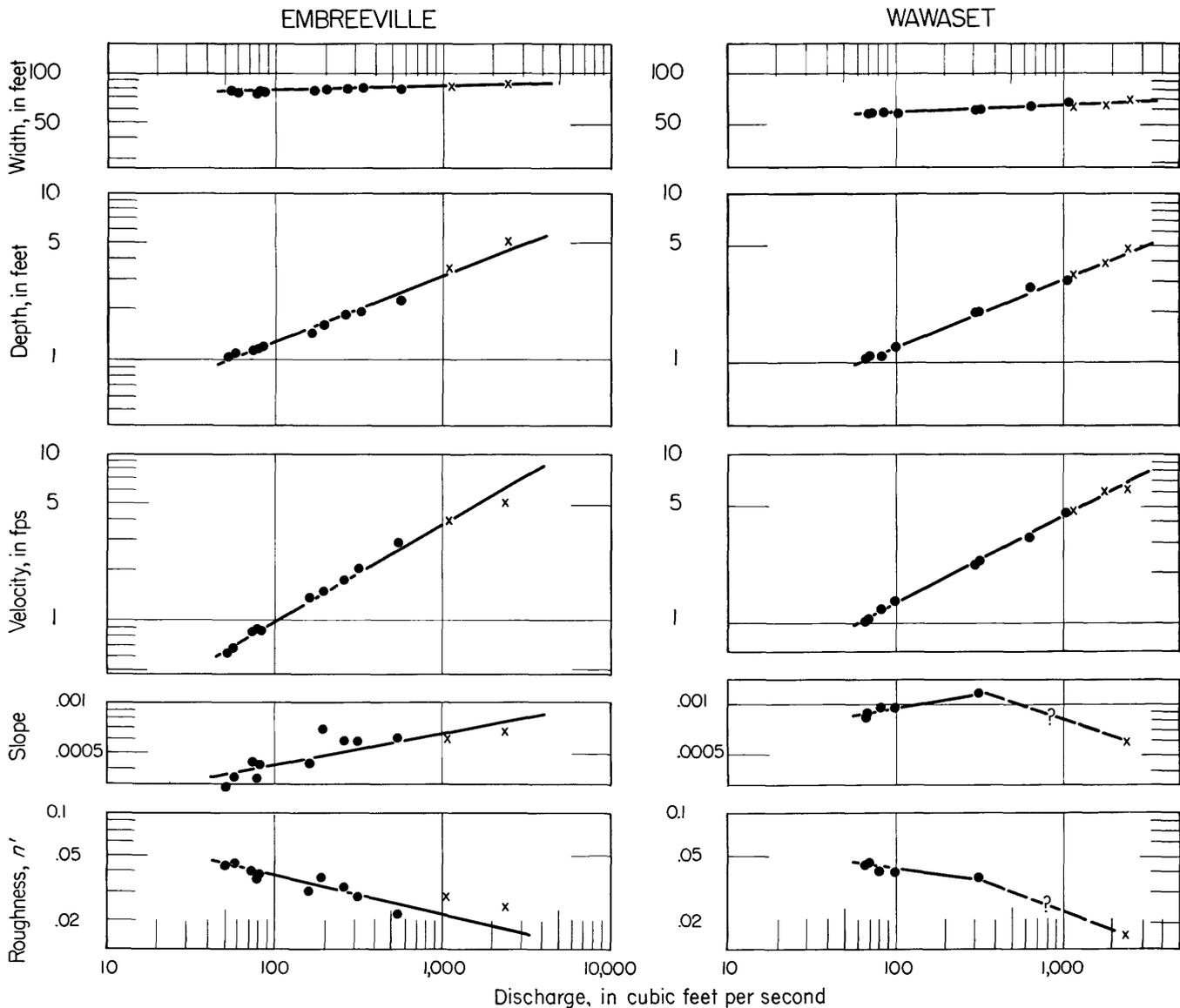


FIGURE 12.—Relation of width, mean depth, mean velocity, water surface slope, and roughness parameter  $n'$  to discharge at Embreeville and Wawaset. Symbol ● denotes velocity measured by current meter. Symbol X denotes velocity measured by floats.

TABLE 1.—Summary of data showing rates of change of width, depth, velocity, slope, roughness, and suspended load in relation to discharge at the principal stations on Brandywine Creek

Station	<i>b</i>	<i>f</i>	<i>m</i>	<i>b+f+m</i>	<i>z</i>	<i>y</i>	<i>j</i>
Cornog.....	.04	.40	.52	0.96	<sup>1</sup> 1.00	— .23	-----
Seven Springs.....	.05	.45	.48	0.98	<sup>1</sup> 1.00	— .22	1.53
South Downingtown.....	.04	.36	.61	1.01	— .03	— .40	1.41
Georgia.....	<sup>1</sup> .00	.32	.69	1.01	<sup>1</sup> .03	<sup>1</sup> — .10	2.65
Embreeville.....	<sup>1</sup> .00	.42	.59	1.01	.19	— .22	1.70
Wawaset.....	.05	.42	.53	1.00	<sup>2</sup> .15	<sup>2</sup> — .15	1.30
Lenape.....	.08	.46	.46	1.00	.00	— .20	2.25
Mean.....	.04	.41	.55	1.00	.05	<sup>3</sup> — 0.20	1.88
Chadds Ford.....	.16	.45	.42	1.03	-----	-----	-----
Coatesville.....	.22	.30	.48	1.00	-----	-----	-----
Wilmington.....	-----	-----	-----	-----	-----	-----	2.37

THE HYDRAULIC FACTORS ARE EXPRESSED BY THESE FORMULAS:

$$w = aQ^b$$

$$d = cQ^f$$

$$v = kQ^m$$

$$s = tQ^z$$

$$n' = rQ^y$$

$$L \text{ (suspended load)} = pQ^j$$

<sup>1</sup> For high discharges only.    <sup>2</sup> For low discharges only.    <sup>3</sup> Georgia not included.

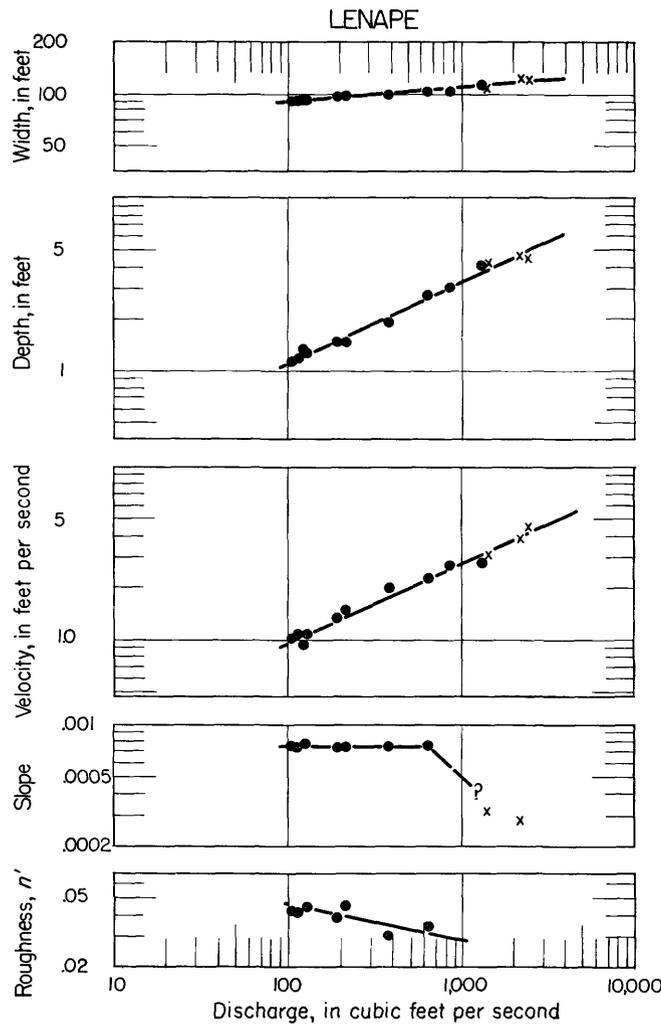


FIGURE 13.—Relation of width, mean depth, mean velocity, water surface slope, and roughness parameter *n'* to discharge at Lenape. Symbol ● denotes velocity measured by current meter. Symbol X denotes velocity measured by floats.

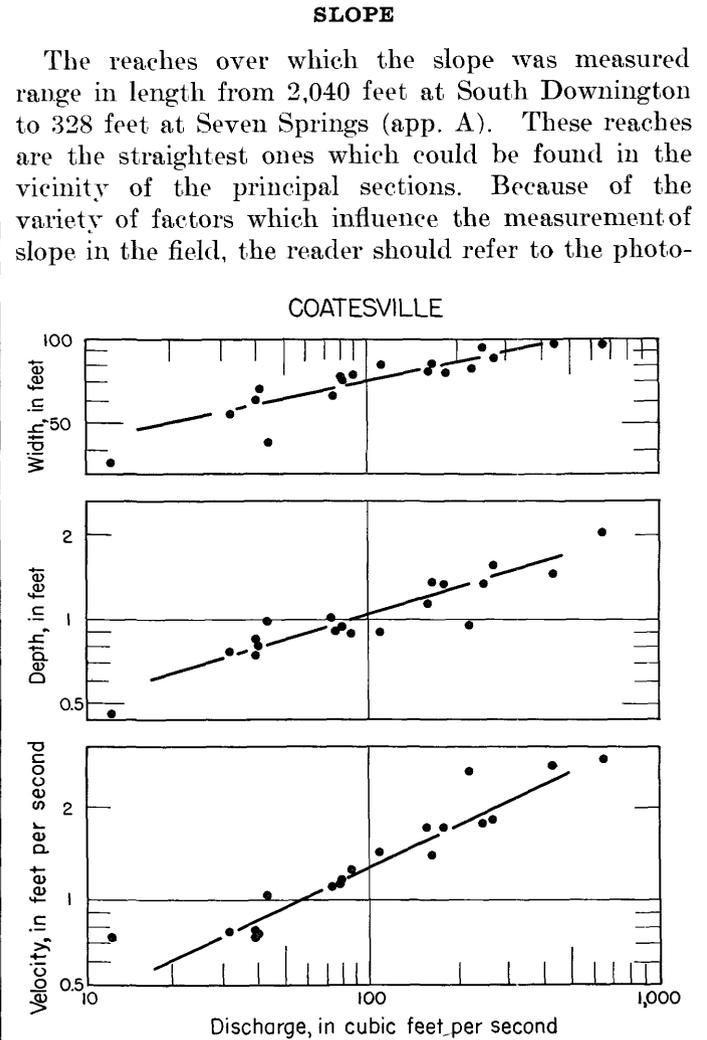


FIGURE 14.—Relation of width, mean depth, and mean velocity to discharge at Coatesville. Data from stream-gaging records.

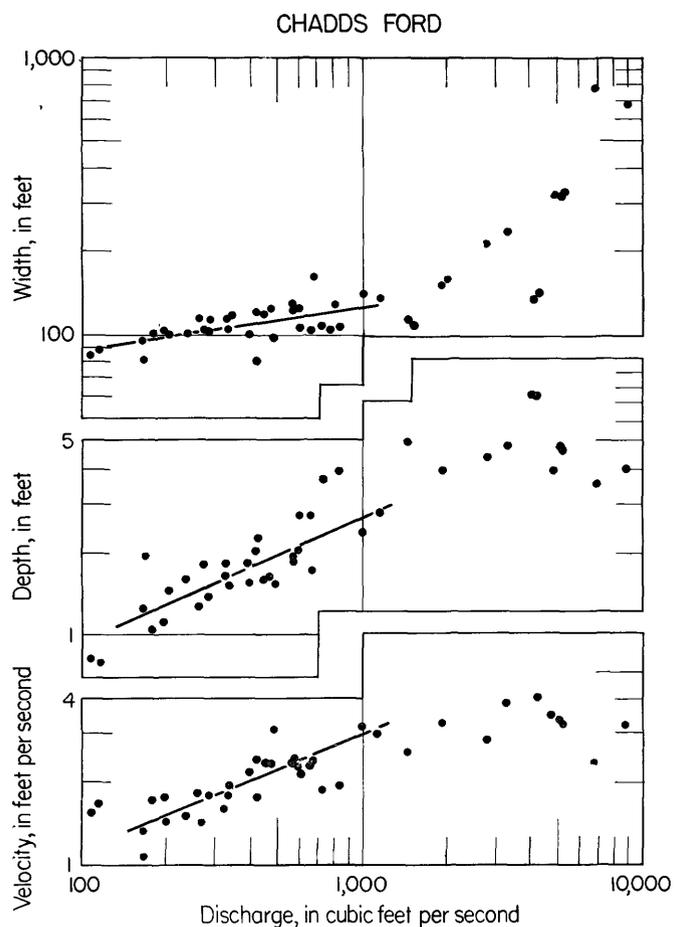


FIGURE 15.—Relation of width, mean depth, and mean velocity to discharge at Chadds Ford. Data from stream-gaging records.

graphs (figs. 3–9) and to the data on local variability given later in the discussion of pools and riffles. Certain general relationships common to all of the reaches and significant peculiarities of individual reaches may be appropriately considered here.

The criteria used in selecting the specific reaches at which the measurements of the water surface were made have already been set forth. To obtain comparable conditions at each of the stations, an attempt was made to select reaches which were, to the eye at least, neither pools nor riffles. Specific attempts to set up objective criteria by which pools and riffles could be distinguished are discussed later.

Generally, in a pool having a section control, as the discharge and stage increase, the slope of the water surface increases; while in a riffle the slope decreases as the stage increases until the section control has been drowned out. The curves of slope plotted against discharge (figs. 10–13) show that reasonable uniformity was obtained in the selection of the individual reaches. At Lenape the slope remained relatively constant throughout the measured range of discharges. A small

increase in the slope of the water surface with increasing discharge was recorded at Seven Springs and at Cornog; at Embreeville the slope of the water surface increased relatively rapidly with increasing discharge. The marked increase in the slope of the water surface at Embreeville is due in part at least to the presence of a minor riffle about 50 feet below the gage at the upstream end of the reach. At low flow this riffle causes a break or step in the water surface between the two gages. After this riffle is drowned out at a discharge of about 150 cfs, the rate of change of slope becomes less.

The curve for Wawaset in figure 12 shows that the water surface slope increases in the lower part of the curve as the discharge increases, but at the highest discharge the slope is markedly reduced. This and the fact that the stage increases very rapidly for small changes in discharge at this point suggest that backwater conditions from the bridge piling and debris may be causing the water elevation to be unduly high at the downstream gage (Corbett and others, 1945 p. 132; Ramser, 1929, p. 26). The backwater which produces this reduction in the slope of the water surface apparently does not become effective except at higher discharges.

At South Downington the decrease in the slope of the water surface with increasing discharge may be related to some expansion in the area of the cross section of the reach and to inflow at the lower end of the reach during the high flow.

Generally, in the reaches studied on the Brandywine, as the discharge increases the slope of the water surface approaches the longitudinal profile of the bed of the stream as determined from topographic maps. The change in slope with changing discharge at-a-station is actually small. This approach toward an approximation of the longitudinal profile of the stream largely represents a smoothing out of the minor variations represented by the pools and riffles. In this sense, the longitudinal profile is associated with the higher flows (Daussé, 1872, p. 320). This relationship, however, represents only one stage of a continuous series of adjustments. These studies indicate that the adjustment of the channel to a group of hydraulic variables is not restricted to the higher flows, but it is an adjustment which prevails over a great range of flows.

#### MATERIALS ON THE BED AND IN THE BANKS

Before a discussion of the computation and significance of the roughness parameter, some information on the physical characteristics of the materials which make up the channel should be presented. These features are essentially geologic variables, or variables wholly or partially independent of the channel itself.

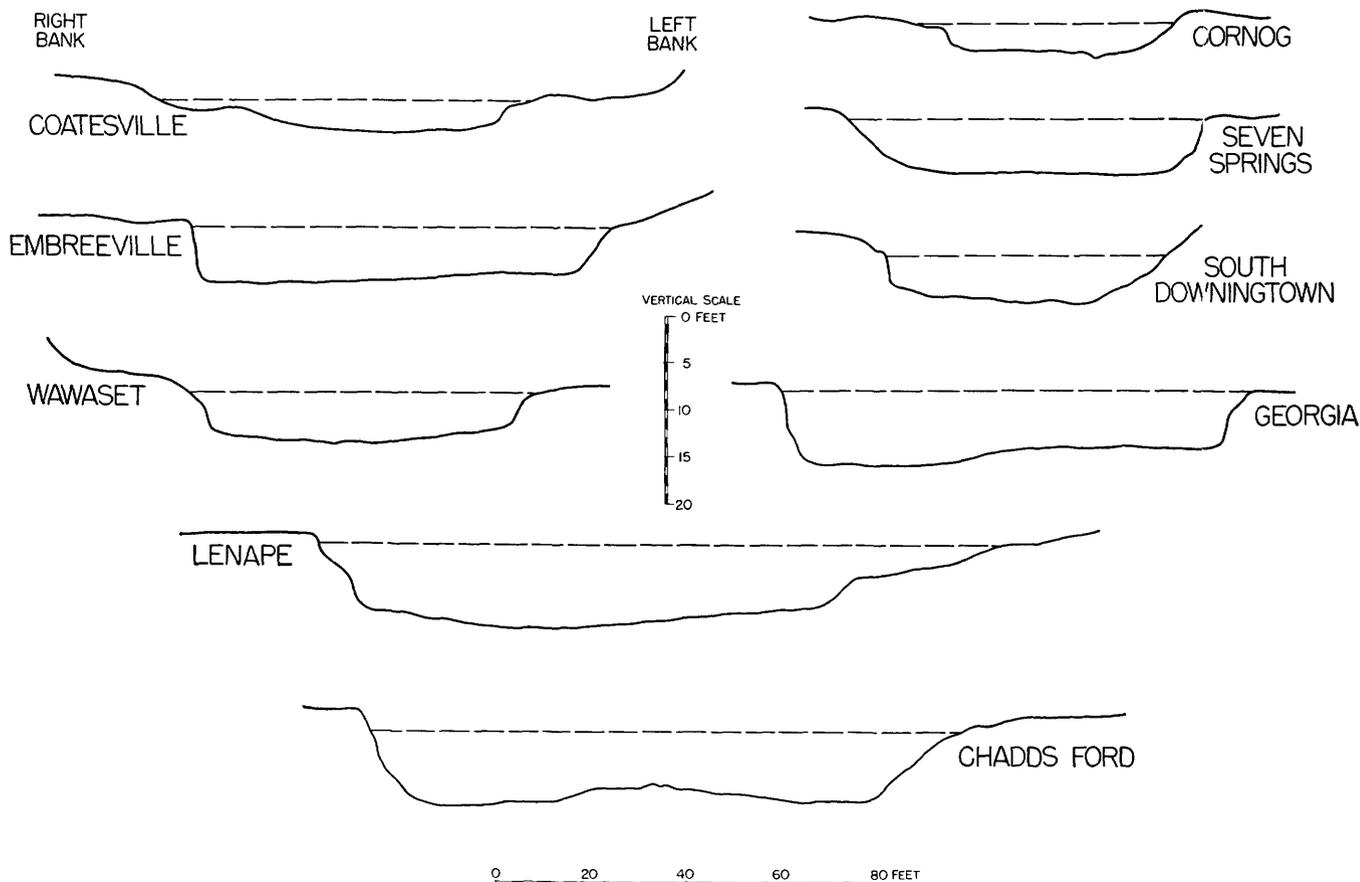


FIGURE 16.—Profiles of cross sections of Brandywine Creek at the principal measuring sections. Dashed line shows bankfull stage at each station.

The method used in this report to describe the bed material is far from satisfactory. Sampling and describing materials from the bottom of a stream are difficult, and the relationship between them and the physical significance of the results has been indefinite. Consequently, in selecting the method, much emphasis was given to the practical problem of sampling from a gravel bed beneath several feet of flowing water.

Had the material been randomly distributed over the bed and had the range of sizes been more restricted, mechanical analyses of large samples weighing several hundred or several thousand pounds would probably have provided the best quantitative description of the bed material. For several reasons this method does not appear to be appropriate to Brandywine Creek. First, because an unrepresentative sample is a distinct possibility in a nonrandom distribution typical of a river reach or cross section, an elaborate analysis of a huge sample from a given location might itself be a detailed but still unrepresentative sample. Second, it is important to know the resistance due to friction produced by the various sizes of material. Therefore, a description of the distribution and the cross-sectional areas of the

materials which obstruct the flow is probably more significant than is a description based simply upon a sample dug out of the bed. Besides these theoretical considerations, it would be almost a practical impossibility to obtain a large sample.

The problem of description is somewhat analogous to that of analyzing the size of particles in thin sections. In microscopic work, a single linear transect is subject to error, but the use of a number of linear transects can be shown to be a valid procedure for sampling. However, because the number of transects required for validity is a function of the variability of the material, it is not easy to determine what that number should be (Postel and Lufkin, 1942). The following method that was used in sampling and describing the sediments of Brandywine Creek is a compromise between theory and practice.

At each of the principal cross sections, a transect of the bed material was made. Every stone greater than 2 inches in median diameter was measured and counted. Measurements were made of the lengths of three mutually perpendicular axes on each of the pebbles. Four samples of the fraction of the bed material less

than 2 inches in diameter were taken from each cross section. These were sieved and a composite cumulative curve of the grain size distribution for this "fine" fraction was drawn. The percent of each grade size used in the composite curve is the mean value of the percents from the four separate analyses. The "pebble counts" and mechanical analyses were combined as follows: The percent of the total width of the bed covered by each of the various sizes 2 inches or larger was computed by dividing the total width by the sum of the median diameters of each of the pebbles within the given grade size. The sum of these percents represents the part of the bed covered by material 2 inches or greater. It was assumed that the remaining part of the bed was described by the composite curves from the sieve analyses. The percents of the various grain sizes less than 2 inches as determined by the sieve analyses were converted to percents of the total width of the bed according to the formula:

$$\frac{W-L}{W}(100)p$$

where  $W$  equals the total width of the bed,  $L$  the width covered by material greater than 2 inches, and  $p$  the percent of the sieved sample in a given grain size. The curves in figure 17 show the percent of the width of the bed covered by each of the grade sizes. It is important to note that statistical parameters (Inman, 1952) derived from these curves cannot be compared with similar measures computed from cumulative frequency curves based on mechanical analyses.

All of the materials sampled (figs. 17, 18, and table 2) are gravels according to the Wentworth scale. At Georgia where the curves indicate that the bed material is the finest, median diameter is 0.73 inch. The

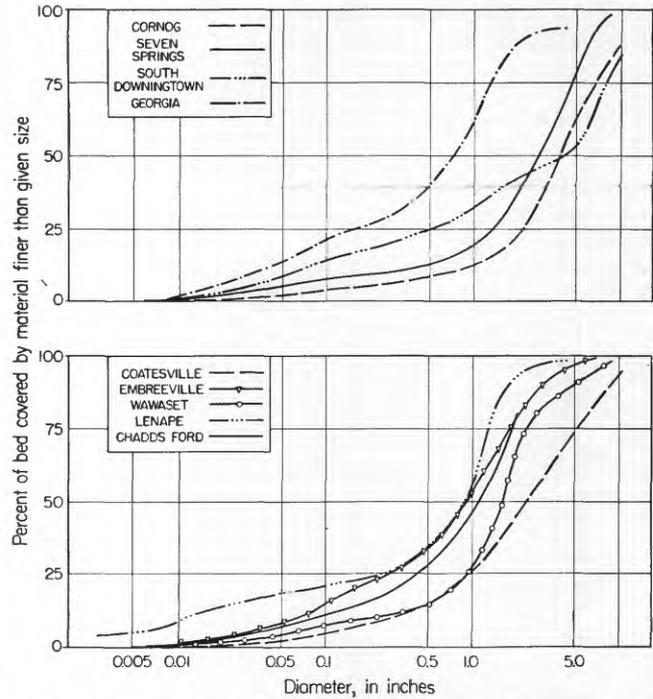


FIGURE 17.—Cumulative frequency curves showing the size of material on the bed of Brandywine Creek at the principal measuring sections.

coarsest material is at Cornog where the median is 3.8 inches; it is composed primarily of cobbles and boulders.

The bank material of Brandywine Creek presents a strong contrast to the bed material everywhere in the watershed (figs. 19 and 20). The four curves marked Taylor Run (table 3) describe four samples taken from a vertical section of the flood plain of the Brandywine about 500 yards above Georgia at the point where Taylor Run traverses the flood plain before emptying into East Branch, Brandywine Creek. These curves,



FIGURE 18.—A, Buck Run at Pomeroy. Median diameter of bed material is 6 inches. B, East Branch near Reeds Road. Cobbles 3 to 5 inches in diameter have been moved by bankfull discharge, which is about 800 cfs.

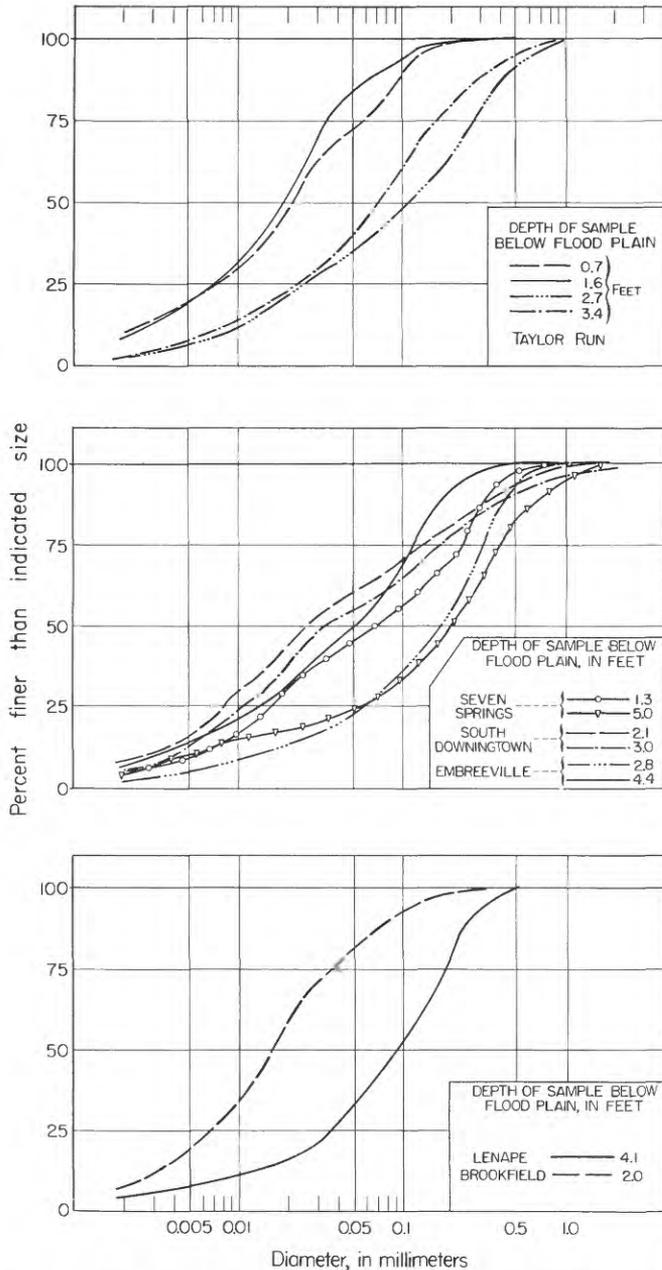


FIGURE 19.—Cumulative frequency curves, based on combined mechanical and pipette analyses, showing the size of material in the banks of Brandywine Creek at six widely separated localities.

as well as the pairs of curves for Seven Springs, South Downingtown, and Embreeville, indicate that regardless of the elevation above the present stream bed, the bank material is vastly different from the bed material, and that bank material is much the same throughout the drainage basin. The variation within a single vertical section, such as at Taylor Run, Seven Springs, South Downingtown, or Embreeville, is as great as the variation among the different cross sections from a

downstream section at Brookfield below Chadds Ford upstream to Seven Springs.

TABLE 2.—Size of material on the bed of Brandywine Creek at the principal measuring sections

Station	Median diameter in inches	Percent of bed covered by material > 2 inches in median diameter
Cornog	3.8	77
Seven Springs	2.8	64
South Downingtown	4.1	57
Georgia	.7	11
Embreeville	.9	21
Wawaset	1.6	33
Lenape	.9	7
Coatesville	2.4	55
Chadds Ford	1.1	23

At only a few places has it been possible to obtain information on the nature and thickness of the coarse gravel fill that makes up the gravel bed of Brandywine Creek. In excavations for a new bridge at Shaws Bridge on the East Branch, 500 yards above the fork of the Brandywine, the fill is 10 to 15 feet thick and rests upon bedrock. The fill is a mixture of sand and 3/4-inch gravel and shows no evidence of stratification or lenses. The particles are subangular to subrounded and include a variety of rock types. These characteristics indicate that this is an alluvial deposit. Records of an excavation for a pipeline indicate that the flood plain at Chadds Ford below the railroad bridge is underlain by at least 8 to 10 feet of sand and gravel. Bedrock was not found in the excavation. No stratification within the fill was reported. Near Lyndell and near Dorlan on the East Branch and near Icedale on West Branch,



FIGURE 20.—Typical bank and flood plain showing uniform silt above gravel, Buck Run at horseshoe bend near Ereildoun.

Brandywine Creek, records of shallow holes beneath the flood-plain deposit indicate the presence of coarse gravel to depths of 3 or 4 feet. It is possible that in some instances weathered bedrock has not been distinguished from gravel. Present information on the depth of the gravel fill throughout the valley is relatively scant. It is important to note in considering the development of the Brandywine profile that the creek does flow upon the gravel fill throughout most of its course. There are isolated points, however, at which the gravel cover is thin or nonexistent over bedrock outcrops.

TABLE 3.—Location and description of samples from the bank of the channel of Brandywine Creek

Station	Depth below surface of the flood plain, in feet	Median diameter, in millimeters	Trask sorting coefficient $\sqrt{\frac{Q_3}{Q_1}}$
Brookfield.....	6.0	0.016	2.36
Lenape.....	4.1	.092	2.30
Taylor Run.....	3.4	.072	2.72
Do.....	2.7	.11	3.41
Do.....	1.6	.019	2.19
Do.....	.7	.021	2.73
Embreeville.....	4.4	.17	2.97
Do.....	2.8	.054	2.26
South Downingtown.....	2.1	.027	4.11
Do.....	3.0	.035	3.93
Seven Springs.....	1.3	.070	3.89
Do.....	5.1	.21	2.79

There is not enough information about Brandywine Creek to permit an adequate discussion of the origin of the flood plain. It is impossible to say whether the coarse material in the bed represents bedload and the fine material in the banks represents suspended load (Mackin, 1937, p. 828; Happ, 1952, p. 3), or whether the coarse material represents the deposit of a fore-runner of Brandywine Creek (Fisk, 1951) which existed under a different climatic or hydrologic regime. During relatively high flows (those occurring possibly 2 to 4 times a year) Brandywine Creek does manage to rework or at least to toss about these older, coarser materials (fig. 18*b*). There is little evidence of deposition of fine overbank deposits during floods except in isolated locations and in extremely thin layers. What the cumulative effect of these irregular thin deposits would be is speculative. On the other hand, if the formation of the flood plain resulted primarily from the accretion of material on the insides of bends and meanders as these migrate laterally, one might expect the flood plain to be composed of alternating silt, sand, and gravel lenses similar to those found in the present channel. Thus far, the data reveal no such changes in grain size (fig. 19). Before any answer to these questions can be

given, much additional work is required to establish criteria which can be used to determine from any deposit its mode of deposition. The writer's present tentative conclusion is that the uniform frequency of overbank flooding throughout the drainage basin and the apparently small amount of overbank deposition, as well as the uniform channel characteristics, indicate that the flood plain (deposit and surface) is closely related to the present channel and hydrologic regime.

#### SUSPENDED LOAD

Leopold and Maddock (1953) have shown that the quantity and concentration of the suspended load are closely related to the shape of a given channel. In addition Vanoni (1941, p. 618) and others (Schnackenberg, 1951, p. 378) have shown that the value of a roughness factor may be influenced by the sediment concentration. Vanoni suggests that a high sediment load may reduce the energy loss in friction by dampening the turbulent eddies. These relationships have been observed primarily in streams carrying large concentrations of suspended material. Unfortunately, little data on the suspended load in rivers in the eastern United States is available for comparison with rivers in other regions. The data on Brandywine Creek are presented here in order to show at least the magnitudes of the suspended loads and, perhaps, the kinds of changes which take place in the suspended load with changing discharge. Although the data on load elsewhere in the Brandywine Creek valley are meager, the sediment rating curve for the Brandywine at Wilmington, Del., is well defined.

At Wilmington the suspended load increases as the 2.37 power of the discharge (fig. 21). This is the value of  $j$  in the equation:

$$L = pQ^j \quad (9)$$

where  $L$  is suspended load in tons per day,  $Q$  the discharge in cubic feet per second, and  $p$  is a constant. The highest load shown on the graph represents a concentration of 2,400 parts per million at a discharge of about 16,000 cfs.

The results of measurements of the suspended load at seven of the principal measuring sections are shown in figure 22. Although the few known values for each station do not justify drawing a curve through the points, at each it is clear that the load increases rapidly with at-a-station discharge. In an ideal example for which curves could be drawn for each station, the curves would presumably be approximately parallel, the most upstream cross section being on the left and to its right the stations successively downstream from it. The proximity of the stations to one another on Brandy-

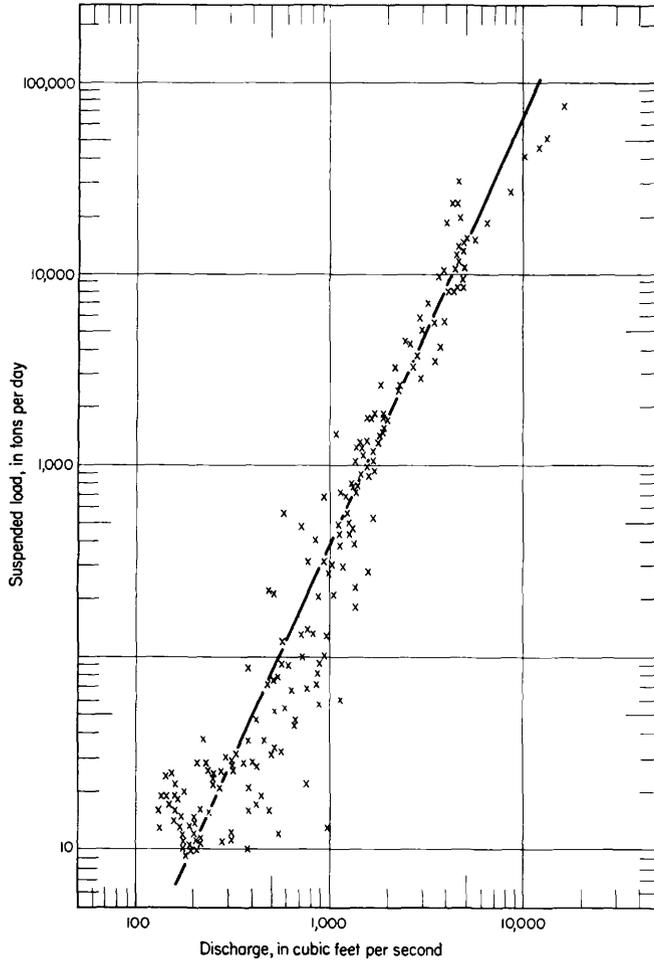


FIGURE 21.—Sediment rating curve for Brandywine Creek at Wilmington, Del. Data collected by U. S. Geological Survey.

wine Creek and the normal scatter of points on the sediment rating curve make it impossible to demonstrate such a relationship here. There is apparently a tendency, however, for the points from the upstream cross sections to lie successively to the left of the straight line representing the sediment rating curve for the Brandywine at Wilmington.

Consideration will be given later to the relationship between suspended load and channel shape in the parts of this report which are concerned with the changes of the variables in the downstream direction.

**ROUGHNESS**

Thus far it has been shown that orderly changes take place in the width, depth, velocity, and slope at-a-station as the discharge increases. Similarly, it has been shown that at every station on Brandywine Creek, the materials on the bed and in the banks are distinctly different. The changes in a roughness parameter which take place at-a-station will now be considered. In a

sense, these changes in roughness reflect the interaction of the hydraulic and geologic variables. Although difficult to explain, this interaction is fundamental to the behavior of all of the variables.

The roughness factor  $n'$  used here represents a coefficient in the equation

$$v = \frac{1.486}{n'} d^{\frac{2}{3}} s^{\frac{1}{2}} \tag{10}$$

where  $v$  is the mean velocity,  $d$  the mean depth, and  $s$  the slope of the water surface. The use of a modified Manning equation in the present study is recommended by its simplicity and its applicability to a wide range of channels. The roughness parameter  $n'$  represents some measure of the resistance to flow in the channel. It differs from the Manning  $n$  for two reasons: the mean depth rather than the hydraulic radius is used in the computation of  $n'$ , and the slope represents the slope of the water surface and not the energy grade line.

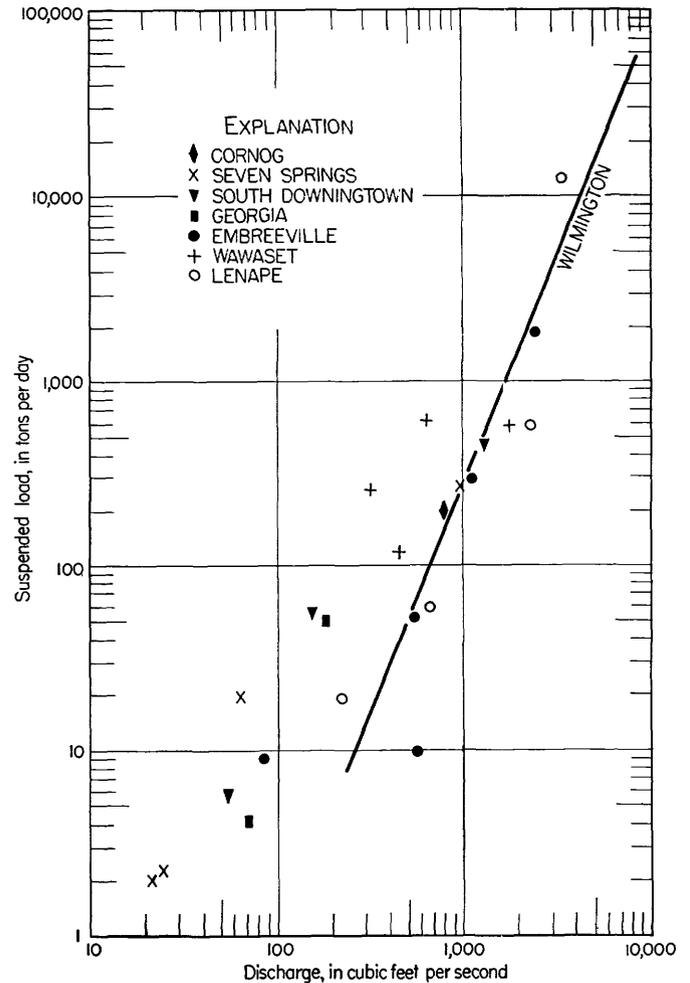


FIGURE 22.—Suspended load at the principal measuring sections at different discharges. Solid line shows sediment-rating curve for Brandywine Creek at Wilmington, Del.

The distinction between  $n$  in equation (2) and  $n'$  in equation (10) is important. In order to determine the value of the Manning roughness parameter  $n$ , it is necessary to obtain as closely as possible a value of the slope of the energy grade line; that is, the rate of dissipation of the energy in the flowing water. In addition to a precise measurement of slope, this requires an evaluation of the changes in velocity head from the upstream to the downstream end of the reach. The slope of the water surface under some conditions may closely approximate this energy slope. The nature of the field conditions in the present investigations, however, did not permit the computation of the energy slope. Therefore, the values of  $n'$  computed for the stations on the Brandywine cannot be substituted for  $n$  in the Manning equation when the latter is used in the indirect determination of velocity in similar channels elsewhere. That this caution is warranted can clearly be seen from the unusually low values of roughness  $n'$  shown for the higher stages on the Brandywine (figs. 10-13).

This study deals only with the relationship between the roughness factor and the size of the material in the bed of the stream. Actually the roughness  $n'$  represents far more than this:  $n'$  is related not only to the bed and bank materials but also to changes in channel shape, the size and distribution of vegetation, bed and bank configuration, the amount of sediment, and other factors. Unfortunately, it is impossible here to evaluate the separate effects of each of these factors. Consideration of this roughness factor, however, does give some way of making the transition between the purely physical characteristics of the channel and the effects which such characteristics have on the flow.

At all the cross sections studied, the roughness  $n'$  decreased with increasing discharge (table 1). This relationship is shown in figures 10-13 and is also illustrated by figure 23 which shows graphs of depth plotted against roughness ( $n'$ ) for four stations on the creek. Smooth curves have been drawn by eye through the plotted points. In each of the examples the roughness decreased with depth; that is, the roughness decreased with increasing discharge. In the cross sections on Brandywine Creek  $n'$  varies approximately as depth to the minus 0.6 power. This decrease in  $n'$  with depth may be associated to some degree with the increasing influence of the relatively fine and possibly smoother bank materials, although vegetation on the banks will usually counteract this effect. The decrease in roughness at higher flows may also be associated with better alignment of the channel with the banks and increasing uniformity of the channel cross sections. Last, and probably most important, the decrease in the roughness

factor  $n'$  may also reflect a decrease in the friction loss accompanying submergence of protuberances on the bed. Such a result is suggested by experiments on pipes and fixed open channels in which the Chezy  $C$  friction factor increases as the ratio of the depth of flow to the size of protuberances increases (Powell, 1950, p. 577; Robinson and Albertson, 1952, p. 882).

Although a number of other factors also affect the value of the roughness parameter, the plot of  $n'$  at a constant depth of 1 foot against grain size in figure 24 for the major stations on the Brandywine indicates that  $n'$  is directly related to the size of the material on the bed. The small range of grain sizes on the Brandywine and the normally narrow range of  $n'$  do not permit a completely satisfactory verification of this relationship. The results, nevertheless, are in accord with those observed by Strickler and others (Lane and Carlson, 1953).

The reader may well wonder at this emphasis on roughness. There are two principal reasons: First, if  $n'$  is to be used in the analysis of river processes because of its "simplicity," it is important to have some understanding of its physical meaning. Second, in a sense  $n'$  is an independent variable in the channel regime. It may be argued that because  $n'$  is simply computed from measurements of width, depth, velocity, and slope, it cannot be used as an independent parameter with which to explain the reactions of the other variables. Regardless of the method of calculation, the roughness  $n'$  is, in part at least, a measure of the physical "roughness" of the channel. Some such roughness factor is essential if we are to relate independent factors such as grain size and sediment to the hydraulic variables which in concert with these independent factors determine the form and gradient of the channel.

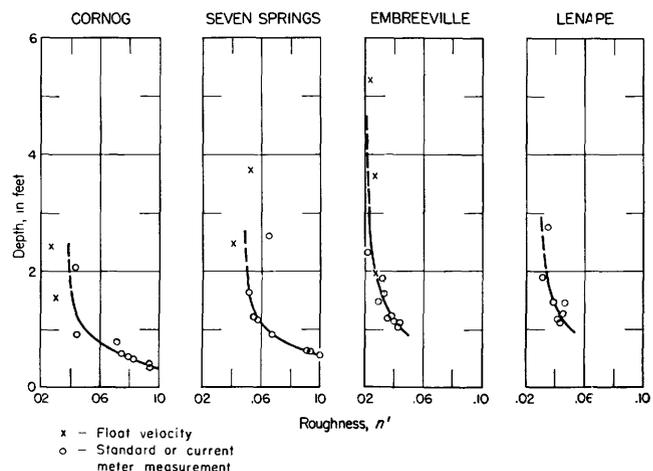


FIGURE 23.—Relation of roughness parameter  $n'$  to mean depth at Cornog, Seven Springs, Embreeville, and Lenape.

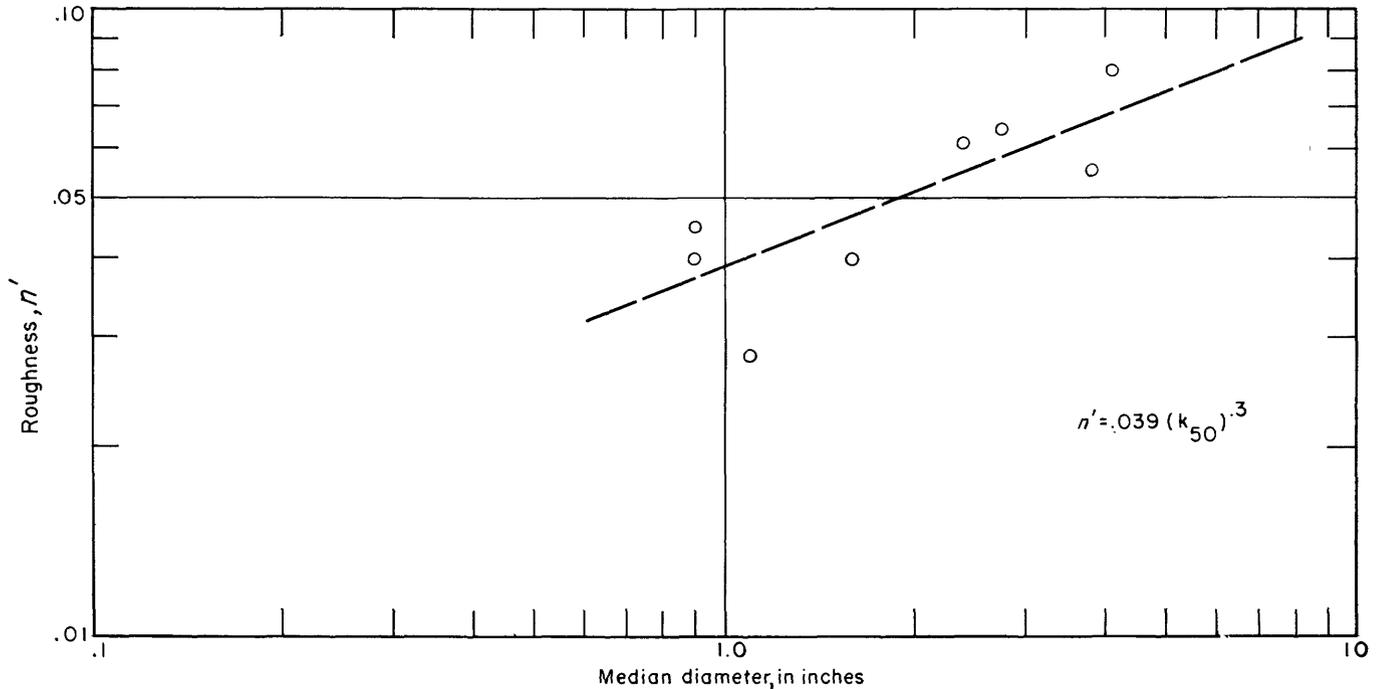


FIGURE 24.—Relation of roughness parameter  $n'$  to median size of bed material;  $n'$  computed for a constant depth of 1 foot.

The problem of choosing the independent variables is a complicated one. Where simultaneous interaction of at least eight designated variables is involved, no simple rules can be laid down by which the independent variables can be separated from the dependent ones. If the process of development of the river channel and profile is a hydrodynamic one, the geologically independent factors must eventually be considered from the standpoint of their behavior and not solely by correlating their effects. Thus, although grain size, one of many geological factors, may be correlated with slope, analysis shows that the hydraulic behavior of "grain size" is a frictional factor, and as such, it is one of several related factors which exercise a frictional resistance. A roughness  $n'$ , although perhaps not the most significant measure of energy and friction losses, is, at present, one which can be obtained under field conditions. As such, it is a useful tool which can at least be applied qualitatively in our effort to find the mechanism by which the river channel and its profile develop.

#### SUMMARY AND INTERPRETATION OF AT-A-STATION CHANGES

At any given reach the shape and profile of the channel appear to be related to the interaction of a minimum of seven variables: discharge, width, depth, velocity, water surface slope, roughness (as a measure of frictional resistance), and load. At each cross section studied on Brandywine Creek an increase in discharge is

associated with the following behavior of the other variables: (1) the width increases slowly or remains relatively constant, (2) both the depth and velocity increase, the velocity as a rule increasing more rapidly, (3) the roughness  $n'$  decreases, (4) the suspended load increases rapidly, and (5) the slope of the water surface remains practically constant.

It could be argued that the hydraulic relationships observed at the cross sections on the Brandywine are simply the changes which would occur in any fixed channel or pipe as the discharge flowing through the cross section increased. In a natural channel, however, the stream determines its own cross section. This cross section is not the only one which would permit the passage of given amounts of water. A given flow could pass at different velocities through a variety of cross sections representing an infinite number of combinations of widths and depths. Despite the possibility that various combinations of width, depth, and velocity would be capable of discharging the water and sediment, natural rivers with self-formed channels have characteristic combinations of these variables. One such combination is observed in Brandywine Creek. The point of interest is that in a specific geologic and climatic setting the river has built for itself a wide, flat channel with steep banks of fine material. In some manner this particular channel was determined by the kinds of hydraulic relationships which have been noted.

If it is assumed that a given drainage basin is under-

lain by a particular type of rock, the shape of a cross section of the stream at any point would presumably be determined by the combined effect of the discharge, suspended load, and roughness (the hydraulic mechanism through which the material expresses its effect). The type of bedrock exercises an influence both within the channel where it affects the roughness of the bed and bank and within the source area from which the suspended load is derived. In a basin in nearly homogeneous rock a hypothetical channel shape imposed by the variables above would be modified where the stream encountered a particularly resistant rock. The latter may cause the width of the channel to expand at the expense of the depth. Nevertheless, the combined effect of the discharge, suspended load, and roughness at that point is to produce a particular channel shape. Although possibly a dominant factor at a cross section, the resistant bed is not the sole factor determining the shape of the channel. That this is true is indicated by the fact that the channel is not inordinately wide where it passes over the resistant bed material. This "constructive" as opposed to "erosive" aspect of channel formation is illustrated by the narrow benches built adjacent to the bedrock walls at South Downingtown and at Embreeville—benches which are constructed to the height of the present flood plain (see fig. 16). There is an infinite variety of factors which may modify the results or interrupt the processes described in this idealized concept of a self-formed channel. Nevertheless, it is probable that these chance irregularities simply obscure but do not suspend the governing principles.

With additional information on the rate of change of suspended load at-a-station, Leopold and Maddock (1953, p. 28) were able to demonstrate that "for a given width, and at a given discharge, an increase in suspended sediment requires an increase in velocity at the expense of depth." Further, they suggest (p. 43) that

The characteristically rapid increase of suspended load with discharge at-a-station requires for transportation of that load, a relatively rapid increase of velocity in comparison with depth or a relatively large value of  $\frac{m}{f}$ . For the cross sections studied this was equal to 0.85. This value, being larger than  $\frac{2}{3}$  requires that  $\frac{s^{1/2}}{n}$  increases with increasing discharge. This is accomplished primarily by the decrease of roughness  $n$ , resulting from the increase of concentration of suspended load with discharge.

The average rates of change with discharge of the width, depth, and velocity at the stations studied on the Brandywine are:

$$b = .04$$

$$f = .41$$

$$m = .55$$

$$j = 2.2$$

$$\frac{m}{f} = 1.34$$

Based on figure 18 (p. 25) of Leopold and Maddock, with  $b = .04$  and  $\frac{m}{f} = 1.34$ ,  $j$  (the rate of increase of suspended load with discharge) should equal approximately 3.2. The measured value at Wilmington of 2.37 is therefore apparently too low to account for the high  $\frac{m}{f}$  ratio.

Aside from the fact that the relationships given by Leopold and Maddock are not meant to be exact, several qualitative explanations can be advanced which may help to account for the differences between the behavior of Brandywine Creek and the western rivers. First, the decrease in roughness with increasing discharge on the Brandywine appears to be related to several factors that include a decrease in the effective roughness of the material on the bed as it becomes submerged, as well as a decrease in channel irregularities. The effect of increasing sediment concentration is probably secondary. Second, the disparity in bed and bank materials may be a cause of the large width at low flows and hence also of the very low rate of change of width with discharge at-a-station. Where the materials are more nearly alike, the increase in width may be greater. In turn, a more rapid increase in width at the same  $\frac{m}{f}$  ratio would require a lower value of  $j$  according to figure 18 in Leopold and Maddock. These suggestions do not explain the operation of the phenomena. Nevertheless, they do point out some additional factors which would be considered in refining the relationship between suspended load and channel shape.

Although the data on the Brandywine show that the behavior of the variables at a number of different locations is very similar, this picture of the process is greatly oversimplified. The adjustments which take place in the channel are simultaneous and not sequential. Our understanding of the processes may be enhanced by this piecemeal approach, but we should recognize that much of the sequence is artificial. In the absence of experiments in which specific variables are controlled and individual ones allowed to vary, it is almost impossible to isolate and consider each factor separately.

**DOWNSTREAM VARIATION IN THE RIVER CHANNEL**

Although the details of the mechanism by which a river channel reach is developed and maintained are not understood, the data in the first part of this study present a picture, at least, of the variables involved and the way in which they behave. The curves (figs. 10-15) showing the relation of the variables to discharge at a given station permit a discussion of the changes which take place in a downstream direction in these variables at comparable flows. Again, there is no assurance that the variables chosen for analysis are physically the most important ones. It is probable, however, that an analysis of these will demonstrate some significant physical phenomena. They may also enable identification of the processes by which the measured effects are brought about. A voluminous geologic literature deals with this question of the way in which a river changes its size and shape in flowing from headwaters to mouth. Most of this literature has dealt with the longitudinal profile, or the way in which the slope of the bed of the river changes in the downstream direction. The purpose here is to describe the changes in the measured variables and to relate whenever possible these changes to changes in the longitudinal profile. The ultimate and as yet unrealized aim is not solely to demonstrate "relationships"; rather the aim is to distinguish causes from results in the observed processes.

Most of the channel characteristics which are being compared at various points along the river are closely related to the amount of water contributed by the drainage basins above the cross sections. In this study the hydraulic characteristics of the separate reaches will be compared when each is experiencing the same duration or frequency of flow. The terms "duration" and "frequency" apply respectively to the percent of time during which a given flow is equaled or exceeded, and to the frequency with which a given flow recurs. The distinction between the two terms is based upon the way in which each is computed (see Linsley, Kohler, and Paulhus, 1949, and other textbooks on hydrology for standard methods of computations; and Leopold and Maddock, 1953).

In the drainage basin of Brandywine Creek the discharge at a given point is directly related to the contributing area at that point. Because the drainage area increases downstream, the discharge at any given frequency of flow likewise increases downstream. In this study, curves showing the hydraulic variables at different stations plotted against discharge for a given frequency of flow (figs. 25, 26, 29, 34, and 35) describe the changes which take place in these variables in the downstream direction.

For the downstream comparison of the width, depth, velocity, slope, roughness, and load, below the bankfull stage (fig. 25), a discharge of a constant duration will be used. Synthetic duration curves have been drawn for each of the seven principal measuring stations established in this study on Brandywine Creek. The method used in constructing these curves is based on a comparison of miscellaneous discharge measurements made at a measuring station with the daily mean flows at a permanent recording gage. In this study measurements made at the seven principal stations were compared with the daily mean flows on the same day at the Chadds Ford gaging station.

The downstream changes of the variables will be discussed in three parts. First, the way in which width, depth, velocity, roughness, and the slope of the water surface change between the principal stations, including Coatesville and Chadds Ford, will be shown. These comparisons will be made for three different durations of flow below the bankfull stage. Second, a comparison of the downstream changes in the variables at the bankfull stage will be made for these same stations. Some consideration will be given here to the question of the frequency with which the bankfull stage is attained at upstream and downstream locations in the drainage basin. Separate consideration is given to the bankfull stage because of its relation to the flood plain and because a different method was used to determine the bankfull discharge. Third, comparisons of the width, depth, and velocity, using all of the measurements covering the entire basin will be made. In these comparisons only two durations will be shown. Because there is usually only 1 and at the most 3 measurements at each of these stations covering the headwater tributaries, to obtain values for the width, depth, and velocity at 2 discharges of specific durations, it was necessary to extrapolate from the discharge which had actually been measured. The methods used in obtaining the discharges for the given durations, as well as the values of the variables, will be given in a later part.

**DOWNSTREAM VARIATION AT LOW AND HIGH FLOWS**

In figure 25 the width, depth, velocity, roughness, and slope are plotted against the discharge which is equaled or exceeded at each station 50 percent of the time. For each station the discharge representing a duration of 50 percent was determined from the synthetic duration curve for that station. The values of the width, depth, velocity, roughness, and slope of the water surface at this discharge were read from the at-a-station curves shown in figures 10-13. Because the values of the variables were read from smooth straight-

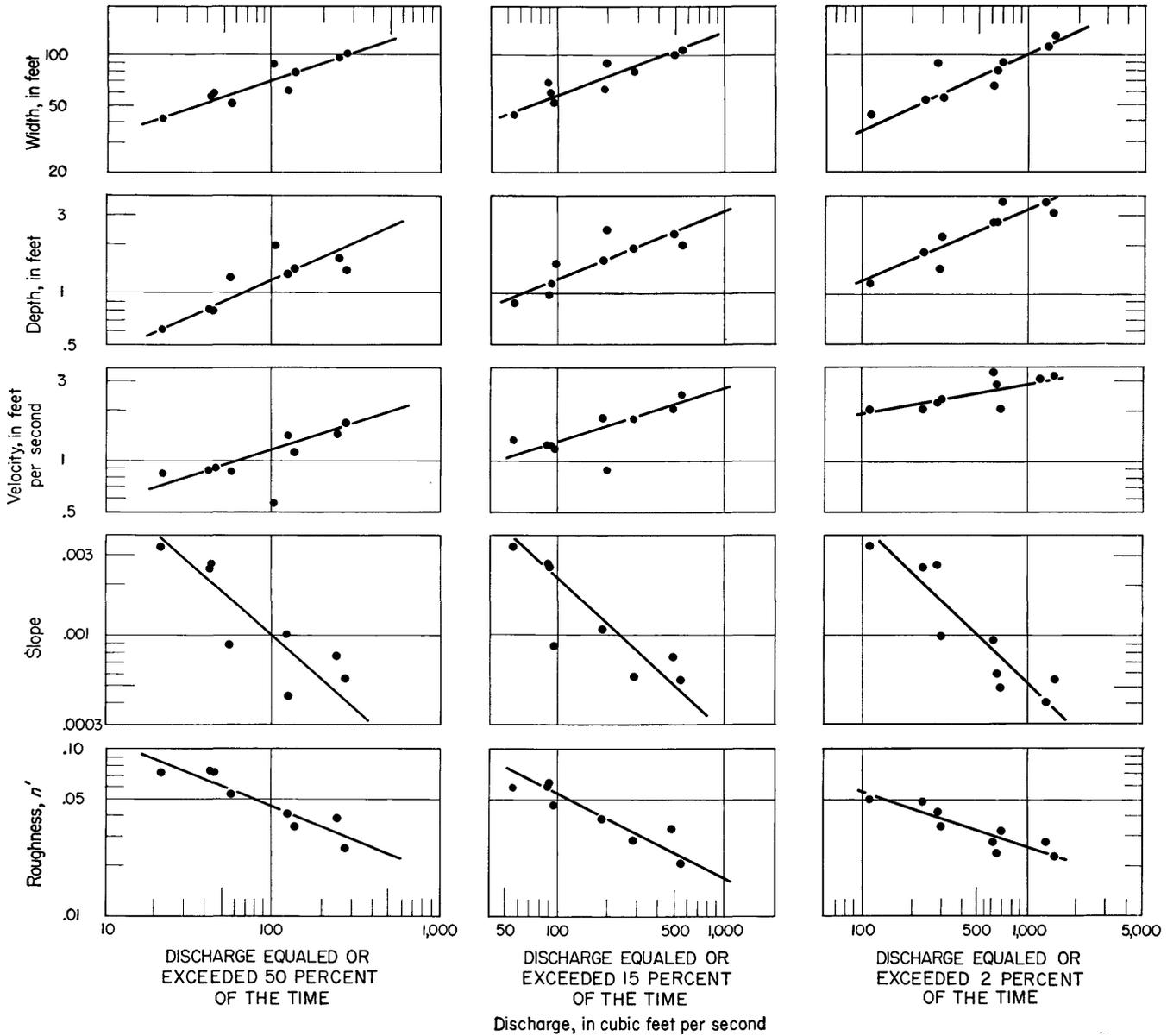


FIGURE 25.—Relation of width, depth, velocity, water surface slope, and roughness parameter  $n'$  to discharge at the principal stations at flows equaled or exceeded 50, 15, and 2 percent of the time.

line curves, equation (10) may not be verified exactly at each of the points on the downstream curves. Each of the variables was plotted against discharge and a straight line was fitted by eye to the plotted points. The slopes of these lines are designated  $b$ ,  $f$ , and  $m$  for width, depth, and velocity, respectively, as they were for at-a-station curves. Similarly, the letters  $\gamma$  and  $z$  are used to denote the slope of the curves of the roughness factor against discharge and of slope of the water surface against discharge, respectively. No data were available for the slopes of the water surface at Coatesville and at Chadds Ford. The slope of the bed as

determined from the profile of the bed elevation was used in the computation of  $n'$  and in the plot of slope against discharge. No great discrepancy between these values and the values of  $n'$  and  $s$  for the other stations, however, is noticeable in the curves showing the changes of the variables in the downstream direction. At Chadds Ford several measurements of the slope of the water surface at low flows indicate that the slope of the bed used in the downstream curves is somewhat greater than the slope of the water surface at Chadds Ford at low flow. The result may be  $z$  values that are somewhat too low; that is, the slope of the curve of  $s$  plotted

against  $Q$  may tend to be too flat. This effect, however, is small.

The other curves in figure 25 were constructed in the same way as was the curve for the discharge experienced 50 percent of the time. A discharge equaled or exceeded 2 percent of the time is a relatively high flow. Although there are no pictures of the river at precisely this discharge, the high flows shown in figures 4-9 represent flows equaled or exceeded less than 1 percent of the time. The flow at Cornog (fig. 3) is somewhat higher and represents a duration of about 0.01 percent, or less than 1 day per year.

The values of the exponents in the equations expressing each of the variables in terms of discharge are given in table 4. Only those values related to curves drawn from data for the main stations at durations equaled or exceeded 50, 15, and 2 percent of the time will be considered now. At every stage the actual values of  $b$ ,  $f$ , and  $m$  closely approximate the equation of continuity in which  $b+f+m=1$ . Width, depth, and velocity all increase as roughness and slope decrease downstream on the Brandywine at all frequencies of flow (fig. 25). At each successively higher flow—that is, decreasing frequency or duration (table 4)—the rate of change of depth downstream remains constant, but the rate of change of width increases, and the rates of change of velocity and roughness decrease. From a qualitative analysis of these changes some understanding of the physical processes which control them can be gained.

TABLE 4.—Summary of changes in width, depth, velocity, slope, and roughness in the downstream direction for discharge of different durations, Brandywine Creek

Duration (percent of time flow is equaled or exceeded)	Rate of change with discharge of:				
	Width b	Depth f	Velocity m	Slope z	Rough- ness y
PRINCIPAL STATIONS					
50-----	.34	.45	.32	-.80	-.40
15-----	.38	.42	.32	-.92	-.51
2-----	.45	.43	.17	-.97	-.32
Bankfull-----	.42	.45	.05	-1.07	1-.28
PRINCIPAL STATIONS AND HEADWATER TRIBUTARIES					
50-----	.57	.40	.03	-----	-----
2-----	.58	.40	.02	-----	-----

<sup>1</sup> Computed from equation (10).

At no stage does the mean velocity decrease downstream. During periods of low flow the mean velocity increases downstream, and at higher flows the velocity apparently remains constant downstream. Values of  $m$  in table 4 decrease with increasing flow. This decrease is probably associated with changes in the rate of decrease of roughness rather than with changes in

the rate at which slope decreases. It is logical to believe that as the change of roughness with respect to changing discharge becomes less—that is, as the difference between the roughness upstream and the roughness downstream is reduced—the difference between the velocity upstream and the velocity downstream will be reduced.

It has already been shown in figure 24 that the roughness factor decreases downstream in company with the decrease in grain size. The curves of  $n'$  plotted against discharge (fig. 25) show that the rate of change of  $n'$  in the downstream direction decreases for three different durations of flow. Several suggestions can be made concerning the decreasing rate of change of  $n'$  at high flows: 1. The range of  $n'$ , as computed in this study, is greater at an upstream station than it is at a downstream one. This may result from the fact that the lowest flow used in this downstream comparison represents a much shallower depth at the upstream stations than it does at the downstream ones. 2. An increase in the concentration of the suspended load downstream at higher flows would tend to reduce  $n'$  slightly in the downstream direction. 3. The absence of sufficient fine material on the bed of Brandywine Creek precludes the possibility that dune formation in the downstream reaches plays an important part in increasing  $n'$  at high flows and thus flattening the curve of  $n'$ .

The slope-discharge curve is steeper for high than for low flows (table 4). This possibly can be attributed to the fact that the relation of discharge to drainage area appears to be different for various durations of flow.

These observations on the changes of the variables at-a-station and in the downstream direction on the Brandywine Creek are combined in figure 26. The relationships described in the figure were collectively termed the "hydraulic geometry" of the stream channel by Leopold and Maddock (1953). In figure 26a the numbered solid lines show the actual changes of the width, depth, velocity, roughness, parameter, and slope at each of the seven major stations established on Brandywine Creek for this study. These curves are the same as the at-a-station curves shown in figures 10-13. The family of curves representing all of the stations indicates the uniformity of the behavior of the variables at the different cross sections. Two points are shown on lines 1 and 7 in each of the groups. The points marked A are the values of the variables at Cornog at a low flow: specifically, at a discharge equaled or exceeded 50 percent of the time. Those designated B are the values of the variables at Cornog at a high discharge, equaled or exceeded 2 percent of the time. A' and B' are the values of the variables at Lenape for the low and high

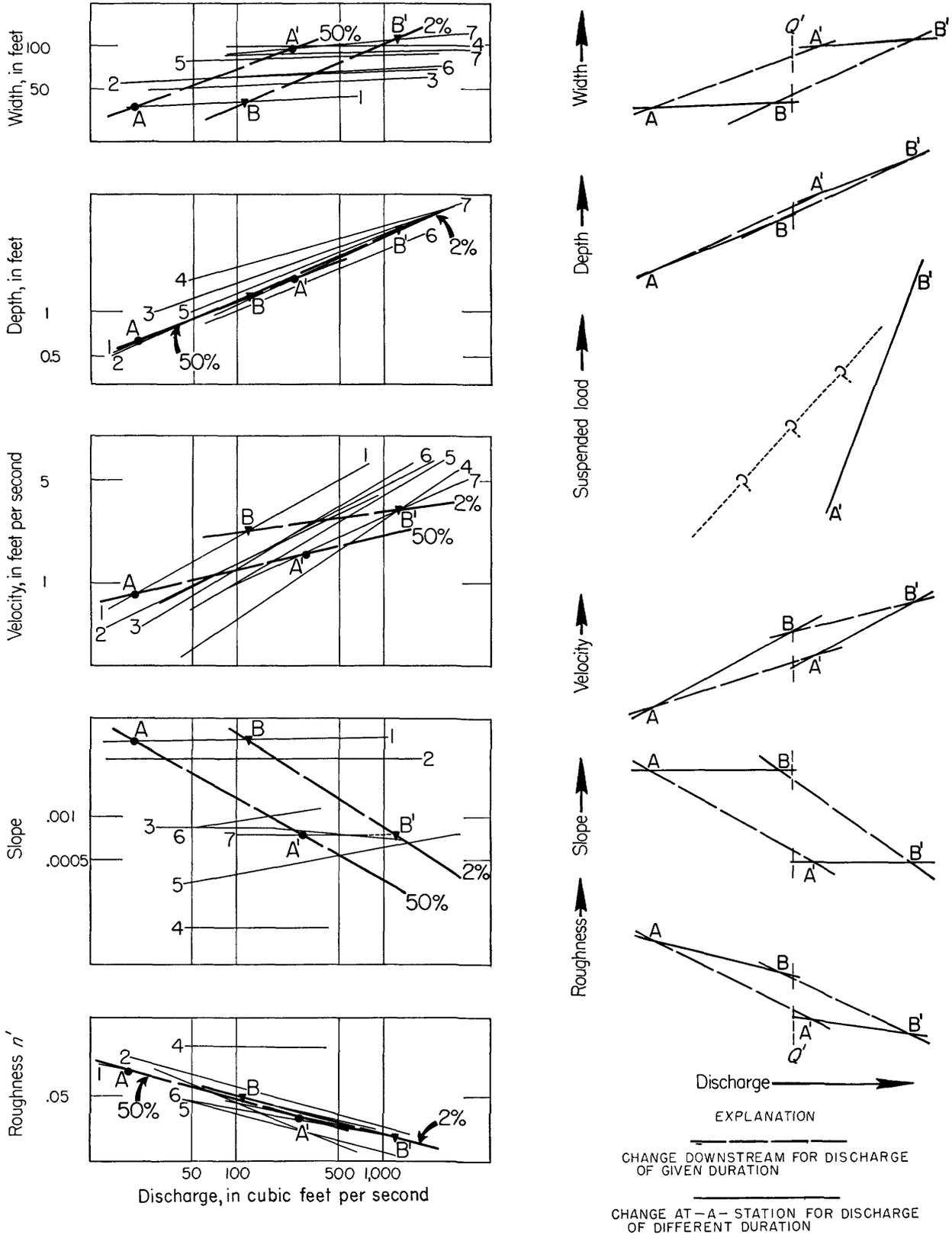


FIGURE 26.—a. At-a-station curves of width, depth, velocity, slope, and roughness parameter for seven principal stations on Brandywine Creek, and downstream variation in these variables between Cornog and Lenape for flows equalled or exceeded 50 and 2 percent of the time. Numbers refer to stations as follows: 1. Cornog, 2. Seven Springs, 3. South Downington, 4. Georgia, 5. Embreeville, 6. Wawaset, 7. Lenape. b. Schematic diagram showing hydraulic geometry of Brandywine Creek: at-a-station and downstream changes in width, depth, velocity, slope, roughness parameter, and suspended load.

flows respectively. The dotted lines A-A' and B-B' therefore show the rates of change of the width, depth, velocity, slope, and roughness  $n'$ , in the downstream direction for durations of 50 and 2 percent, as shown in the figure. They are near replicas of the downstream curves shown in figure 25. The slopes of the lines are somewhat different since the lines connect 2 points only and do not represent the average lines drawn through all 7 points. Figure 26a summarizing both the at-a-station and the downstream changes in the variables with changing discharge shows the following results: First, whereas width and slope change little at-a-station, width increases and slope decreases very markedly in the downstream direction. Second, depth increases at-a-station at nearly the same rate as it does in the downstream direction. Third, roughness decreases at-a-station somewhat more rapidly than it does downstream. And last, velocity increases slightly or remains constant in the downstream direction, while at-a-station velocity increases rapidly with increasing discharge. These are the fundamental changes which take place in the measured variables.

These same relative rates of change of the variables at-a-station and in the downstream direction are shown diagrammatically in figure 26b. Although the diagram closely resembles the actual results shown beside it in figure 26a, they are not identical. The slopes of the curves represent the mean at-a-station and downstream change in the variables. Also, a diagram has been added, based on the information available, showing the changes in the suspended load with increasing discharge at-a-station. This diagram is a replica of figure 22. Figure 26b therefore shows the complete hydraulic geometry of the Brandywine Creek.

A comparison of the hydraulic geometry of the Brandywine Creek with that of the Bighorn River in Wyoming shown by Leopold and Maddock (1953, fig. 12, basin A) reveals some similarities in the behavior of the variables. The relative rates of change of the variables at-a-station and in the downstream direction are the same in the two rivers. Why these parallels exist is not clear. The similarities however do suggest two conclusions: First, the rates of change of the variables are not specifically related to regional differences. Second, the processes which are manifested in these changes of the variables are similar in different regions.

The hydraulic geometry of the Brandywine Creek illustrates a generalization noted by Leopold and Maddock (1953, p. 27). At a constant discharge, marked  $Q'$  on the diagram in figure 26b, the conditions which prevail at an upstream station are represented by the intersection of the 2 percent curve with the vertical line for the given discharge. The conditions at the downstream station are given by the inter-

section of this line with the downstream curve for the 50 percent duration. At a constant discharge the width of the downstream cross section is greater than the width of an upstream section. The depth is very nearly the same at both, while the velocity is higher at the upstream section when it is experiencing the same discharge as a downstream section. When both the upstream and the downstream cross sections are experiencing the same discharge, the concentration of the suspended load is apparently greater at the upstream station.

Figure 27 constitutes an attempt to elaborate this relationship between suspended load and channel shape. The diagram generalizes some complex relationships and at the present time is meant to be suggestive rather than conclusive.

In figure 27 the width-to-depth ratio has been plotted against the suspended load from data collected on the Brandywine. The numbers beside each point are the discharges, in cubic feet per second, which occurred with the given load and width-to-depth ratio. The two lines marked 50 percent and 2 percent are mean lines drawn through points showing the relation of discharge to the width-to-depth ratio as discharge increased in the downstream direction for those two durations of flow. The diagonal lines are isopleths of discharge. The numbers beside the points indicate that the discharge in general increases from left to right. The spacing of the isopleths is only approximate. It is based upon the numbers shown on the figure and upon the known decrease in the width-to-depth ratio with increasing discharge at each station. This decrease in the width-to-depth ratio at a particular station, for example, would be given by a diagonal line from E on the 50 percent curve to H on the 2 percent curve. The slopes of the isopleths of discharge are based upon the points shown and upon the spacing determined above. They are also only approximately

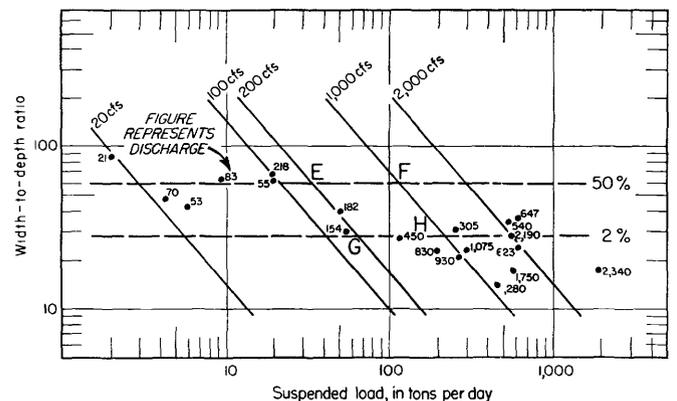


FIGURE 27.—Relation of suspended load to width-depth ratio. Comparison of channel shapes at constant discharge with different suspended loads.

correct. Assuming that figure 27 presents a fair picture of the available data for the principal stations on Brandywine Creek, one can make several observations concerning the behavior of suspended load on the Brandywine.

At the principal stations on Brandywine Creek the width-to-depth ratio remains relatively constant with increasing discharge in the downstream direction. This relationship is shown by the horizontal lines labeled 50 and 2 percent. (Some increase in this ratio is indicated by the data from the headwater tributaries.) The downward trend from left to right of the plotted points indicates that small loads are associated with low discharges and high width-to-depth ratio, and large loads with high discharges and a low width-to-depth ratio. Such an association might be expected simply because at any cross section an increase in discharge is accompanied by an increase in suspended load and by a decrease in the width-to-depth ratio.

For any given discharge the graph also indicates that a narrower cross section may be associated with a greater suspended load. For example, a flow of 200 cfs represents a flow equaled or exceeded 2 percent of the time at an upstream station (G), while the same flow downstream (at E) is equaled or exceeded 50 percent of the time. When the station at G is experiencing a flow of 200 cfs the width-to-depth ratio is about 28 and the suspended load about 65 tons a day. When the downstream station at E is experiencing the same flow, the width-to-depth ratio is about 60 and the suspended load 35 tons a day. A similar general relationship can be shown for points H (upstream) and F (downstream) when each is experiencing a flow of 1,000 cfs. These comparisons suggest that at the principal sections on the Brandywine a narrower section seems to be associated with a greater concentration of suspended load, as well as with a higher velocity (fig. 26b).

At a constant discharge the width-to-depth ratio might have remained constant between the two stations. Because the upstream channel has a smaller area of cross section, for a given discharge the velocity would be higher. In this instance the larger suspended load might be associated with an increase in velocity alone, the channel shape being unchanged. For the width-to-depth ratio to remain the same at a given discharge, however, would require the isopleths of discharge in figure 27 to be horizontal. The points on the figure indicate that the isopleths of discharge cannot be drawn horizontally. Thus, not only in the average data from western rivers (Leopold and Maddock, 1953, p. 24), but in a single eastern stream the increased suspended load apparently is accommodated not only by an increased velocity but also by a change in channel shape.

### THE BANKFULL STAGE

The bankfull stage is very close to the stage at which overbank flooding occurs. For this study the bankfull stage has been defined as that stage at a given cross section at which, in a plot of the width-to-depth ratio against stage, the curve breaks sharply and the width becomes exceedingly large. Thus the bankfull stage at South Downingtown in the example in figure 28 is at 6 feet. Because the author was unable to make any discharge measurement in the field at this bankfull stage, the discharge at each station was determined by extrapolating a curve of discharge against the width-to-depth ratio. Because measurements had been made very near the bankfull stage at most of the principal sections, the extrapolations required were not large. At Coatesville and at Chadds Ford, the width, depth, and velocity at the bankfull discharge were read from the smooth curves (figs. 14 and 15). At the seven stations established for this study, the values of the width and mean depth at the bankfull stage were determined from the cross sections (figure 16). The mean velocity at the

bankfull stage was then  $v = \frac{Q}{wd}$

Figure 29 shows a plot of each of the variables against discharge in the downstream direction at the bankfull stage. The values of the slopes of the lines are listed in table 4. For the graph of slope plotted against discharge, the slope used is the slope of the bed at Chadds Ford and at Coatesville; at each of the other seven stations it is the measured slope of the highest flow. After the slopes of the curves of width, depth, velocity, and water surface slope in relation to discharge had been determined, the relation of the roughness  $n'$  to discharge was computed as follows:

From the equation

$$v = \frac{1.486}{n'} d^{2/3} s^{1/2} \quad (10)$$

substituting equations (4), (5), and (6) for  $d$ ,  $v$ , and  $s$ , respectively, and letting

$$n' = rQ^y \quad (7)$$

then

$$rQ^y = \frac{1.486}{kQ^m} (cQ^l)^{2/3} (tQ^x)^{1/2}$$

and, substituting from table 4,

$$y = -.05 + \frac{2}{3}(.45) + \frac{1}{2}(-1.07)$$

$$y = -.28$$

A comparison of these curves for the bankfull stage with those in figure 25 shows that the trends in the

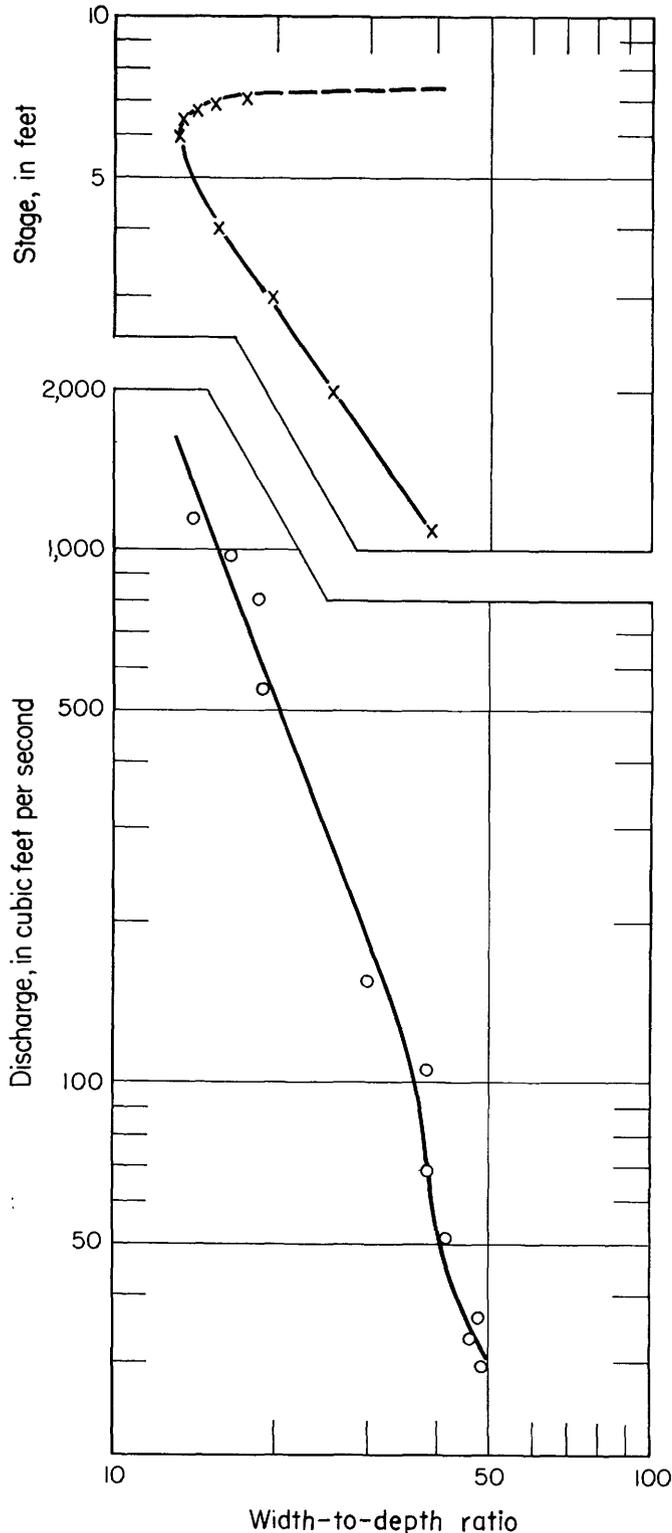


FIGURE 28.—Example of method used to determine bankfull stage at each of the cross sections. Width-to-depth ratio plotted against discharge and stage at South Downingtown.

rates of change of the variables which took place at lower and more frequent flows are continued to the bankfull stage. The rate of change of width in relation to discharge continues to increase. The mean velocity at this higher stage becomes very nearly constant downstream, and the depth and slope continue to increase downstream at the same rates as they did during the lower flows. The rate of change of roughness is somewhat less than the rate during the more frequent lower flows.

The bankfull discharge was computed without regard to frequency in order to find out whether the bankfull stage represented a particular frequency of flow at both upstream and downstream stations. It was felt that the available data might be of some use in approaching the question of the significance of the flood plain and the frequency with which it is flooded. The frequency which the bankfull discharge represents at each of the nine stations was computed as follows:

Records were available at Coatesville and Chadds Ford from which curves showing the relation between discharge and recurrence interval could be drawn. The annual-flood method was used in analyzing the 8-year record at Coatesville and the 19-year record at Chadds Ford; this means that only the highest instantaneous peak recorded in a year was used in plotting the high flows. (See Jarvis and others, 1936, for standard methods of determining flood frequencies.)

At Cornog, Seven Springs, South Downingtown, Georgia, Embreeville, Wawaset, and Lenape, where no records existed, curves were drawn for each station by plotting the discharge per square mile at the station against the discharge per square mile at Chadds Ford. Figure 30, which uses the South Downingtown station as an example, illustrates how this was done.

The computed discharge at the bankfull stage at South Downingtown is 1,600 cfs, or 18.6 cfs per square mile. Figure 30 shows that this discharge corresponds to a discharge of 28 cfs per square mile at Chadds Ford (see arrows). The Figure indicates that this flow has a recurrence interval of 2.7 years at Chadds Ford; that is, the annual maximum flow will equal or exceed 1,600 cfs, the discharge at bankfull stage, on the average of once every 2.7 years.

In constructing the curve in part *a*, measured low-flow discharges per square mile of drainage area at South Downingtown were plotted against the daily mean flow at Chadds Ford; at high flows, discharge values at peak stages, obtained from a crest-stage recorder at South Downingtown, were plotted against corresponding peaks obtained from the continuous rec-

ord at Chadds Ford. This same method was used in calculating the recurrence interval of bankfull stage at Cornog, Seven Springs, Georgia, Embreeville, Wawaset, and Lenape.

The method is obviously not exact, because the assumption of similar frequencies being experienced upstream and downstream is not strictly true for infrequent flash flows. Too, the extrapolation required to obtain the discharge at the bankfull stage introduces the possibility of error. Nevertheless, the following results may be of some significance.

Figure 31 is a graph of drainage area plotted against the recurrence interval of the bankfull stage for nine locations on Brandywine Creek. The graph shows that at most places the annual maximum discharge will equal or exceed the bankfull stage on the average of every 1.2 to 2.7 years. The average recurrence interval is 2.2 years. The curve also suggests that the bankfull stage may be reached somewhat more often downstream than it is upstream.

The overbank stage was not used here because of the difficulty in obtaining a consistent definition of the overbank stage, as well as the problem of obtaining discharge measurements at overbank stage. A close examination of most of the cross sections (fig. 16) shows that near the overbank stage the channel begins to flare out and thus departs from the more rectangular shape prevailing below this stage. Because of this and because, as already shown, the increase in velocity is not constant, the increment of discharge within this flaring cross section between the bankfull stage and the overbank stage is difficult to calculate. This change in the shape of the channel may itself be related to the same variables, such as the load, width, and depth, which determine the height and structure of the flood plain.

Because the difference in discharge between the bankfull stage and the overbank stage is small, a tentative conclusion concerning the frequency of flooding of the flood plain may be made. The flood plain is attained or just over-topped by a discharge corresponding to the annual flood that is equaled or exceeded once every 2.2 years. When expressed in terms of a partial duration series—that is, a series of floods above a given base—the bankfull stage can be expected to recur about once every 2 years (Langbein, 1949, p. 880). Although there is a faint suggestion that the frequency with which overbank flooding takes place decreases upstream, the frequency with which overbank flooding takes place both upstream and downstream ranges between narrow limits. The fact that flooding of the flood plain occurs periodically indicates that the flood-plain height is closely related to the regime of the stream which traverses the flood plain.

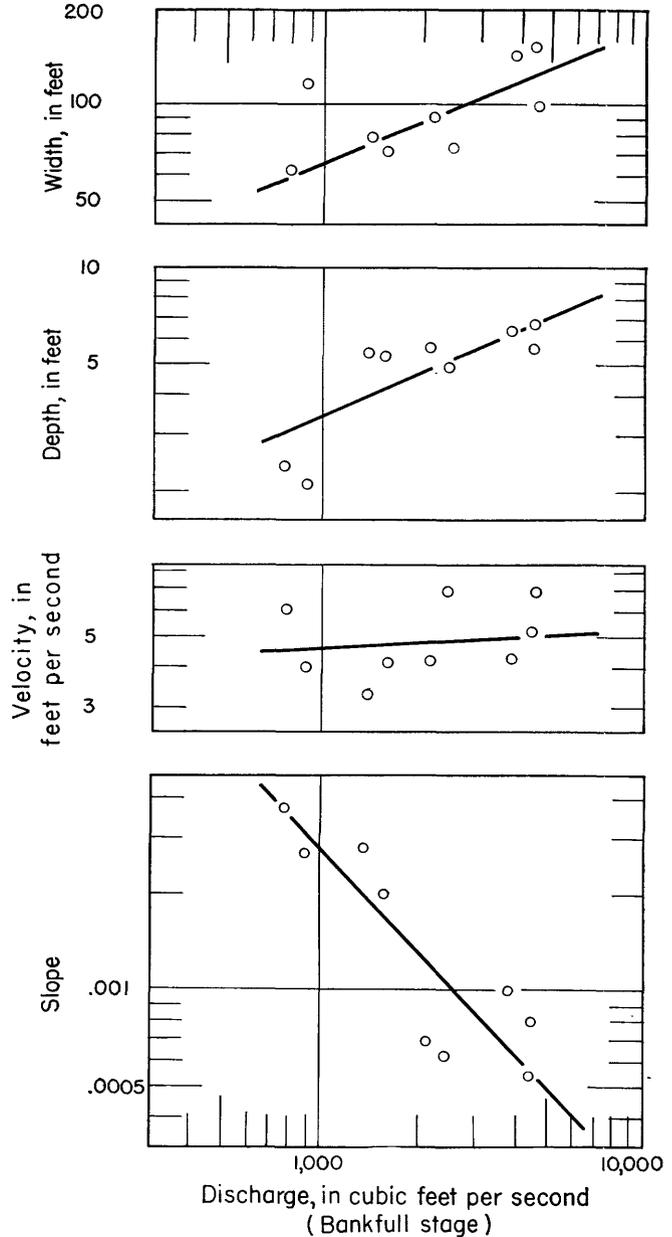


FIGURE 29.—Variations of width, mean depth, mean velocity, and slope of the water surface with discharge in the downstream direction at the bankfull stage.

If the periodicity of flooding actually proves to be uniform throughout a particular region, a corollary conclusion might be that a stream does not gradually entrench itself below its flood plain until over-bank flooding becomes impossible. A given flood plain will cease to be flooded only as a result of fundamental alterations in the regime of the stream such as climatic or tectonic changes might induce.

**THE HEADWATER TRIBUTARIES**

Thus far only the data from nine cross sections on the Brandywine Creek ranging in drainage area from

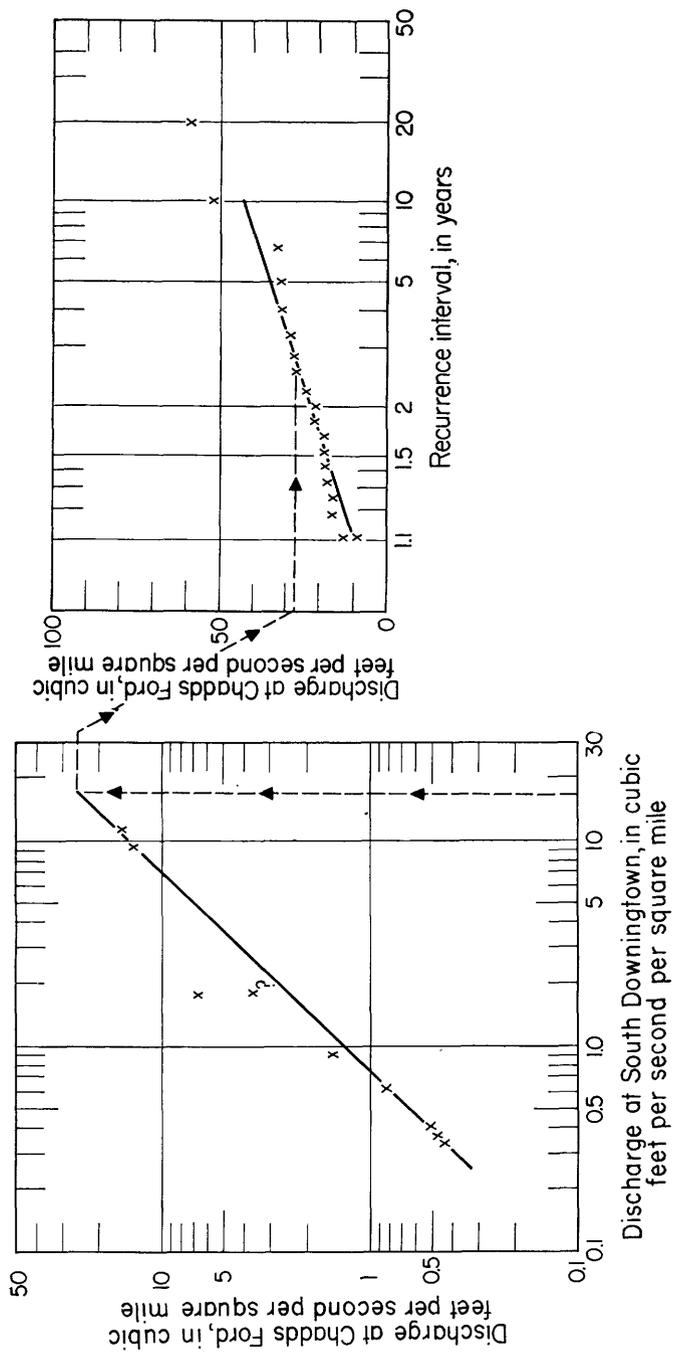


FIGURE 30. Graphs showing method of determining recurrence interval of the bankfull stage at seven of the principal measuring stations. Discharge per square mile at South Downingtown compared with related discharge per square mile at Chadds Ford having known recurrence interval.

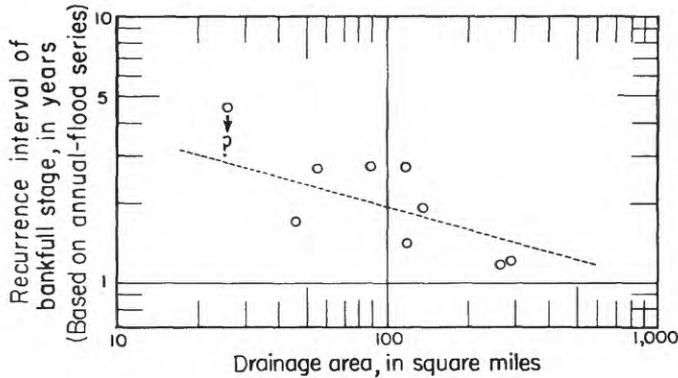


FIGURE 31.—Relation of drainage area to recurrence interval of the bankfull stage at the principal stations on Brandywine Creek.

25 square miles at Cornog to 287 square miles at Chadds Ford have been considered. Measurements of the discharge have been made at upstream locations on tributaries having drainage areas as small as 0.1 square mile (fig. 32) and downstream at places having drainage areas as much as 298 square miles (fig. 1). With the exception of the major cross sections and of five locations on Buck Run at which three measurements at different discharges were made, a single measurement of the discharge has been made at each of the other points. These measurements of the discharge, width, depth, and velocity are tabulated in appendixes B and C.

In order to compare the changes in the width, depth, and velocity which take place at two durations of flow as the discharge increases in the downstream direction, it was necessary to determine two discharges at each point: One was the flow at the section which is equaled or exceeded 50 percent of the time; and the other, the discharge at the same point which is equaled or exceeded 2 percent of the time. These durations were arbitrarily selected as representing low and moderately high flows on Brandywine Creek. For this purpose graphs of discharge plotted against drainage area for the nine stations were made. On this graph (fig. 33) discharges at two separate durations of flow plot as two nearly parallel lines. At each cross section of the stream where a measurement of the discharge was made, the contributing drainage area was calculated by outlining it on a topographic map and measuring the area with a planimeter. The discharges corresponding to this drainage area for durations of 50 and 2 percent were read from the curves in figure 33. It is possible that for small drainage areas the extrapolation of the relationship of discharge to drainage area as a straight line is not accurate. Because no other method is available and because positive data are lacking which indicate that a curve rather than a straight line should be

drawn, the method may be adopted for present purposes.

The required discharges for each station having been determined, it was necessary to extrapolate the measured widths and depths to the specified discharges. The width was obtained by extrapolating from the measured width plotted against discharge, using a straight line having a slope of .04. This is the mean  $b$  value obtained from all of the at-a-station curves of width and discharge. Similarly, the depth was extrapolated from the measured point along a straight line whose slope was .41 or equal to the mean  $f$  value of the at-a-station curves. The following example illustrates the method used in determining the discharges of the given durations and the corresponding width, depth, and velocity.

A measurement of the discharge was made near Marlboro, Pa., on Pocopson Creek, at which point the discharge was 1.45 cfs, the width 7 feet, and the depth 0.35 foot. In figure 33 the width and depth are plotted against this discharge. At this point on Pocopson Creek the contributing drainage area is 4.07 square miles. Figure 33 indicates that for a drainage area of 4.07 square miles the discharge equaled or exceeded 50 percent of the time is 3.3 cfs, and the discharge equaled or exceeded 2 percent of the time is 18.0 cfs (see arrows). Extrapolating from the known points along the lines with slopes equal to  $b=.04$  for the width and  $f=.41$  for the depth, one finds that for a discharge of 3.3 cfs the width is 7.2 feet and the depth 0.49 foot. Thus the velocity is  $\frac{Q}{wd}=0.94$  foot per second. Similarly, at a discharge of 18.0 cfs the width is 7.7 feet, the depth is 0.99 foot, and the velocity is 2.36 feet per second. These



FIGURE 32.—Typical small tributary at low flow. Illustrates smallest channels included in comparison of downstream changes in channel characteristics shown in figures 34 and 35.

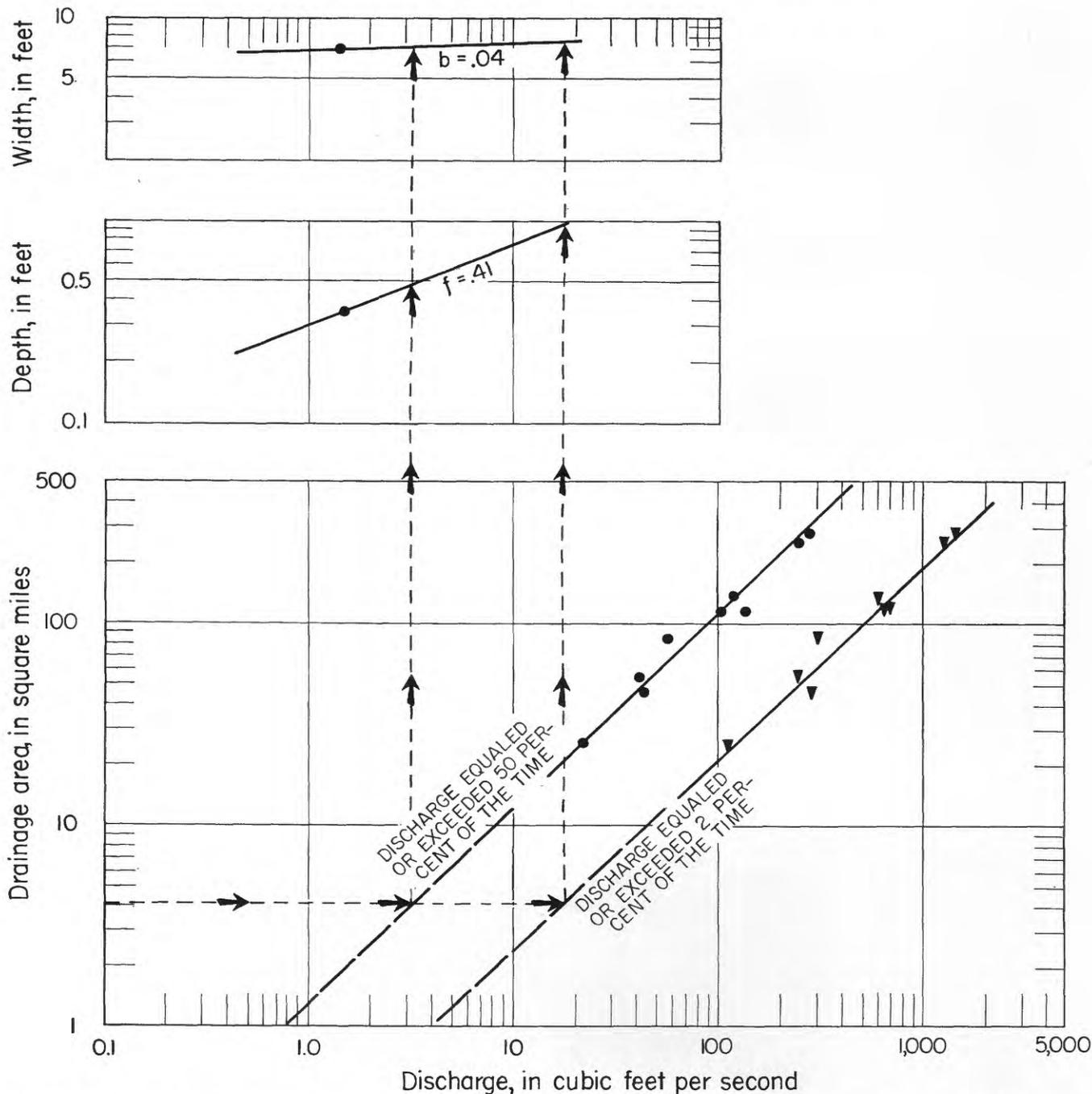


FIGURE 33.—Graphs showing method of extrapolation used to obtain discharges equaled or exceeded 50 percent and 2 percent of the time and corresponding values of the width, depth, and velocity at individual measuring sections. Method illustrated by extrapolation of measurement made on Pocopson Creek below Marlboro.

are the values of the variables at this point which are subsequently plotted on the downstream curves in figures 34 and 35. Some justification for the use of this approximate method is provided by the consistency of the at-a-station curves and by the behavior of the downstream curves of width, depth, and velocity.

Figure 34 shows width, mean depth, and mean velocity plotted against discharge in the downstream direc-

tion for a duration of flow equaled or exceeded 50 percent of the time. Figure 35 shows these same curves at a duration of flow equaled or exceeded 2 percent of the time. The points from the nine principal stations have been differentiated from the others in order to emphasize the difference between the curves based on main-station data alone, and those based on data from the widespread stations shown in figure 1. These

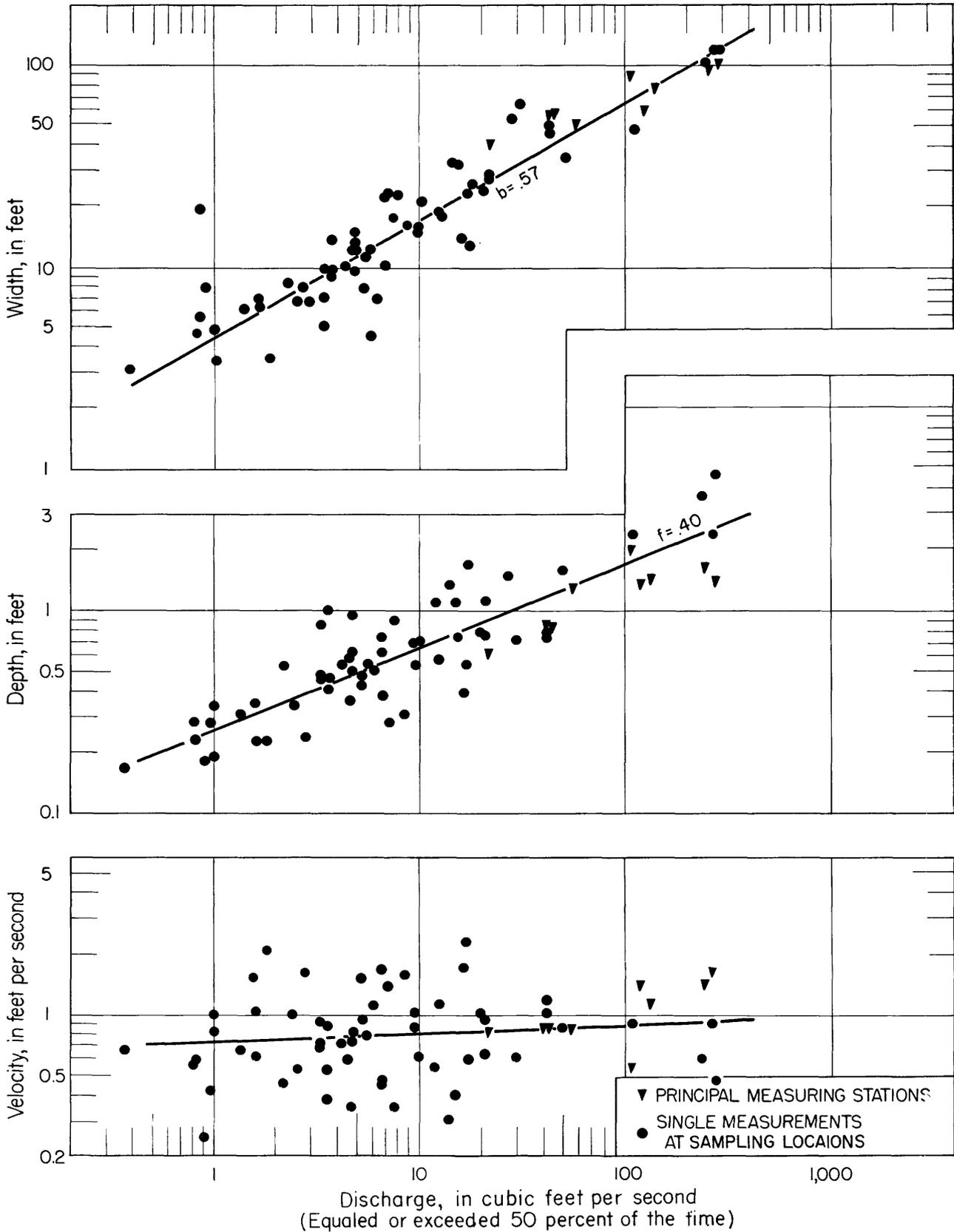


FIGURE 34.—Relation of width, mean depth, and mean velocity to discharge in the downstream direction from headwater tributaries to Wilmington, Del., for flow equaled or exceeded 50 percent of the time.

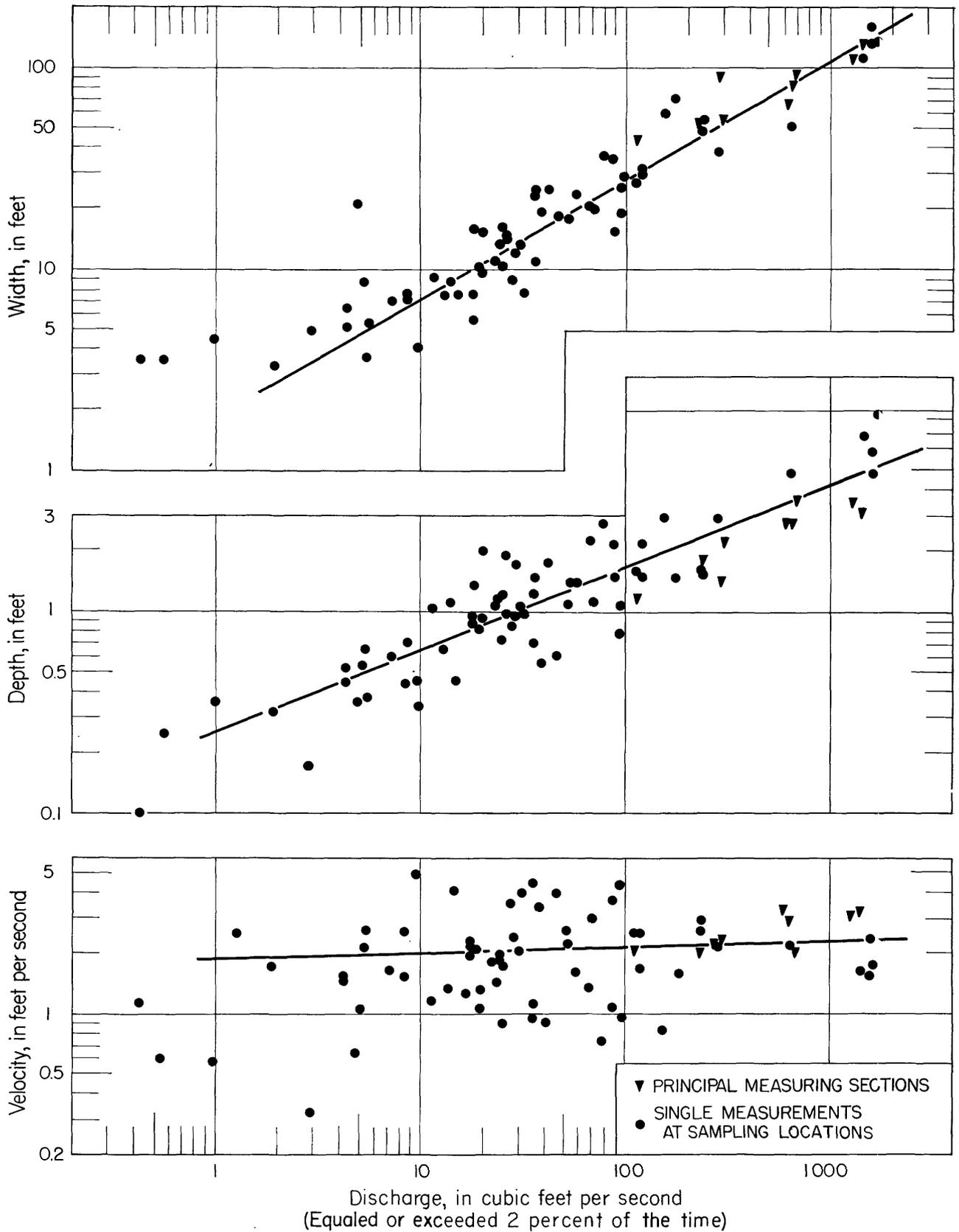


FIGURE 35.—Relation of width, mean depth, and mean velocity to discharge in the downstream direction from headwater tributaries to Wilmington, Del., for flow equalled or exceeded 2 percent of the time.

differences are also shown by a comparison of the  $b$ ,  $f$ , and  $m$  values listed in table 4.

Data from the entire watershed (figs. 34 and 35) indicate that changes in the variables in the downstream direction are the same at high flows as they are at low flows. The figures also show, however, that the rates of change of the variables are somewhat different when smaller drainage areas are included on the graph. Thus, the value of  $b = .57$  in figure 34 indicates that the rate of increase of width downstream with increasing discharge, all stations considered, is greater than is the rate of increase of width between Cornog and Chadds Ford. An  $f$  value of .40 shows, however, that the depth increases downstream over the entire drainage area at approximately the same rate as it does between Cornog and Chadds Ford. In consonance with the increased rate of change of width downstream, the rate at which the mean velocity increases is reduced. The mean velocity hence remains practically constant downstream.

The scatter of the points on these downstream curves of width, depth, and velocity will be considered later. Despite the extrapolations and the amount of scatter, particularly on the graph of velocity against discharge in the downstream direction, the general direction and relative amounts of the changes can be seen.

#### SUMMARY OF VARIATIONS IN THE DOWNSTREAM DIRECTION

At all stages of flow, the width and depth increase as the discharge increases in the downstream direction. Also, the velocity increases or remains constant. At all frequencies or stages of flow, the slope of the water surface decreases in the downstream direction.

The physical factors that control these downstream changes are the same as those that fix the behavior of the geologic and hydraulic variables at-a-station. The discharge and the suspended-sediment load are functions of the climate and of the drainage area contributing to the stream. There is some evidence which suggests that these independent factors play a part in determining the ratio of width to depth in the channel. As the contributing drainage area increases downstream, there is a change in the proportion of suspended material to water in the channel. The rate of increase of width in the downstream direction may be closely related to changes which take place in the concentration of the suspended load (fig. 27). Concomitantly with these changes in the discharge and suspended load the material on the bed of the stream is being abraded, sorted, and removed. At any cross section the composition, size, and shape of the bed material are originally independent of the channel itself. Once within the channel, however, the material is altered by the processes taking

place within the channel. Thus the hardness, initial shape, and sorting of the bed material are independent; yet in any given reach the actual properties of the material are the initial ones which have been modified both in place and in movement by the flowing water.

At any reach, therefore, the channel is given a specific volume of water, a specific suspended load, and a local type of rock modified by other types of rock brought in from upstream. The tendency on the Brandywine is for the aggregate size of the bed material to decrease downstream. This can be seen in figure 36 on the graph showing slope plotted against size of bed material. The data on figure 36 represent slope, bed material size, and other characteristics of all the principal measuring stations except Georgia, which was eliminated because computations of the roughness parameter were meaningless owing to the small values of water surface slope associated with the pool reach where the measurements were made.

Local influences such as the slabby material at South Downingtown may modify this tendency for bed material to decrease in size downstream. When the discharge, channel shape, and channel roughness are fixed, the depth and velocity of the water are determined. Assuming complete adjustment where the flow is steady and uniform, the slope would be adjusted in accordance with the given velocity, depth, and roughness. Despite the correlation between grain size and the slope, both of which decrease downstream (see fig. 36), neither the mean velocity nor the computed velocity at the surface of the bed decreases in the downstream direction. Therefore, it cannot be assumed that because slope and grain size are correlative, the slope is simply adjusted to the velocity required to carry the smaller particles.

It must be emphasized that the classification of a variable as being independent rests upon certain rational assumptions. The data for the at-a-station and downstream changes simply suggest a number of correlations among the variables. Which of them are the independent ones cannot be determined from the data alone.

#### VARIABILITY OF CROSS SECTIONS IN A SHORT REACH: THE POOL AND RIFFLE PROBLEM

Anyone who has looked at a natural stream knows that there is rarely a reach along it which is truly uniform. Even where the stream looks straight, it almost always possesses minute distortions. The channel is alternately wide and narrow, straight and winding, deep and shallow. These deviations within a given reach will here be called local variations. While all cross sections are experiencing the same discharge, the areas of these cross sections are different

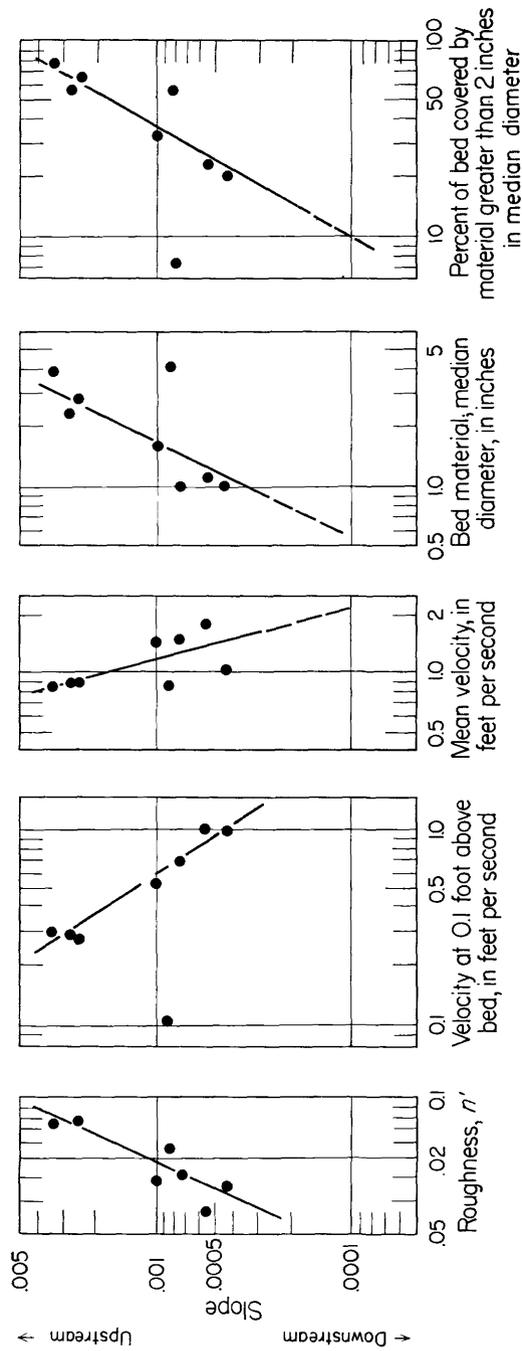


FIGURE 36.—Relation of water surface slope (which decreases downstream) to the size of the material on the bed, to mean velocity, to velocity near the bed, and to the roughness parameter  $n'$ . Data represent 6 of the measuring stations established for this study and the 2 gaging stations, all at a duration of flow equaled or exceeded 50 percent of the time.

and hence their mean velocities are also different. We have seen above that the scatter of points on the curves of width, depth, and velocity plotted against discharge at Chadds Ford and at Coatesville results from the measurements not always having been made at the same cross section. The scatter is therefore closely related to the choice of the measuring section. In making comparisons of changes in the downstream direction it is important to be consistent in this choice. Theoretically, one should compare pools with pools, riffles with riffles, or "in-betweens" with "in-betweens." Thus, the problem of scatter is partly a problem of pools and riffles. These in turn become partly a problem of bends and meanders.

The following brief discussion of the importance of the local variations in the river channel consists of two parts. The first is an analysis of the magnitude of the local variations in width, depth, and velocity compared to the magnitude of the total change of these variables in the downstream direction. The second part concerns criteria which could be used to assure a consistent choice of cross sections.

**AMOUNT OF LOCAL VARIATION WITHIN A GIVEN REACH**

In addition to the measurement of the cross-sectional areas of certain stations where discharge was determined, the areas of the flowing water at 10 other cross sections near each station were also measured. These stations are: Cornog, Seven Springs, and South Downingtown on the East Branch, Brandywine Creek; Embreeville and Wawaset on the West Branch; and 5 locations on Buck Run. At Embreeville and Wawaset the 10 cross sections were at 200-foot intervals, 5 above and 5 below the discharge-measuring section. At each of the other locations the interval between cross sections was 100 feet. Thus, for each discharge measurement in a reach, there are 11 measured values of width, depth, and velocity.

Based on these measurements, figure 37 shows the effect of local variations in cross-sectional area on the downstream curves of width, depth, and velocity. It also indicates, within each reach, the range of values of the variables, as well as the value of each variable at the regular discharge-measuring section. The method of computation is now described.

Because the measurements in the field were made at different frequencies of flow, it was necessary to reduce both the discharge and some measure of the range of the variables to a constant frequency. A discharge having a frequency of 50 percent was chosen. The range delineated by the vertical lines on figure 37 was computed as follows.

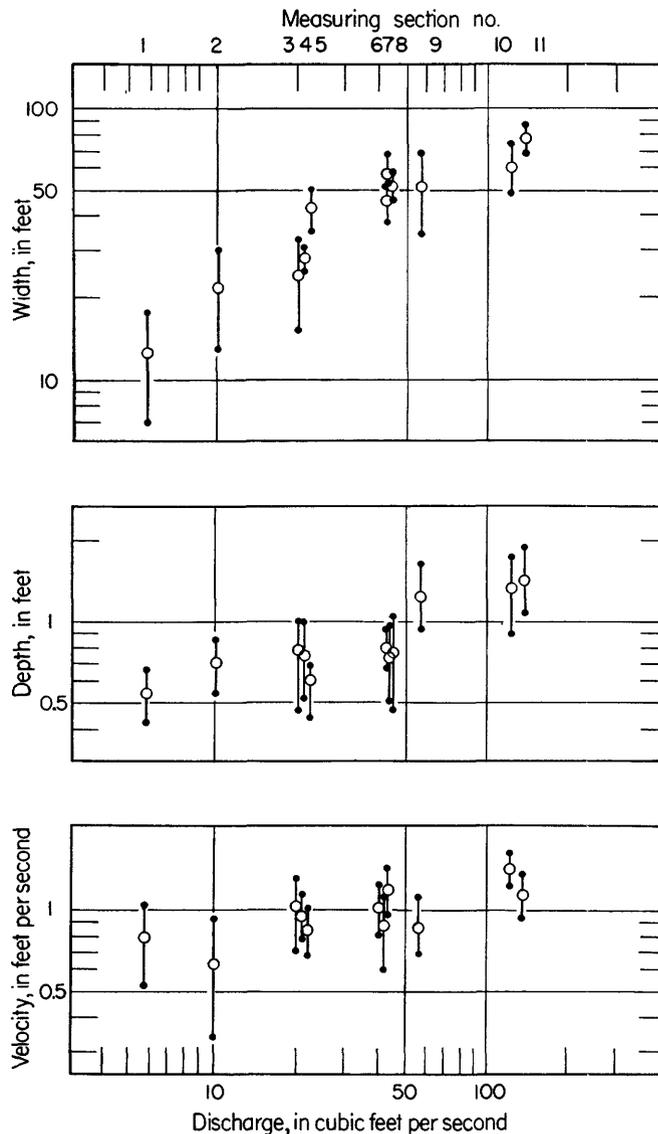


FIGURE 37.—Effect of local variations in cross section on downstream changes in width, depth, and velocity.

(NO. OF MEASURING SECTION)

1. Buck Run at Stottsville. 2. Buck Run near Newlin. 3. Buck Run near Ereildoun. 4. Buck Run above Doe Run. 5. Brandywine Creek at Cornog. 6. Buck Run at mouth. 7. Brandywine Creek at Seven Springs. 8. Buck Run 1 mile above mouth. 9. Brandywine Creek at South Downingtown. 10. Brandywine Creek at Wawaset. 11. Brandywine Creek at Embreeville.

A standard deviation from the mean of the 11 values of the width, depth, and velocity at each cross section was estimated. The standard deviation for each variable was expressed as a percent of the width, depth, or velocity at the cross section at which the discharge was made. The length of the vertical lines both above and below the mean is equal to this standard deviation expressed in percent multiplied by the width, depth, or velocity prevailing at the median-day dis-

charge. The method of computation for the width alone is as follows:

$s_w$  = estimated standard deviation of the 11 measurements of width from the mean

$w$  = measured width

$\frac{s_w}{w}(100) = p$  = standard deviation, in percent

$w_{50}$  = width at a discharge equaled or exceeded 50 percent of the time at a regular measuring section

$w_{50}(p)$  = standard deviation of width at discharge equaled or exceeded 50 percent of the time

$w_{50} \pm w_{50}(p)$  = plotting position of solid circles in figure 37

In using this method to reduce the discharge to a constant frequency, it must be assumed that the irregularities in the channel are the same at high and at low flow. Within the range of discharges involved in this correction, the assumption is probably true. Regardless of the merits of the assumption, however, it would be extremely difficult to obtain the actual measurements in the field at a time when each cross section was experiencing the same frequency of flow. Some such method of making the correction therefore is required.

The scatter of the points on the curves of width, depth, and velocity plotted against discharge in the downstream direction is increased by the addition of the local irregularities. Nevertheless, the trends of the variables in the downstream direction are similar to those shown above on figures 25, 34, and 35. The width increases downstream more rapidly than does the depth, and the velocity increases slightly or remains constant in the downstream direction. These data indicate that the apparent rates of change of the variables in the downstream direction can be influenced by the choice of the measuring section within each reach. They also indicate, however, that their effect is to modify but not to mask the changes observed above.

Unfortunately, because the widths, depths, and velocities in a natural channel do not have a normal, random distribution, no simple statistical computation procedure can be used to test the reliability of the 11 measurements. Similarly, because there is a real variation in natural-channel cross sections an infinite number of measurements would not reduce the error to zero. This fact was demonstrated in part by comparing the scatter of points on downstream curves in which the means of the 11 measured cross sections at each reach were used with the scatter on similar curves based upon data from the single discharge-measuring cross sections. By this method it was possible to bring a few of the points into better alinement. For example, the cross sections regularly measured at Wawaset

and at South Downingtown are somewhat narrower than the mean for those reaches. Similarly, the discharge-measuring section at Embreeville is wider than the average for the reach. The improvement—that is, reduction in scatter—brought about by using the mean cross sections in the downstream curves, was not great, however.

#### CRITERIA FOR DISTINGUISHING POOLS FROM RIFFLES

Straub (1942, p. 616), Fargue (1908), and many others have pointed out that pools and riffles in an alluvial channel are associated with the bends and crossovers of meanders. Bends and pools and riffles are characteristics of almost all natural channels. True meanders are rare on the Brandywine Creek, but where the channel is not tightly confined by sheer rock walls, truly straight reaches are equally rare. Detailed mapping of apparently straight reaches on Buck Run and on Valley Creek indicated that the thalweg of the flow winds back and forth within the channel. Because pools and riffles, whether at bends or elsewhere, are natural features of any channel, any comparisons which are made between the shapes and gradients of the channel at different points must take into account systematic variations related to pools and riffles.

Analysis of the scatter on the at-a-station curves for Chadds Ford and Coatesville indicates that the changes of the variables in the downstream direction would probably show fewer irregularities if a rational method of differentiating pools from riffles had been devised.

By use of a plot of  $\frac{v^2}{d}$  against discharge, because pools are distinguished by low velocities and large depths, an arbitrary separation of pools, riffles, and "normal" cross sections could be made. Such a division, however, is arbitrary because of the continuous range of values of  $\frac{v^2}{d}$  between the obvious riffles at the maximum values of  $\frac{v^2}{d}$  and the obvious pools at the minimum values of  $\frac{v^2}{d}$ . Using only "normal" cross sections in

plotting the downstream curves of width, depth, and velocity, it was possible to eliminate some extremely aberrant points such as Georgia (fig. 25). There is a limit, however, to the amount of scatter which can be removed from the downstream curves in this way. Aside from the arbitrary nature of the selection, the method does not take into account the changes in width which accompany changes in velocity and depth. Many of the cross sections shown in figures 34 and 35 may have relatively low depths, but because of the greater width of the cross section, the velocity is not

exaggerated. The measurements made thus far show no correlation of width with either pools or riffles. Although one might expect the pools to be somewhat wider, the coarser material on the bed of the riffles may result in widening at the riffles at the expense of the more erodible banks. The mutual adjustment of the three variables is likely to make any simple differentiation of pools and riffles unsatisfactory.

Besides the effects of pools and riffles on the cross section, innumerable other factors produce irregularities in natural channels. Trees, brush, weeds, plants in general, single stray boulders or erratics, and all works of man modify the channel. Their effect cannot be eliminated and will undoubtedly be present in the kinds of downstream comparisons which are made in this study. In addition, at least in figures 34 and 35, the necessity of using extrapolated data may add to the amount of the scatter.

From this analysis of channel irregularities it can be seen that on a stream such as the Brandywine, and probably on most natural channels, it is impossible to eliminate all of the scatter of the points on the curves of the width, depth, and velocity plotted against discharge in the downstream direction. In homogeneous material where the stream is able to adjust all of the variables with equal ease, it is conceivable that the scatter would be reduced to a minimum. Nevertheless, where bends are allowed to form and pools and riffles exist, no random choice of sections will provide perfect curves of the change in width, depth, and velocity in the downstream direction. As the following pages will show, the variability of the downstream curves is closely related to the ideas of regime, equilibrium, and grade, as these terms are used by geologists and engineers.

#### EQUILIBRIUM, GRADE, AND THE LONGITUDINAL PROFILE

The analysis of the interrelationships of the width, depth, mean velocity, roughness, and slope, both at-a-station and in the downstream direction on Brandywine Creek, indicates that within a single small drainage basin the behavior of these variables conforms to the generalizations derived by Leopold and Maddock (1953) from the study of a number of larger rivers. Most of those rivers, to the eye at least, are very different from Brandywine Creek; yet between those larger rivers and Brandywine Creek there is a striking similarity in the relationships among the variables. This similarity and the fact that changes of several variables take place harmoniously and simultaneously suggest that a kind of equilibrium controls the development of the river channel and its longitudinal profile. Con-

sideration of the Brandywine may permit one to extend the understanding of equilibrium along the lines suggested by Leopold and Maddock.

The geologist's concept of a graded river also involves a consideration of the phenomena that tend to produce and maintain equilibrium. The observations on Brandywine Creek suggest to the author certain relationships between the geologist's concept of grade in rivers and the type of quasi-equilibrium which exists on the Brandywine. This discussion includes several parts. First, it is argued that the cross section and the gradient of Brandywine Creek represent the spontaneous or nearly spontaneous adjustment of a number of variables to external controls; second, that these adjustments suggest at least a phase of an equilibrium process which is not related to the length of time during which a given set of conditions exists; third, there is an analogy between behavior of self-adjusted irrigation canals and Brandywine Creek which indicates that given a sufficient period during which conditions remain unchanged, a river channel will tend to reach a steady state; fourth, the characteristics of this stable state or dynamic equilibrium can be gleaned from the definitions used by engineers and geologists to describe channels in equilibrium; fifth, the geologist's concept of grade has overemphasized the importance of a uniform concave longitudinal profile as evidence of the existence of channel equilibria. While the measurements on the Brandywine do not provide ample evidence with which to support each of these points, the creek does provide a starting point from which to consider these broader problems.

#### SIMULTANEOUS ADJUSTMENTS OF THE VARIABLES

It has been shown that on Brandywine Creek, as in other rivers, the suspended load is related to the channel shape. On the Colorado River near Grand Canyon, Ariz., on the Rio Grande at Bernalillo, N. Mex., and on the San Juan River near Bluff, Utah, Leopold and Maddock (1953, p. 30) described the sequence of changes in the width, depth, velocity, and suspended load which take place during the passage of individual floods. These analyses indicate a direct correlation between changes in the suspended load and changes in the width, depth, and velocity of the water in the channel. The measurements on Brandywine Creek show that during the passage of a flood the concentration of suspended material changes and the values of the other variables also change. Despite the fact that the specific rates of change of these variables differed from place to place, the relative magnitudes of the changes in the variables and their direction were the same at all of the stations. It is clear that despite these changes the channel shape is

much the same before and after the flood. In Brandywine Creek the changes in width, depth, and velocity accompanying changes in discharge are not associated with scour and fill of the channel bed to nearly the same extent as in the large rivers in the West referred to above.

Because of the relatively small amount of scour and fill observed in Brandywine Creek the observed changes in suspended load in a given reach must be attributed primarily to changes in the suspended load supplied to the reach from upstream. The load, therefore, constitutes as external control. As the observed changes in width, depth, and velocity with discharge are similar to the changes observed in the large rivers, they also appear to represent on the Brandywine rapid simultaneous adjustments to changes in external controls.

Several tentative conclusions are suggested by these observations. First, factors which control the shape and profile of the channel are the same everywhere along the channel. Second, the data on Brandywine Creek and analogies drawn from data on some other rivers indicate that the channel and the flood plain associated with it are the result of the simultaneous interaction of a number of variables. Third, presumably this quasi-equilibrium prevailed during the development of the channel and the flood plain (the walls of the channel are the flood plain). The changes in the variables which occur at-a-station with increasing discharge in the present channel presumably continue to reproduce the pattern of changes which led to the development of the channel. This adjustability, which has been demonstrated both in the large rivers and in Brandywine Creek, is one aspect but not the only aspect of stream equilibrium.

#### COMPARISON OF BRANDYWINE CREEK WITH STABLE IRRIGATION CHANNELS

In addition to the orderly changes of the variables which take place with changing discharge at a given cross section, it has been shown that these same variables change in an orderly progression downstream on the Brandywine Creek. These changes in turn must be related to the succession of changes which take place in the discharge, sediment load, and material on the bed. Despite the scatter of points, the consistency of these downstream changes as illustrated in figures 25, 34 and 35 is such as to suggest that a quasi-equilibrium prevails in the downstream direction. This is to be expected because the discharge and sediment load, as well as the volume, density, and size of the bed material at any reach, are intimately related to the reaches above and below it. Regardless of local

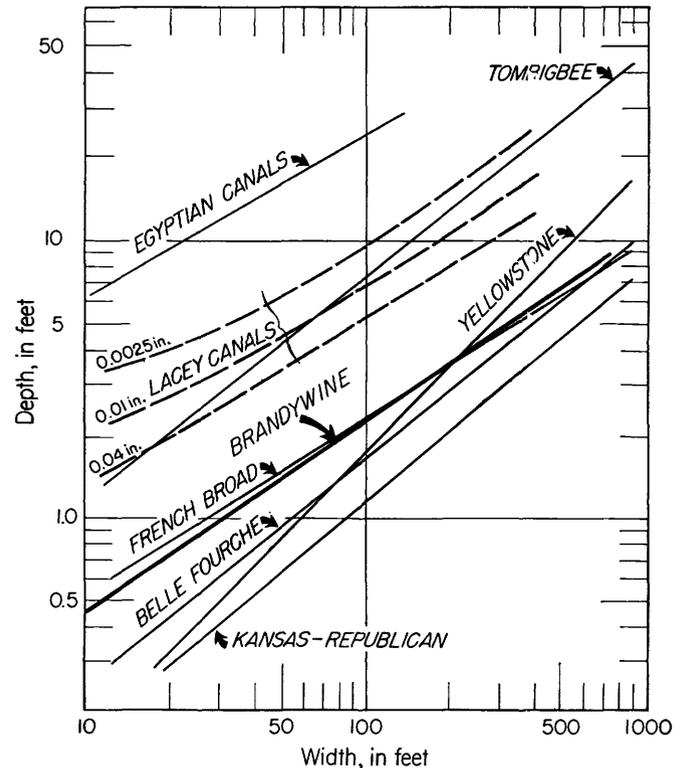


FIGURE 38.—Relations of width to depth in canals and rivers in alluvial material compared to relation of width to depth in Brandywine Creek.

influences, certain variables, such as the amount of water and sediment, maintain their continuity downstream. As Gilbert (1877) termed it, "There is interdependence throughout the system." As the following comparison of Brandywine Creek with unlined irrigation canals will show, if the controlling variables remain constant over a long period of time stability might be attained throughout the entire river system.

The similarity between these downstream changes in a natural river channel and those which take place in stable irrigation canals has already been described in detail (Leopold and Maddock, 1953, p. 43). For comparison with the Brandywine data, table 5 shows several formulas derived primarily from canals and rivers in India. Additional comparative data are provided by figure 38 which shows the relationship between width and depth in a number of different rivers and canals (Lane, 1937, p. 130). The curve shown for Brandywine Creek in figure 38 was computed using values of  $b = .57$  and  $f = .40$ . These are the rates of change of the width and depth with discharge, respectively, and are derived from the curves shown in figure 34 in which all sections were used in making the down-

stream comparisons. The slopes of the lines in figure 38 were derived from the equations as follows:

$$w = aQ^b \quad (3)$$

$$d = cQ^f \quad (4)$$

Then

$$Q = \left(\frac{w}{a}\right)^{\frac{1}{b}} = \left(\frac{d}{c}\right)^{\frac{1}{f}}$$

and

$$d = \frac{c}{a^{\frac{f}{b}}} w^{\frac{f}{b}}$$

Substituting the values from figure 34,  $d = .089w^7$ .

Figure 38 includes three separate curves derived by Lacey for canals in India in which the bed material is of three different sizes. These lines are nearly parallel to the width-depth curve for Brandywine Creek. A comparison of these curves with the one shown for the Brandywine suggests that the coarser material on the Brandywine is responsible for its lower intercept at a given width. Additional data, however, do not support this conclusion. The curves of width plotted against depth for the five rivers shown in figure 38 were derived from the data presented by Leopold and Maddock (1953, p. 15). The material on the bed of the Kansas-Republican River is of the order of 0.00025 inch. On the Tombigbee River the grain size is also small, while on the Yellowstone-Bighorn-Wind River system the material ranges from gravel in the upstream areas to silt downstream. As figure 38 shows, the width-depth curves of the Kansas-Republican and the Tombigbee Rivers are nearly parallel to the curve of Brandywine Creek. The intercept of Brandywine Creek falls between the two. As the grain size on the Kansas-Republican River is 1,000-fold smaller than on Brandywine Creek, it might be expected that its intercept would be higher. Instead, it has the lowest intercept. This specific illustration, combined with the random distribution of all of the intercepts shown in figure 29, demonstrates that grain size alone does not fix the position of the curve. Presumably the rate of change of roughness in the downstream direction, the relative erodibility of the bed banks, and the nature and amount of the suspended load, also help to fix the position of the width-depth curve.

The rivers and canals compared in the figure have vastly different topographic, lithologic, and climatic environments. The striking feature of figure 38 then is the close agreement shown between the behavior of the width and depth in rivers and in canals. This agreement indicates that the processes which determine the shapes of natural alluvial channels are much

the same regardless of their geographic location. These same relationships are illustrated by the formulas in table 5.

It is interesting to note (table 5) the similarity of the rate of change of width with discharge in the downstream direction. In the Indian examples  $b = .5$ , while Schoklitsch (1937, p. 151) gives a value of .6 for natural rivers. These values are very close to the value  $b = .57$  found applicable to all stations on the Brandywine. Although the hydraulic radius increases to the one-third power of the discharge alone—that is,  $f = .33$ , in the Punjab Research Institute equation, the Lacey equations involving the velocity, depth, and slope include in addition to the discharge a silt load factor or a grain size factor. Nevertheless, the changes of these variables with discharge are in general of the same magnitude as those experienced on the Brandywine. The difference in the actual values may be related to the concentration of the suspended load and to the relative erodibility of the bed and bank, factors which have not been sufficiently studied on the Brandywine to allow detailed comparisons to be made with the Indian data.

Although the data for Brandywine Creek indicate that the channel of the creek behaves very much like a “stable” irrigation canal, the amount of scatter of points on the curves is greater for the Brandywine. The multiple causes of this scatter have already been discussed. It is not due solely to departure from so-called equilibrium, although the deviation of some few points may be attributed to this cause.

Unlike the Brandywine, the canals are all compared at a single discharge, generally a so-called “dominant discharge” (Blench, 1951a, 6.9). Thus a family of width-discharge curves represents (ibid., 5.30) the relation of width to discharge at one frequency for channels flowing in a number of different materials. A similar family of curves, as in figures 25, 34, and 35 for the Brandywine Creek, represent different frequencies of flow in the same materials. The concept of “dominant discharge” may be applicable to natural streams, such as the Brandywine. (See Mackin, 1948, p. 477, for geologic observations on the relative significance of various stages of flow.) The adjustments which have been observed at all stages of flow, however, indicate that the quasi-equilibrium prevails at more than one frequency of flow. A complete comparison of the canals with the Brandywine from the standpoint of scatter would require a family of curves to be drawn for each frequency of flow and for each type of rock through which the Brandywine flows. This is an impossible task. Unless it were possible first to eliminate the predictable or regular causes of the scatter on the downstream curves, such as the pools and

TABLE 5.—Formulas for stable channels in alluvial material for comparison with similar formulas for Brandywine Creek, Pennsylvania

Author	Location or other data	Formula <sup>1</sup>	Remarks
Lindley (1919)	India, canals	$P=1.984Q^{.506}$	
Lacey (1939)	India, Punjab	$V=1.138R^{.4995}$	
Do	India, Madras	$V=0.790R^{.508}$	Finer silt than above item.
Do	India, Rivers	$V=16.0R^{\frac{2}{3}}S^{\frac{1}{3}}$	
Do	India, Punjab	$P=2.80Q^{\frac{1}{2}}$	
Do	India, Jamrao	$P=2.50Q^{\frac{1}{2}}$	Only five points on curve.
Do	Rivers and torrents, India	$P=2.63Q^{\frac{1}{2}}$	
Do	India, canals	$V=1.155f^{\frac{2}{3}}R^{\frac{1}{2}}$	
Do	do	$S=0.000542 \frac{f^{\frac{2}{3}}}{Q^{\frac{1}{2}}}$	
Punjab Research Institute (see Lacey, 1939, p. 24)	India	$R=0.470Q^{\frac{1}{2}}$	
Do	do	$S=0.00209 \frac{m^{.86}}{Q^{.29}}$	
Woods, F. W. (see Lane, 1937, p. 127)	Canals	$B=d^{2.305} - \frac{1}{2}d$	
Molesworth and Yenidunia (see Lane, 1937, p. 127)	do	$d=(9060S+.0725)B^{\frac{1}{2}}$	
Schoklitsch (1937)	Rivers	$W=kQ^{.6}$	
Wolman	Brandywine Creek	$W=aQ^{.57}$	Tributaries included.
Do	do	$W=aQ^{.40}$	Average between principal stations.
Do	do	$d=.089W^{.7}$	Tributaries included.
Do	do	$V=kd^{.75}$	Do.

<sup>1</sup> The following use of symbols applies in this report to this table only:

$a$  = a coefficient.  
 $B$  = bed width, in feet.  
 $d$  = mean depth for wide channels, in feet.  
 $f$  = a silt factor.  
 $k$  = a coefficient.  
 $m$  = diameter of silt particle, in millimeters.

$P$  = wetted perimeter in feet.  
 $Q$  = discharge, in cubic feet per second.  
 $R$  = hydraulic radius, in feet; approximately equal to  $d$ .  
 $S$  = slope, in feet per foot.  
 $V$  = mean velocity, in feet per second.  
 $W$  = width, in feet; approximately equal to  $P$  and  $B$ .

riffles, the amount of scatter would not in itself be a significant measure of departure from equilibrium. The significant observation at this point is the close parallelism between data on Brandywine Creek and the canals.

#### CHARACTERISTICS OF STABLE CHANNELS

These stable channels in alluvial material (Lane, 1937), or regime channels as they have often been called, have been studied in considerable detail. Some understanding of the kind of equilibrium which prevails in these channels, and by analogy the kind of equilibrium which is suggested by the downstream curves on the Brandywine Creek can be gained by reviewing a number of definitions of regime and its synonyms.

According to Lacey (1939, p. 1),

Regime flow in silt transporting channels excavated in alluvium connotes physical stability, a balance between silting and scouring, and a dynamic equilibrium in the forces generating and maintaining the channel cross-section and gradient.

He adds (p. 4),

It is evident that Lindley's concept of regime which postulates a delicate and precise dynamic balance in all the dimensions of an open alluvial channel can very seldom be perfectly achieved. Effectively there are too many variables, and in practice not all are equally free to vary.

For regime to be established the fundamental requirements are that the discharge should be constant, the channel flowing uniformly in unlimited incoherent alluvium of the same character as that transferred, and the silt grade and silt charge a constant.

To rivers and torrents Lacey (p. 12) applied the term "quasi-regime."

Blench (1951a, 4.1) states that,

A channel is said to be in regime when it is capable of adjusting its cross-sectional form and/or longitudinal slope by means of alterations that the flow can impose on the solid, and exhibits an average equilibrium between the actions and counteractions there.

He states elsewhere (1951b, p. 2) that,

\* \* \* [if] a "specific gage" record shows a trend in time the river is not in regime but [its] behavior is consistent with classification as a "regime type." If the record shows no trend, the river is practically certain to be in regime during the period for all factors provided it is of regime type \* \* \*. Regime implies a time scale.

Several other words have also been used to convey much the same meaning as regime. Clom (1950, p. 4) points out that,

When the determining factors (slope, load, discharge, character of materials on the bed) have remained within reasonable limits for a considerable period, over-all cutting or filling of the stream bed ceases and the stream is said to be "poised."

Bryan (1922, p. 89) defined the "regimen of a stream" as,

The system or order characteristic of a stream—in other words, its habits with respect to velocity and volume, form of and changes in channel capacity to transport sediments, amount of material supplied for transportation, etc. The term is also applied to a stream which has reached an equilibrium between corrasion and deposition or, in other words, to a graded stream.

Kesseli (1941, p. 568) however, quotes Surell to the effect that "regimen" is a final period in the development of a river "during which the water overflows and returns into an invariable bed."

Mackin (1948, p. 484) uses a number of apparently synonymous terms in discussing the engineering literature concerned with transportation by running water. Thus he states that the " \* \* \* concept of the 'adjusted' or 'stable,' or 'regime' (in the sense of equilibrium) cross-sectional form" is emphasized in that literature. Very much earlier, however, Davis (1902, 1909 ed. p. 391) suggested that, " \* \* \* 'regimen' may better be used as meaning the rule of river action under which the balanced condition is developed and maintained \* \* \* ," rather than as the condition obtained.

Webster's dictionary (1941, p. 2,097) indicates that there is a distinction between *regime* and *regimen* in rivers as some of these definitions suggest. Presumably *regimen* originally implied stability or regularity and related to a particular state of a river system, whereas *regime* meant simply the prevailing mode or condition of a river. Bryan's dual use of the term, however, seems to weaken this distinction. In the engineering use of *regime* to describe a particular type of canal or river, *regime* is virtually synonymous with *regimen*. In the present paper, where *regime* is used it is intended to apply to a particular state or condition of the river and hence is used synonymously with most of the definitions of *regimen* given here. The comparisons to be made with canal data unfortunately require this usage.

Despite the wealth of terms, these collected definitions have several fundamental points in common. First, *regimen*, *regime*, *equilibrium*, and *poised*, as used in these definitions, all imply a certain amount of stability in time. Second, they imply flexibility in the adjustments made by the channel itself. Third, there are no simple fixed criteria by which we can tell whether a given stream is or is not "in regime," or is "poised," or "exhibits an average equilibrium." Fourth, the definitions do not imply that the adjustments of slope are more important than the adjustments in the cross section, or vice versa. It is also important to note that the "ideal" or perfect equilibrium is seldom attained because of many irregularities present in natural materials.

#### THE BRANDYWINE: A CHANNEL IN EQUILIBRIUM

On Brandywine Creek it has been seen that rapid adjustments take place in the geologic and hydraulic variables within a given cross section. The various channel shapes and the similarity in the behavior of the variables in the channels suggest that the channel has been free to adjust itself. These adjustments have apparently been made in both the form of the cross section and its longitudinal profile. The data in figures 25, 34, 35, and 38 and in table 5 indicate that through these adjustments the Brandywine has developed a channel which is very similar to the channels found in regime canals. This particular channel shape and profile apparently represent a stable configuration. Unless the measurements on the Brandywine were repeated over a period of years, one could not say with assurance that the channel is truly stable. By analogy with the canals, however, and bearing in mind that there are no simple fixed criteria for determining whether a given stream is "in equilibrium," it is suggested that Brandywine Creek falls within the broad definitions of regime or quasi-equilibrium.

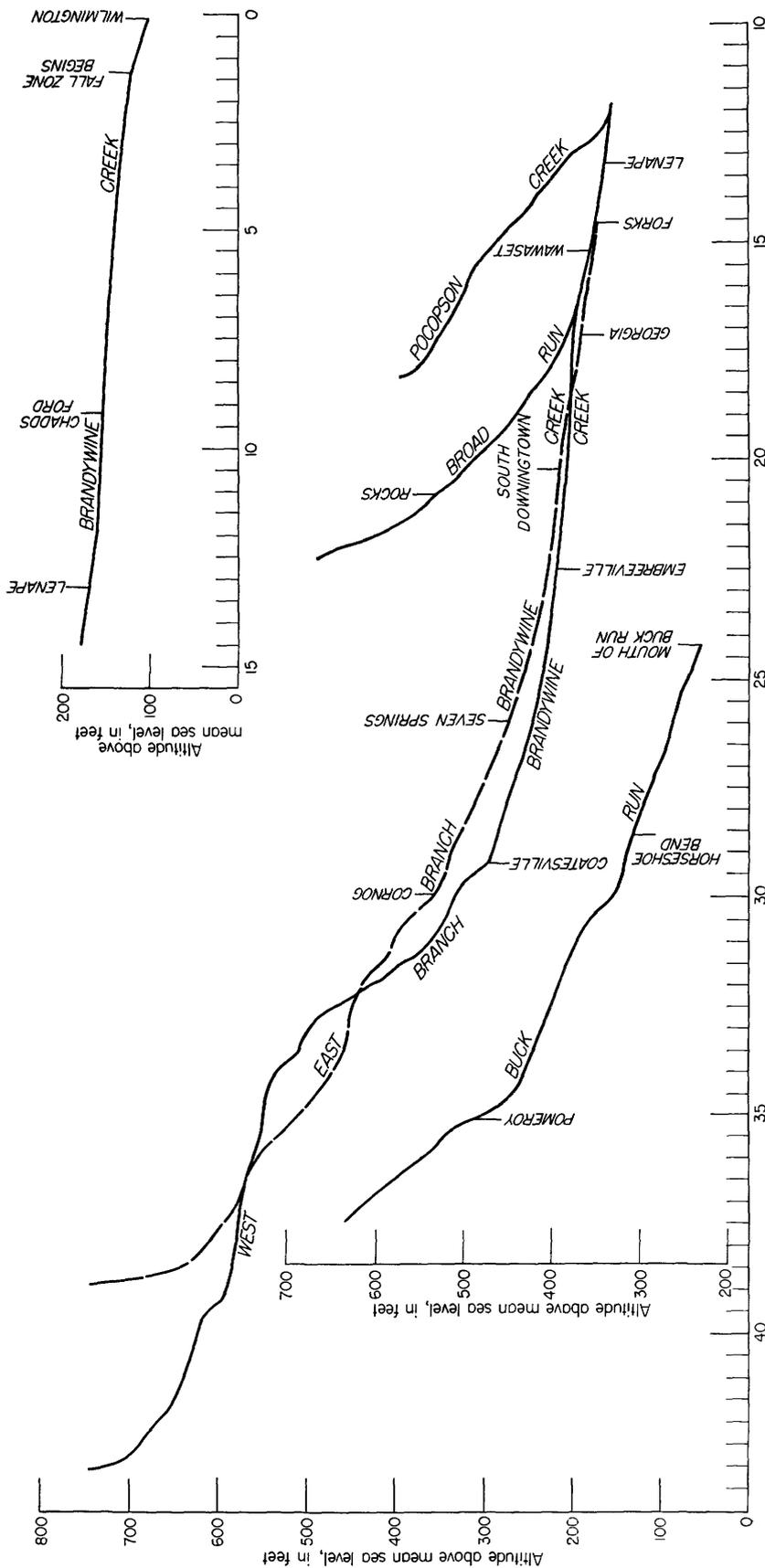
#### EQUILIBRIUM AND THE LONGITUDINAL PROFILE

The word "equilibrium" has also been much used by the geologists to describe a particular condition attained by natural rivers during the "normal cycle of erosion." In contrast to the emphasis given to adjustments in channel shape by the engineers, the geologic literature has emphasized the "equilibrium" slope almost to the exclusion of the cross-sectional form. Mackin (1948), Rubey (1952) and Leopold and Maddock (1953) have all emphasized the need for consideration of both cross section and profile in defining equilibrium. Brandywine Creek fits several definitions of equilibrium or quasi-equilibrium. In the minds of many geologists a condition of equilibrium is associated with a smooth concave-upward longitudinal profile. The conditions of quasi-equilibrium on the Brandywine, however, are not accompanied by a regular longitudinal profile (fig. 39). Because the geological emphasis on the profile stems from the introduction of the word "grade," it is of some interest at the outset to trace the history and meaning of this term.

Gilbert (1877, p. 114) in his original discussion of the mutual adjustment of velocity, discharge, declivity, and load in natural river channels referred to the attainment of an "equilibrium of action." Similarly, Davis (1902, p. 389) spoke of the "balanced condition." To express the equilibrium or balanced conditions Davis (p. 395) adopted the word "grade." He pointed out that,

River action in such a graded reach may justly be said to be

THE NATURAL CHANNEL OF BRANDYWINE CREEK, PENNSYLVANIA



Miles above Henry Clay Bridge, Wilmington, Delaware  
 FIGURE 39.—Longitudinal profile of Brandywine Creek showing main stream, East and West Branches, and several tributaries.

organized, inasmuch as a change in form or action at any one point involves a change at every other point.

Davis's words such as "balanced," "adjusted," and "organized" permitted a wide latitude of interpretation. From his own and other geologists' writings, however, it was clear that "grade" was very nearly, if not exactly, synonymous with "graded slope" (p. 399). Davis also spoke of "graded reaches" and of "graded river systems." Presumably, a graded river is a graded river system; that is, one which is graded from headwaters to mouth.

The original association of "equilibrium," "grade," and "graded slope," has led to the emphasis which has often been given to the correlation of a longitudinal profile that is concave upward with the idea of a river at grade (Cotton, 1941, p. 73; von Engel, 1942, p. 133). Both Baulig (1925) and Mackin (1948) have demonstrated, however, that the term should be applied not to a stream with a particular profile but to a stream in equilibrium. Baulig referred to the "profile of equilibrium" and pointed out that the profile of equilibrium may well depart from a regular parabolic curve. Mackin indicated that grade was a condition of equilibrium maintained primarily by adjustments in slope but added that these adjustments did not necessarily produce a uniform concave profile.

Although Rubey (1933, 1952) stressed the importance of considering both the cross-sectional form and the slope of the channel, the geologic literature has continued to emphasize the profile. Mackin (1948, p. 487), after considering the importance of changes in channel section, suggested that,

\* \* \* while adjustments in section may or may not accommodate the effect of a change in control, slope is always modified, by the stream itself, in such a manner as to absorb the effect of the stress \* \* \*. In the case of grade it means simply that the stream normally reacts to a change in controls calling for an increase or decrease in energy required for transportation by increasing or decreasing the total energy through modification of slope rather than by effecting economies in the energy dissipated in friction.

There is another reason besides Mackin's rational one for emphasizing slope in the maintenance of equilibrium in rivers. In the comparison of past streams, inferred from the reconstruction of the profiles of rivers from terrace remnants, with modern streams, and in the interpretation of geologic history from such comparisons, the longitudinal profile is the only tool available to the geologist.

The downstream curves on Brandywine Creek and the canal and river data shown in figure 38 suggest that the adjustment of the channel shape may be as significant as the adjustment of the longitudinal profile. There is no way in which one could predict that the

effect of a change in the independent controls would be better absorbed by a change in slope rather than by a change in the form of the cross section. This observation does not alter Mackin's (1948, p. 463) point that "grade is a condition of equilibrium in streams." It suggests, however, that slope is not necessarily the primary mechanism by which such an equilibrium is maintained. Leopold and Maddock (1953, p. 39) have also shown that a change in the governing factors may result in changes in channel characteristics without a change in river slope. Similarly, Odom (1950) points out that, because of the complexity of the relationship among discharge, bedload, and suspended load, the reduction in discharge of the Mississippi River below the Atchafalaya cutoff has not resulted in changing the slope of the Mississippi River below the cutoff. It is not yet possible to say with any degree of confidence what proportion of the adjustment to a change in external control will be taken up by slope as compared with cross-sectional form. Undoubtedly the characteristics of the country rock through which the stream is flowing have an effect upon the relative ease with which such adjustments can be made.

From these considerations one must conclude as Mackin did (1948, p. 507) that if we are going to continue on the present path, the meaning of *grade* should be synonymous with *equilibrium*. If grade is to be used synonymously with equilibrium, the emphasis on the longitudinal profile results in confusion because the existence of an equilibrium is not solely dependent upon slope. Further, the definition of *grade* whether it is applied to reaches or to river systems, should show no preference for adjustments in slope to adjustments in cross-sectional form.

As figure 39 shows, the profile of Brandywine Creek and several of its tributaries is broadly concave upward. Actually the overall profile consists of segments which are concave to the sky and separated by knickpoints. Analysis of the changes of the variables in the downstream direction suggests that at those cross sections in which measurements have been made the channel is in a state of equilibrium or quasi-equilibrium. Thus, by definition, throughout those sections the stream is at grade if grade is used synonymously with quasi-equilibrium. Figures 34 and 35 (see fig. 1 for location of stations) show then that a large proportion of the mainstream and its tributaries is at grade. Few measurements have been made at the knickpoints, but one measurement at Rocks, a bedrock outcrop on Broad Run is significant. At this point (app. C; figs. 34 and 39)  $Q=0.9$  cfs and  $w=20$  feet. A low flood plain at the measuring section composed of fine material which the stream is obviously capable of moving

indicates that the channel is adjustable. In addition, the velocity is consistent with that at points above and below Rocks on Broad Run. The velocity is not accelerating through the knickpoint reach. A consistent velocity is maintained through this reach of steeper slope by an increase in width and concurrent decrease in depth. Thus, the adjustability of the section and the continuity of the velocity indicate that the presence of bedrock at this point does not preclude the possibility of the stream being in quasi-equilibrium and, therefore, at grade.

The same case can be made for the knickpoint which occurs above Pomeroy on Buck Run. Here the stream passes through resistant sandstones where the channel is somewhat wider and the velocity somewhat greater than upstream or downstream. The reach is not a waterfall and possesses a flood plain throughout much of its length. Again, the adjustability of the cross section is evidence that a quasi-equilibrium exists at this section. Elsewhere in the basin, because of the presence of manmade dams, it has been impossible to study in detail the sections at which the knickpoints appear on the longitudinal profile. At Wagontown on West Branch Brandywine Creek, however, the knickpoint appears to be due to the presence of resistant amphibolite. Several miles below this point, about 1 mile above the city of Coatesville, Pa., resistant sandstones occur along the banks of the Brandywine and in its bed. There is no evidence to suggest that at either of these cross sections the channel cross section is adjustable.

The final form of the graded longitudinal profile may indeed be a smooth parabolic curve such as the curve derived by Unwin (1892, p. 503). Unwin showed that if the discharge increased downstream, and the width-to-depth ratio and the mean velocity remained constant, the longitudinal profile would be a smooth parabolic curve. Leopold and Maddock (1953, p. 48) also showed that the concavity of the smooth hypothetical profile was closely related to the rate of change of roughness in the downstream direction. Because of the fact that contributions of discharge and load to a natural river by its tributaries may be highly erratic, an absolutely smooth curve is not likely to conform precisely to the actual longitudinal profile of a natural river (Davis, 1902, p. 400).

Many studies have demonstrated the close relationship which exists between the slope of the profile of a river and the size of the material in the bed (Shulits, 1941; Pettijohn, 1949, references in chapter on transportation). As Mackin (1949, p. 483) pointed out, however, the causal relationship often expressed between large grain size and steep gradient is undoubtedly an over simplification.

On Brandywine Creek there is a direct relationship between grain size and slope (fig. 36). It is clear from the figure, however, that specific velocities "required" for the transport of materials of different sizes are not responsible for the correlation of slope and grain size. The mean velocity remains constant despite changes in slope and grain size. Not only does the mean velocity remain constant, but the computed mean bed velocity (O'Brien, 1937; Leopold, 1953) increases downstream. Neither mean velocity nor mean bed velocity may be the factor effective in moving materials. As the shear decreases in the downstream direction, it is possible that this factor may be more closely related to the decrease in caliber of material and decline in slope in the downstream direction than is the velocity. The general term "velocity," in any event, does not appear to be an appropriate parameter with which to explain the change which takes place in a river channel in the downstream direction.

With the exception of a few reaches in which the channel is flowing wholly within bedrock, Brandywine Creek is a graded stream. That is, it is a stream which has attained and maintains an equilibrium or quasi-equilibrium through the mutual adjustment of the discharge, load, bed material, slope, width, depth, and velocity. The adjustment may actually involve combinations of these variables or adjustments of other variables of a similar kind. In addition, the mutual adjustments of the variables take place whether the stream is at grade, aggrading, or degrading. Hence the processes of aggradation and degradation are not recognizable at any given time by maladjustments in the variables. Save over a long period of time during which finite changes may be detected in the shape or profile of a given reach, the differences between the river at grade, degrading, and aggrading may be indistinguishable. This dilemma forces consideration of another aspect of grade and equilibrium.

Strictly speaking, equilibrium is a stable condition. Running water carrying material in suspension and on the bed of a channel can erode the bed and by removing material continue to help erode its basin. Thus, in keeping with Davis's original conception, and Mackin's observation (1948, p. 500), the attainment of grade does not preclude denudation of the land. Mackin (1948) has emphasized the perfection of the dynamic equilibrium in a truly graded reach of a channel. He distinguished between a graded reach and one which is either slowly aggrading or degrading. This distinction emphasizes the aspect of stability of grade or equilibrium. Because true stability is difficult to reconcile with continued degradation of the land, Mackin uses "over a period of years" to define the length of time implied in the word stable. On the other hand, as the

time span is reduced, attention is focused upon the "equilibrium" processes of adjustment within the channel and less upon the stability of a particular shape and profile. The processes of adjustment are the same whether the stream is at grade, degrading, or aggrading. During each period, in accordance with the requirements of an equilibrium process, the variables may change so as to counteract the effect of a change in the external control. The concept of equilibrium implies both stability and the ability to make adjustments.

#### SUMMARY AND CONCLUSIONS

From this study of Brandywine Creek in Pennsylvania and from the previous discussion of the concept of grade, the following general conclusions may be drawn:

1. So far as it has been possible to measure them, the behavior of the variables both at-a-station and in the downstream direction on Brandywine Creek conforms to the generalizations derived by Leopold and Maddock (1953) from a larger number of rivers in the western United States.

2. In many respects the channel of Brandywine Creek also bears a remarkable resemblance to regime canals.

3. When casually observed in the field, the channel of Brandywine Creek is a picture of disorder. To the contrary, however, measurements of the discharge, width, depth, velocity, slope, and suspended-sediment load, and the computation of a roughness factor indicate that the cross-sectional form and the gradient of the channel actually approach a dynamic equilibrium.

4. The actual physical mechanisms by which adjustments in the variables are made are not known. The gradient and shape of the channel at any given reach are assumed to be primarily a function of the discharge and load supplied to the reach, and of the material supplied to the bed of the stream both within the reach and from upstream.

5. Throughout most of its length Brandywine Creek is a stream at grade, if grade is used synonymously with equilibrium. Like any other natural channel, however, neither the adjustability nor the stability of any parts of the channel conforms to the rigorous requirements of a stable equilibrium. Hence, quasi- or semi-equilibrium more nearly describes these conditions.

6. If grade is used synonymously with quasi-equilibrium, the data on Brandywine Creek confirm the observations of Baulig (1925), Mackin (1948), and others that neither a reach at grade nor a graded river system necessarily has a uniform longitudinal profile which is concave to the sky.

7. A decrease in slope in the downstream direction is associated with a decrease in size of the bed material. No specific functional relationship has been found to

explain this association. The problem of cause and effect is complicated by the fact that the frictional resistance of the bed material is not a function of grain size alone, but is also related to the shape, distribution, form, and packing of the particles on the bed.

8. The problem of grade, or equilibrium, in natural river channels is complicated by the number of variables involved and by the fact that the reactions are simultaneous. Quantitative, as well as qualitative, reasoning forces one to reject the over-simplified concepts often used to explain the morphology of rivers. Unhappily at the present time, an exposition such as this illustrates the complexity of the processes without offering a comparable or more nearly complete explanation. Because of the complex interrelationships of the variables, however, no explanation is likely to be complete that rests solely upon an analysis of a simple sequence of events.

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**APPENDIX**

**APPENDIX A.—Measurements of discharge and hydraulic variables at the seven principal stations established for this study on Brandywine Creek**

Date	Discharge <i>Q</i> (cfs)	Width <i>w</i> (feet)	Mean depth <i>d</i> (feet)	Mean velocity <i>v</i> (fps)	Area of cross sec- tion <i>A</i> (square feet)	Slope of water surface <i>s</i>	Roughness parameter <i>n'</i>	Remarks
<b>Cornog</b>								
Drainage area: 25.7 square miles. Length of reach over which slope was measured: 645 feet.								
Aug. 1, 1951-----	10.3	23.5	0.82	0.53	19.3	0.0033	0.140	Not at regular cross section.
Aug. 6, 1951-----	8.6	41.0	.41	.50	17.0	.0033	.094	
Aug. 11, 1951-----	12.9	41.0	.48	.64	19.6	.0033	.082	Velocity measured by float and corrected to mean velocity. Do.
Sept. 7, 1951-----	14.7	41.5	.52	.69	21.4	.0033	.080	
Sept. 10, 1951-----	5.7	39.5	.34	.44	13.6	.0033	.094	
Apr. 27, 1952-----	304	49.6	1.56	3.92	177.6	.0034	.030	
Apr. 28, 1952-----	830	55.8	2.44	6.10	136.0	.0037	.027	
May 27, 1952-----	73.4	44.0	.90	1.86	39.5	.0035	.044	
July 8, 1952-----	19.4	43.0	.57	.79	24.4	.0034	.075	
July 9, 1952-----	322.4	47.7	2.09	3.22	100.0	.0033	.043	
July 11, 1952-----	35.0	42.5	.79	1.04	35.0	.0034	.071	
<b>Seven Springs</b>								
Drainage area: 54.2 square miles. Length of reach over which slope was measured: 328 feet.								
July 27, 1951-----	20.7	57.0	0.64	0.57	36.4	0.0025	0.091	Velocity measured by float and corrected to mean velocity. Do.
Aug. 3, 1951-----	20.8	58.0	.64	.56	37.0	.0026	.093	
Aug. 20, 1951-----	53.8	58.0	.92	1.01	53.4	.0024	.067	
Sept. 10, 1951-----	13.8	56.5	.53	.46	30.1	.0025	.105	
Apr. 27, 1952-----	472	66.6	2.47	2.87	164.8	.0019	.041	
Apr. 28, 1952-----	930	70.2	3.73	3.56	261.5	.0028	.053	
June 2, 1952-----	212.7	62.8	1.62	2.09	101.9	.0027	.051	
July 9, 1952-----	411.0	67.6	2.60	2.33	176.2	.0029	.065	
July 11, 1952-----	152.6	62.0	1.41	1.75	87.2	.0026	.055	
Aug. 7, 1952-----	100.8	58.5	1.18	1.46	69.2	.0025	.057	
<b>South Downingtown</b>								
Drainage area: 86.0 square miles. Length of reach over which slope was measured: 2,026 feet.								
July 27, 1951-----	36.1	53.0	1.10	0.62	58.2	0.00073	0.067	Velocity measured by float corrected to mean velocity. Do.
Aug. 3, 1951-----	33.7	50.0	1.07	.63	53.6	.00086	.072	
Aug. 13, 1951-----	52.0	51.0	1.20	.85	61.3	.00086	.058	
Aug. 20, 1951-----	154.4	53.0	1.78	1.63	94.6	.00086	.039	
Sept. 8, 1951-----	29.5	50.5	1.05	.56	53.2	.00086	.080	
Apr. 27, 1952-----	550	54.5	2.85	3.55	155.0	.00071	.022	
Apr. 28, 1952-----	1,280	61.5	4.39	4.74	270.0	.00066	.022	
July 7, 1952-----	69.5	50.0	1.30	1.07	64.8			
July 8, 1952-----	105.5	56.5	1.47	1.27	83.3	.00083	.044	
July 9, 1952-----	809.0	56.7	3.06	4.67	173.5	.00074	.018	
July 10, 1952-----	978.0	57.2	3.64	4.69	208.2	.00071	.020	

APPENDIX A.—Measurements of discharge and hydraulic variables at the seven principal stations established for this study on Brandywine Creek—Continued

Date	Discharge $Q$ (cfs)	Width $w$ (feet)	Mean depth $d$ (feet)	Mean velocity $v$ (fps)	Area of cross section $A$ (square feet)	Slope of water surface $s$	Roughness parameter $n'$	Remarks	
<b>Georgia</b>									
Drainage area: 118 square miles. Length of reach over which slope was measured: 489 feet.									
July 24, 1951	67.3	88.5	1.80	0.42	159.6	0.0007		Slope too low to measure with gages used. Do. Do. Do. Velocity measured by float and corrected to mean velocity. Do.	
July 27, 1951	57.9	89.0	1.79	.36	159.5				
July 31, 1951	61.8	89.0	1.78	.39	158.7				
Aug. 3, 1951	54.4	89.0	1.69	.36	150.8				
Aug. 20, 1951	182.2	91.0	2.26	0.89	205.5				
Apr. 27, 1952	877	93.0	3.82	2.47	<sup>1</sup> 355.0	0.0006			
Apr. 28, 1952	2,430	95.0	5.74	4.46	<sup>1</sup> 545.0	.0008			
July 9, 1952	670.0	92.0	3.43	2.13	315.1	.0005	0.035		
Aug. 8, 1952	924.0	94.7	3.99	2.45	376.9	.0005	.025		
<b>Embreeville</b>									
Drainage area: 117 square miles. Length of reach over which slope was measured: 850 feet.									
July 30, 1951	73.8	76.5	1.15	0.84	88.0	0.00043	0.040	Velocity measured by float and corrected to mean velocity. Do.	
Aug. 3, 1951	57.0	76.0	1.11	.69	84.0	.00036	.044		
Aug. 14, 1951	81.6	77.5	1.21	.87	93.5	.00042	.038		
Sept. 7, 1951	79.7	78.0	1.18	.86	92.1	.00035	.036		
Sept. 10, 1951	51.9	78.0	1.05	.63	82.2	.00031	.043		
Sept. 15, 1951	160.5	79.0	1.49	1.36	118.4	.00043	.030		
Apr. 28, 1952	2,340	88.0	5.26	5.05	463.1	.00069	.024		
May 26, 1952	1,075	84.0	3.64	3.98	305.7	.00060	.028		
June 6, 1952	319.2	81.0	1.98	2.00	160.4	.00059	.028		
Aug. 7, 1952	259.8	81.0	1.87	1.71	151.9	.00060	.032		
Sept. 19, 1952	540.0	81.0	2.31	2.91	186.8	.00061	.022		
Dec. 16, 1952	191.7	80.0	1.62	1.47	130.0	.00070	.037		
<b>Wawaset</b>									
Drainage area: 134 square miles. Length of reach over which slope was measured: 403 feet.									
July 24, 1951	98.5	60.0	1.23	1.34	73.6	0.00098	0.040		Velocity measured by float and corrected to mean velocity. Do. Do.
July 30, 1951	80.2	61.0	1.10	1.20	67.0	.00097	.041		
Aug. 3, 1951	68.2	60.0	1.10	1.04	66.0	.00090	.046		
Aug. 20, 1951	312.0	64.0	2.05	2.38	131.1	.00120	.038		
Aug. 27, 1951	66.4	60.5	1.08	1.02	65.1	.00085	.045		
April 27, 1952	1,117	67.0	3.42	4.80	<sup>1</sup> 229.0				
Apr. 28, 1952	2,400	75.5	4.93	6.46	<sup>1</sup> 372.0	.00062	.017		
May 26, 1952	1,750	69.0	4.02	6.33	<sup>1</sup> 277.0				
July 10, 1952	1,036.0	71.6	3.12	4.61	223.5				
Aug. 7, 1952	293.9	64.5	2.02	2.26	130.0				
Sept. 19, 1952	623.0	67.5	2.84	3.24	192.4				

<sup>1</sup> Area from stage-area relationship.

APPENDIX A.—Measurements of discharge and hydraulic variables at the seven principal stations established for this study on Brandywine Creek—Continued

Date	Discharge $Q$ (cfs)	Width $w$ (feet)	Mean depth $d$ (feet)	Mean velocity $v$ (fps)	Area of cross sec- tion $A$ (square feet)	Slope of water surface $s$	Roughness parameter $n'$	Remarks
<b>Lenape</b>								
Drainage area: 259 square miles. Length of reach over which slope was measured: 578 feet.								
July 21, 1951	218.4	98.0	1.47	1.51	144.4	0.00075	0.034	Weeds in measuring section.  Velocity measured by float and corrected to mean velocity. Do. Do.
July 31, 1951	124.9	93.0	1.34	.94	124.5			
Aug. 7, 1951	128.7	93.0	1.28	1.08	119.0	.00078	.045	
Aug. 13, 1951	191.5	97.0	1.47	1.35	142.6	.00074	.039	
Aug. 20, 1951	646.7	101.0	2.78	2.30	281.0	.00076	.035	
Aug. 27, 1951	115.8	92.0	1.17	1.08	107.6	.00074	.042	
Sept. 8, 1951	107.2	91.0	1.14	1.04	103.7	.00076	.043	
Sept. 15, 1951	385.7	100.0	1.90	2.00	190.1	.00076	.031	
Apr. 27, 1952	1,410	109.0	4.17	3.10	<sup>1</sup> 455.0	.00032		
Apr. 28, 1952	2,420	121.0	4.42	4.52	<sup>1</sup> 535.0			
May 26, 1952	2,190	125.0	4.52	3.87	<sup>1</sup> 565.0	.00028		
July 10, 1952	1,310.0	113.0	4.14	2.80	468.2			
Aug. 8, 1952	864.3	103.4	3.02	2.75	312.9			

APPENDIX B.—Discharge measurements at nine points on Buck Run.

Station	Drainage area (square miles)	Date	Discharge $Q$ (cfs)	Width $w$ (feet)	Mean depth $d$ (feet)	Mean velocity $v$ (fps)	Area of cross section $A$ (square feet)
Scrogie Route 222 and Route 30	10.6	June 6, 1952	1.11	6.0	0.26	0.71	1.57
		Aug. 25, 1951	1.70	8.5	.76	.26	6.46
Near Sadsburyville Stottsville	4.5	June 6, 1952	5.01	11.5	.58	.75	6.68
		July 2, 1952	2.44	11.5	.54	.39	6.20
		Aug. 25, 1951	2.56	9.0	.88	.32	7.93
Newlin	6.9	Aug. 25, 1951	4.81	12.5	.53	.73	6.57
		June 6, 1952	15.4	13.3	.72	1.61	9.62
		July 2, 1952	8.26	13.5	.63	.98	8.48
Horseshoe bend near Ercildoun	12.4	Aug. 24, 1951	8.19	22.0	.59	.63	12.9
		July 2, 1952	15.3	22.0	.99	.70	21.8
		Aug. 24, 1951	15.6	24.5	.65	.98	15.9
Forks	22.6	June 6, 1952	71.8	22.5	1.36	2.35	30.5
		July 1, 1952	31.9	20.0	1.07	1.49	21.4
		Aug. 24, 1951	18.7	28.0	.72	.43	20.1
Road fork	25.0	July 1, 1952	37.1	31.0	.99	1.21	30.6
		Aug. 24, 1951	34.2	51.5	.68	.98	35.0
		June 6, 1952	142.3	52.0	1.24	2.20	64.7
Mouth	48.1	June 30, 1952	77.6	52.0	1.06	1.38	56.3
		Aug. 22, 1951	42.8	46.5	.78	1.19	36.0
		June 6, 1952	153.2	48.0	1.24	2.58	59.3
		June 30, 1952	80.6	46.5	1.03	1.69	47.6

## APPENDIX C.—Discharge measurements at miscellaneous stations on Brandywine Creek and its tributaries

Stream and location	Drainage area (square miles)	Date	Discharge $Q$ (cfs)	Width $w$ (feet)	Mean depth $d$ (feet)	Mean velocity $v$ (fps)	Area of cross section $A$ (square feet)
West Branch:							
Brandamore	20.6	July 30, 1951	3.96	25.0	0.92	0.17	23.0
Rocklyn	5.8	Aug. 2, 1951	2.55	13.0	.75	.26	9.81
Birdell	18.5	do	6.88	14.0	.53	.92	7.45
Siousea	35.2	do	15.6	65.0	.55	.44	35.6
Modena	57.3	do	30.4	35.0	1.20	.72	42.0
Northbrook	124.4	do	66.6	48.0	1.97	.71	94.3
Cedar Knoll	25.0	Aug. 6, 1951	8.90	28.0	.76	.42	21.4
East Branch:							
Honeybrook	3.2	Aug. 6, 1951	1.10	8.0	.40	.34	3.20
Lewis Mills	8.0	Aug. 1, 1951	4.78	21.0	.38	.59	8.05
Do	8.0	Aug. 9, 1951	3.26	25.0	.65	.20	16.3
Glenmore	16.5	Aug. 6, 1951	5.68	33.0	.94	.18	31.2
Reeds Road	31.8	Aug. 9, 1951	13.9	55.0	1.11	.23	61.0
Main stream:							
Brookfield	292.6	July 28, 1951	152.0	145.0	2.40	.44	348.1
Do	292.6	Aug. 4, 1951	123.6	142.5	1.97	.44	281.0
Do	292.6	Aug. 13, 1951	226.0	144.0	2.49	.63	359.4
Below Pocopson	267.5	Aug. 8, 1951	146.0	103.0	2.98	.48	307.4
Above Wilmington	312.0	Aug. 30, 1951	132.0	120.0	3.44	.32	412.2
Smiths Bridge	298.0	Aug. 8, 1951	160.6	121.0	1.94	.69	234.2
Doe Run:							
Near Cochranville	.3	May 29, 1952	1.82	4.7	.46	.83	2.18
Below Cochranville	2.0	do	9.23	7.8	.72	1.65	5.58
Above Springdell	6.4	do	19.7	12.0	.83	1.97	9.97
Springdell	11.4	do	24.2	16.0	1.05	1.45	23.2
Taylor Run:							
Mouth	5.6	Aug. 21, 1951	5.12	15.5	.38	.87	5.87
Mile above mouth	4.1	do	5.33	10.5	.56	.90	5.30
Tributary near Route 322	1.7	do	1.31	6.5	.30	.67	1.96
Above West Chester		do	1.30	7.5	.19	.90	1.44
Mouth	5.6	July 25, 1951	4.07	20.0	.43	.48	8.50
Valley Creek:							
Below Route 322	20.0	Aug. 14, 1951	8.64	12.5	.41	1.72	5.08
Mile above Route 322	19.6	do	9.04	23.5	.30	1.30	6.93
Smiths Bridge	15.0	do	6.14	18.0	.43	.80	7.70
Pennsylvania R. R.	14.1	do	6.07	19.0	.84	.38	15.95
Glad Acres	11.3	do	5.28	16.0	.43	.77	6.85
Whiteford	10.0	Aug. 15, 1951	4.37	17.0	.23	1.13	3.86
Exton	5.7	do	1.94	9.5	.44	.46	4.23
Ship Rd., north tributary	.74	do	.25	4.5	.57	.96	2.56
Ship Rd., south tributary	2.3	do	.24	3.5	.10	.67	.36
Glen Loch	.12	do	.12	3.5	.06	.60	.20
Broad Run:							
Below "Rocks"	1.26	July 8, 1952	13.3	8.0	.69	2.40	5.50
Headwater	.15	June 3, 1952	.16	3.5	.17	.27	.59
"Marsh"	.50	June 5, 1952	2.48	3.5	.36	1.96	1.26
Buck Hill Farm	1.03	June 3, 1952	4.47	5.2	.55	1.54	2.90
"Rocks"	1.17	do	7.80	19.5	.43	.93	8.36
Below "Rocks"	1.26	do	8.92	8.0	.68	1.64	5.44
Near Romansville	3.05	do	22.3	7.0	.83	3.81	5.84
Below Route 162	4.48	June 5, 1952	31.0	15.5	1.12	2.34	17.4
Edwards Farm	5.47	do	30.8	12.7	1.30	1.85	16.6
Above mouth	5.77	do	33.7	14.8	1.11	2.05	16.4
Do	5.77	Aug. 22, 1952	3.22	8.0	.46	.88	3.66
Rock Run, 1/2 mile above mouth	8.0	Aug. 7, 1951	2.04	22.0	.46	.20	10.2
Marsh Creek, near Milford	18.1	Aug. 9, 1951	5.72	32.0	.73	.24	23.5
Indian Run, mouth	6.3	Aug. 22, 1952	1.85	8.0	.28	.82	2.27
Brinton Run, mouth	1.3	do	1.05	3.5	.34	.89	1.18
Radley Run, mouth	4.1	do	2.02	5.2	.57	.68	2.96
Plum Run, mouth	3.5	do	2.10	7.0	.21	1.46	1.44
Culbertson Run, mouth	4.4	do	2.51	9.5	.36	.80	3.39
Pocopson Creek:							
Mouth	9.2	Aug. 8, 1951	5.20	23.0	.76	.30	17.50
1.5 miles above mouth	8.6	Aug. 17, 1951	7.16	18.0	.28	1.41	5.07
2 miles west of Lenape	7.9	do	2.75	7.0	.37	1.06	2.92
Tributary at Pocopson Home	2.0	do	1.18	6.5	.20	.92	1.28
Near Pocopson Home	5.2	do	2.24	10.0	.42	.53	4.24
Near Marlboro	4.1	do	1.45	7.0	.35	.59	2.44
Headwater	1.3	do	.65	5.0	.16	.83	.78
Pocopson Home	7.3	do	2.75	7.0	.37	1.06	2.61

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