Effects of orographic precipitation variations on the concavity of steady-state river profiles

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ABSTRACT

The concavity, or curvature, of river profiles has long been taken to be a fundamental indicator of the underlying processes governing fluvial erosion, and thereby of landscape evolution. However, erosion laws have generally been derived without accounting for the strong orographically driven gradients in precipitation typically found in mountainous regions. In addition, field measurements have found discrepancies between the form of measured stream profiles and theoretically derived values. Introducing a simple physically based feedback, we find that orographically induced variations in precipitation strongly affect the curvature of steady-state river profiles. This feedback complicates efforts to infer the form of erosion laws from observed profile concavities, but could help explain discrepancies between observations and theory. Our results demonstrate a strong feedback through which climate influences the form of river profiles and show how such climatic effects act to limit the relief of unglaciated mountain ranges.

Keywords: fluvial erosion, orography, precipitation, mountain building, relief.

INTRODUCTION

The longitudinal profiles of mountain river systems are set by the interplay of tectonic processes driving rock uplift and erosional processes that govern the ability of the river channel to incise into bedrock (Mackin, 1948; Seidl and Dietrich, 1992; Whipple and Tucker, 1999). River profiles have therefore been the subject of extensive investigation for what they reveal about how such processes act over geologic time. It has long been recognized that the ability of a channel to transport sediment is a function of discharge and slope (Gilbert, 1877). Processes of bedrock river incision have been represented in a similar form (Seidl and Dietrich, 1992; Howard et al., 1994; Stock and Montgomery, 1999; Whipple and Tucker, 1999). The sensitivity of river profiles to the particular form of the erosion law governing long-term river incision has led to efforts to infer the specific form of erosion laws from channel profiles (e.g., Stock and Montgomery, 1999; Snyder et al., 2000), and some studies have addressed river profile response to spatial variability in tectonically driven rock uplift (e.g., Kirkby, 1997; Kirby and Whipple, 2000). However, whereas none of these previous studies accounted for variability of precipitation, a fundamental characteristic of mountainous terrain is its dramatic impact on the distribution of precipitation. Because rivers are fed by precipitation, there is a clear potential for a feedback on mountain evolution, which we here incorporate into models of river profile development. We show that orographic precipitation variability strongly influences both river profile concavity and the relief of mountain drainage basins. Direct inference of exponents in the erosion law is therefore complicated by this feedback. However, because it has the potential to explain a large fraction of the observed range of river profile concavities, the existence of this feedback may support the notion that simple governing relationships characterize fluvial incision into bedrock.

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THEORY

For the portion of a stream channel in which fluvial erosion dominates, the stream power erosion law has the following general form:

$$E(x) = KQ(x)^{\tilde{m}}S(x)^{n}, \tag{1}$$

where E is the local erosion rate at a point x along the channel. The erosivity, K, is a dimensional constant incorporating information about rock strength, sediment loading, and the processes and mechanisms causing river incision (e.g., Whipple and Tucker, 1999). $Q(x)$ is the local stream flow (or discharge); $S(x)$ is the local along-channel slope; and $\tilde{m}$ and n are real, positive exponents set by the channel incision process.

Whereas equation 1 is the physical formulation of the erosion law, the erosion law is generally developed further by positing a discharge-area relationship of the following form:

$$Q = \mathrm{k_q} A^{\mathrm{c}}, \tag{2}$$

where $\mathrm{k_q}$ and c are constants, and A is the upstream drainage area. The value of c is frequently set to one, implying uniform precipitation (e.g., Pazzaglia et al., 1998; Snyder et al., 2000). Limited observations give values of between 0.7 and 1.0 for c, values attributed to relative size differences between drainage basins and rain-producing storm cells (Dunne and Leopold, 1978). Specifying discharge in this way implies a strong functional constraint on the precipitation and precludes the possibility of a feedback between the orography and precipitation.

Substituting equation 2 into equation 1 yields

$$E = \mathrm{K}'A^{\mathrm{m}}S^{\mathrm{n}}, \tag{3}$$

where $\mathrm{m} = \tilde{m}\mathrm{c}$ and $\mathrm{K}' = K\mathrm{k_q^{\tilde{m}}}$. In the case of a steady-state mountain range, the local erosion rate everywhere balances the uplift rate, U (hereafter assumed uniform). That is,

$$S = (U/\mathrm{K}')^{1/\mathrm{n}} A^{-\mathrm{m/n}}. \tag{4}$$

The general erosion law therefore predicts a linear relationship between log S and log A, with a proportionality constant equal to m/n. The theoretical value of m/n depends on the assumed physics: For $\mathrm{c} = 1$, if the erosion rate is proportional to total stream power, then $\mathrm{m/n} = 1$; if it is proportional to unit stream power or the basal stress at the stream bed, then $\mathrm{m/n} = 0.5$ (e.g., Whipple and Tucker, 1999).

Observations of actual stream and river profiles can be fit to an empirical relationship with the form:

$$S = \mathrm{k}A^{-\theta}, \tag{5}$$

where θ is defined as the concavity index (Flint, 1974). If the assumptions made in the erosion model are correct, then θ should equal m/n for steady-state river profiles.

ADDING OROGRAPHIC PRECIPITATION

By definition, steady-state stream flow is the integral of the precipitation over the upstream drainage area:

$$Q(x) = \int_0^x p(x') \frac{dA(x')}{dx'}\, dx'. \qquad (6)$$

where the precipitation distribution, p, is specified as a function of the along-channel distance x, x' is a dummy variable of integration, and $x = 0$ is taken to be at the drainage divide. Equation 6 reflects a simple basin geometry and implies that tributaries have not captured significant precipitation from areas far from the main channel, and that longitudinal variations in precipitation dominate. Note also that equation 6 means that the often assumed area-discharge relationship in equation 2 implies a singularity in the precipitation at $x = 0$ for values of $c < 1$.

So equating erosion rate and uplift rate again, rearranging equation 1 (with $\tilde{m} = \mathrm{m}$), and taking the log of both sides yields

$$\log S = \log\left(\frac{U}{K}\right)^{1/\mathrm{n}} - \left(\frac{\mathrm{m}}{\mathrm{n}}\right)\log Q. \qquad (7)$$

Empirical data show that across the full range of basin sizes, drainage area can be specified as a function of channel length: $A(x) = x^2/3$ (Montgomery and Dietrich, 1992). Thus, differentiating equation 7 with respect to log A gives:

$$\theta(x) = -\frac{d \log S}{d \log A} = \left(\frac{\mathrm{m}}{\mathrm{n}}\right)\frac{A}{\frac{dA}{dx}}\frac{1}{Q}\frac{dQ}{dx}. \qquad (8)$$

Substituting in from equation 6 results in

$$\theta(x) = \left(\frac{\mathrm{m}}{\mathrm{n}}\right)\frac{A(x)p(x)}{Q(x)}. \qquad (9)$$

Therefore, if precipitation varies as a function of stream channel position, then the observed concavity index will not in general be equal to m/n, even in steady-state conditions and uniform rock type. Moreover, the concavity index will vary as a function of position along the channel. Using equation 6 again, equation 9 can be integrated by parts and rearranged to yield:

$$\theta = \left(\frac{\mathrm{m}}{\mathrm{n}}\right)\left[\frac{1}{1 - \dfrac{\int_0^x A \frac{dp}{dx'}\, dx'}{pA}}\right]. \qquad (10)$$

So if dp/dx is negative over the entire channel (i.e., precipitation increasing toward the channel head), then θ will be smaller than m/n. This situation applies to the windward side of a range where the prevailing winds are forced upslope. However, if the precipitation regime is such that precipitation is decreasing with elevation, then θ will be larger than m/n.

Furthermore, the average of the concavity index over the fluvial portion of the stream channel (i.e., $x_c < x \leq L$) is obtained straightforwardly from equation 7:

$$\theta_{\mathrm{av}} = -\overline{\frac{d \log S}{d \log A}} = \left(\frac{\mathrm{m}}{\mathrm{n}}\right)\frac{\log Q(L) - \log Q(x_c)}{\log A(L) - \log A(x_c)}. \qquad (11)$$

PRECIPITATION FEEDBACK

It is impossible to account for all the variations in precipitation that occur over the time scales of stream-channel evolution. However, in certain meteorological regimes, such as where prevailing mid-latitude westerlies impinge on a coastal mountain range, the qualitative nature of the precipitation pattern will not change and the physical mechanisms are robust. First, the moisture content of an air column decreases with elevation due to the decrease in temperature with height. For constant relative humidity the moisture content of an air column is approximated well by the saturation vapor pressure at the surface, $e_{\mathrm{sat}}(T_s)$, which is an exponential function of the surface temperature, T_s, and is given by the Clausius-Clapeyron relation (e.g., Holton, 1992). Second, the direction of the prevailing wind is important. Where the winds are forced upslope, the air column cools and saturates. Moisture convergence in excess of saturation then rains out. Conversely, prevailing downslope winds dry out the air column, and precipitation is suppressed (the so-called rain shadow). Both effects are robust and well observed in mountain ranges in today's climate (e.g., Barros and Lettenmaier, 1994).

Letting $-\nabla \cdot \vec{F}$ be the convergence of the column moisture flux, a parameterization accounting for this is

$$-\nabla\cdot\vec{F} = \left(\alpha_0 + \alpha_1 \bar{v}\frac{dz_s}{dx}\right)e_{\mathrm{sat}}(T_s), \qquad (12)$$

where α_0 and α_1 are constants, and $\bar{v}$ is the prevailing wind. Moisture convergence, however, does not translate directly into precipitation: there is a finite formation time for raindrops, and rain can be advected a significant distance before reaching the ground. Therefore a Gaussian-shaped upstream weighting function is applied to equation 12 to give the precipitation rate (e.g., Alpert, 1986):

$$p(x) = \frac{\Delta x\sqrt{\pi}}{2} \times \int_x^\infty -\nabla\cdot\vec{F}(x') \times e^{-[(x-x')/\Delta x]^2}\, dx'. \qquad (13)$$

Note that Δx is a smoothing scale and can be regarded as also incorporating some variation of the wind speed over time; $T_s(x) = T_s(L) - \Gamma z(x)$, where Γ is the assumed atmospheric lapse rate (-6.5 °C km^{-1}); α_0 and $T_s(L)$ are chosen to give a precipitation rate of 1 m yr^{-1} for $z = dz/dx = 0$, typical of mid-latitude climates; and $\alpha_1 e_{\mathrm{sat}}(T_s(L)) = 110$ m yr^{-1}/m s^{-1} (Roe, 1999).

We stress that Δx and $\bar{v}$ are parameters tuned to give precipitation rates consistent with observations. In addition, the formulation does not take into account the influence of ridge topography, which influences precipitation on interbasin scales. Although this is of undoubted importance in reality, the reduced model presented here is capable of qualitatively representing observed precipitation regimes: i.e., that steeper slopes and increasing prevailing upslope winds lead to increased precipitation. Although equations 12 and 13 are much simplified, they are the basis of more complex parameterizations that also require tuning to the area of interest (e.g., Barros and Lettenmaier, 1994). Furthermore, because detailed knowledge of the atmospheric circulation (and the orography) is very limited on long time scales, the use of a more complex treatment may be unwarranted.

IMPLEMENTATION IN EROSION MODEL

The precipitation feedback can be incorporated into equation 6, and it is straightforward to solve for the steady-state stream channel profile. We take $L = 5$ km and $x_c = 400$ m, and between $x = 0$ and $x = x_c$ we simply extrapolate the channel slope at $x = x_c$. The solutions are not sensitive to this choice. At $x = L$, the height is fixed at zero. Unless otherwise noted, we use $U = 2$ mm yr^{-1}, $K = 4.0 \times 10^{-5}$ yr$^{-2/3}$, m = 1/3, and n = 2/3.

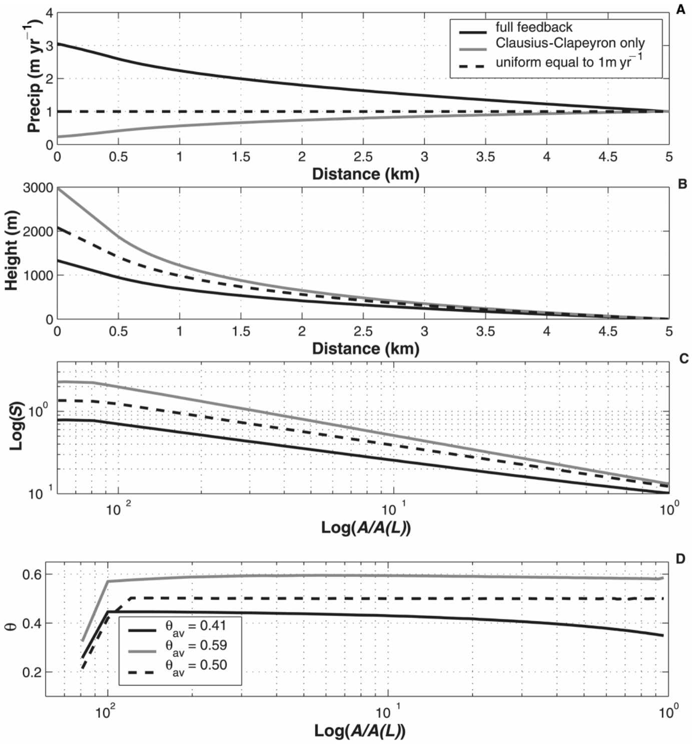

Figure 1. Stream profiles including precipitation feedback. A: Precipitation variations for different orographic feedbacks, explained in text. B: Steady-state stream profile relief. C: Log *S*-log *A* relationships. D: Concavity indices. Note that in C and D abrupt changes in values near channel head are results of choice of upper boundary condition explained in text.

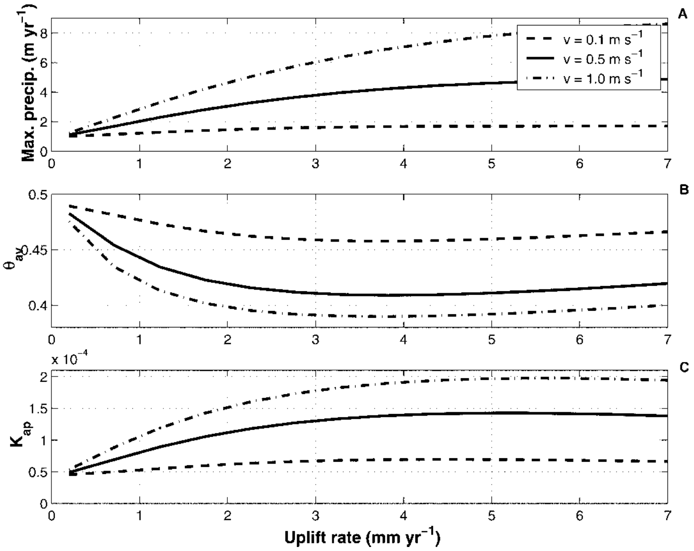

Figure 2. Concavity indices versus uplift rate for several different feedback strengths. A: Maximum precipitation (i.e., at x = 0). B: Average channel concavity index. C: K_{ap}, apparent value of erosivity, as defined in text.

A uniform precipitation rate of 1 m yr^{-1} (i.e., with no feedbacks) gives a total relief for the channel of ~2100 m (Fig. 1B). When the precipitation feedback is included (equation 13 with $\Delta x = 30$ km and $\bar{v} = 0.5$ ms^{-1}) , the stream channel relief is reduced dramatically to ~1300 m (Fig. 1B). High precipitation rates occur in the upper reaches of the channel, maximizing at ~3.0 m yr^{-1} (Fig. 1A), and the resulting enhanced stream flow causes preferentially greater local erosion. While the profile looks quite linear when plotted on a log *S*-log *A* graph (Fig. 1C), the concavity index, θ, departs significantly from the m/n value of 0.5 used in the erosion law (Fig. 1D), and it decreases uniformly with distance down the channel, consistent with equation 10. The mean value is ~0.41, 82% of the value of the governing m/n value.

An alternative precipitation regime might be one in which the prevailing winds were negligible, and the precipitation distribution controlled solely by the moisture content of the air column (i.e., according to the Clausius-Clapeyron relation). In this case $\alpha_1 = 0$, and we take $\Delta x = 100$ m. Figure 1A shows that precipitation rates are reduced by ~70% at the highest elevations. This concentrates precipitation on the lower flanks of the profile, and the consequently weaker stream flow in the upper portion of the profile allows an increase in the elevation of the mountain of ~900 m, relative to the uniform precipitation case. Although the log *S*-log *A* plot looks linear, θ increases with downstream distance and has an average value of 0.59 (Fig. 1D).

The effect of the orographic precipitation feedback does not depend much on domain size: varying the channel length affects θ_{av} only weakly (θ_{av} [av is average] increases monotonically to 0.43 as L is increased to 30 km). The magnitude of the slope-dependent part of feedback may be controlled by varying the prevailing wind strength (keeping Δx fixed), but $\theta_{av} < 0.45$ even for $\bar{v} = 0.15$ m s^{-1}, for which the precipitation maximum is 1.7 m yr^{-1}, a fairly moderate orographic effect.

An interesting consequence of the feedback is that if all else is equal, different uplift rates imply different values of θ_{av}. This effect is shown in Figure 2B for a variety of feedback strengths. There are two distinct regimes. At low values, any increase in uplift rate causes steeper slopes, thereby enhancing the precipitation and reducing the river profile concavity. At high uplift rates, an increase still produces greater relief, but the maximum precipitation now begins to level off as the higher elevations become starved of moisture due to the Clausius-Clapeyron relation (equation 12), and this effect offsets the steeper channel slopes. The result is that θ_{av} actually increases with uplift rate, although it remains below the assumed value of m/n. For a strong precipitation feedback, this leveling off of precipitation occurs in the model for mountains as low as 2500 m, and the effect has been shown to explain precipitation variations in the Sierra Nevada (Alpert, 1986).

Equation 4 predicts that the *y*-axis intercept on a log *S*-log *A* plot is equal to $\log(U/K)^{1/n}$, and this has been used to deduce information about K and n in regions with known uplift rates (e.g., Snyder et al., 2000). However, small systematic changes in θ_{av} have a large impact on the intercept point. We can define an apparent erosivity, K_{ap}, by interpolating to the *y*-axis from the mid-point of the fluvial channel (in log A space) using the calculated value of θ_{av}:

$$\log\left(\frac{U}{K_{ap}}\right)^{1/n} = \log S_1 + \theta_{av}\log A_1, \tag{14}$$

where $\log A_1 = 0.5[\log A(x_c) + \log A(L)]$, and S_1 is the slope at the point corresponding to A_1. K_{ap} varies significantly as a function of uplift rate and feedback strength (Fig. 2C), and is due to changes in both the magnitude and distribution of precipitation. We emphasize that there is no actual change in the erosivity of the channel, and that the effect is purely a consequence of the precipitation feedback.

DISCUSSION

We have demonstrated that longitudinal variations in precipitation necessarily imply that concavities in steady-state stream profiles are not the same as the values of m/n in the erosion law driving the incision. This complicates inferring the erosion laws from observations of drainage areas and slopes derived from river profiles or digital elevation models. The distribution of precipitation has to be accounted for, and the magnitude of the precipitation feedback depends on the temperatures and prevailing winds, which in turn are consequences of large-scale climate processes.

Observed concavities typically have values between 0.4 and 0.8 (e.g., Hack, 1957; Flint, 1974; Tarboton et al., 1989; Moglen and Bras, 1995; Slingerland et al., 1998; Snyder et al., 2000), although various methods were used in obtaining these values. Note also that not all the profiles can be considered to be in steady state, and they do not share uniform lithology or uplift rates. The calculations presented here suggest that orographic feedback can yield values for θ_{av} of between 0.4 and 0.6 for a controlling m/n value of 0.5. While this range in θ_{av} values is less than the observed variability in river profile concavities, and other influences such as nonsteady-state or transient profiles may have an even greater effect on θ_{av}, we have shown that the orographic feedback is a first-order factor in setting river profile concavities. Our results therefore lend support to the contention that the variability of profile concavities is attributable to local variations in physical factors acting in concert with a simple erosion law.

The precipitation feedback causes an apparent covariance between the uplift rate and the erosivity, K_{ap}, as was found by Snyder et al. (2000) for the King Range in northern California. For uplift rates of 0.5 and 4 mm yr^{-1}, their observations suggested that K differed by a factor of between 2 and 6, depending on the assumed value of n. Snyder et al. (2000) considered orographic precipitation and concluded that plausible, but uniform, precipitation changes could account for their observations only for $n \sim 2$, which is at the high end of estimates from theoretical considerations (e.g., Whipple and Tucker, 1999). However, Figure 2C suggests that the changing distribution of precipitation could account for these intriguing results without invoking any changes in erosional process or rock strength, although we emphasize that a more careful and direct comparison should be made.

While the profiles we have considered are essentially one dimensional, we anticipate that richer interactions between landscape evolution and climate are possible if the extra horizontal dimension is included. For example, models of optimal drainage networks predict that differences in θ lead to different styles of branching network architecture (e.g., Howard, 1990). In addition, drainage network evolution models (Rodriguez-Iturbe and Rinaldo, 1997) typically equate discharge with drainage area (i.e., assume uniform precipitation), whereas the results presented here show that including orographic feedback necessarily means that the slope-area distribution does not obey a strict power-law relationship. Including the orographic feedback therefore may mean that the spatial structure of channel networks varies as a function of position in mountain drainage basins. The specific inclusion of finite-scale valleys and ridges may yield other interesting meteorological effects. In particular, there is the potential for a feedback that encourages the growth of larger valleys, which preferentially capture atmospheric moisture flux. Including the effects of temporal variability in precipitation, such as those due to differences in seasonality or to flood recurrence characteristics (e.g., Tucker and Bras, 2000; Snyder, 2001), may also influence river profiles.

The strong impact on headwater relief suggests that orographic feedback is a fundamental constraint on relief. For example, for the simple case where $m = n = 1$ and no feedback, an eightfold increase in uplift rate causes an eightfold increase in relief. With the feedback included, increased uplift steepens the slopes; precipitation is enhanced, local erosion increases, and the change in relief is reduced to about half that of the no-feedback case. Hence, the orographic feedback is an important component of the climate-erosion-uplift system that ultimately controls the topographic (and to some degree geological) evolution of mountain ranges.

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