

Estimating exhumation rate and relief evolution by spectral analysis of age–elevation datasets

Jean Braun

Research School of Earth Sciences, The Australian National University Canberra, ACT 0200, Australia

Dating topography of the Sierra Nevada, California, using apatite (U–Th)/He ages

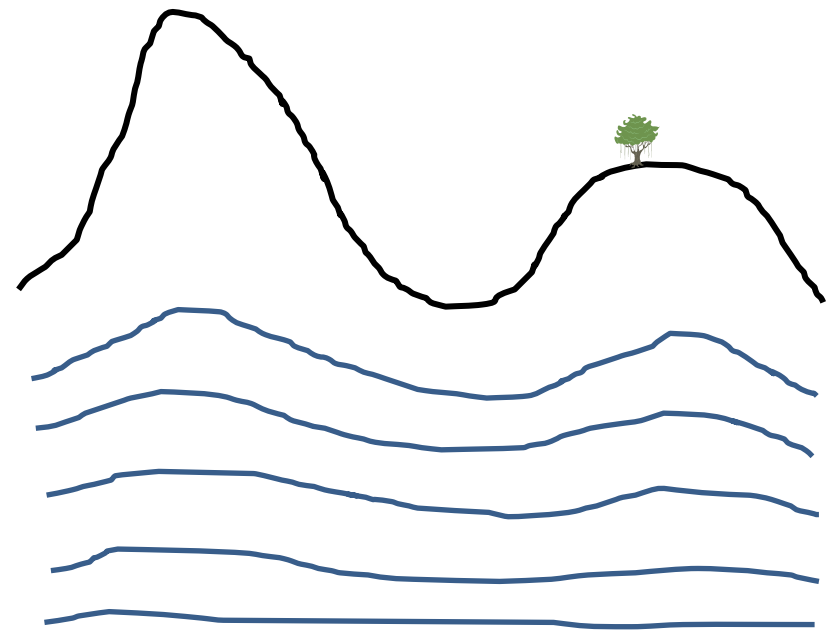
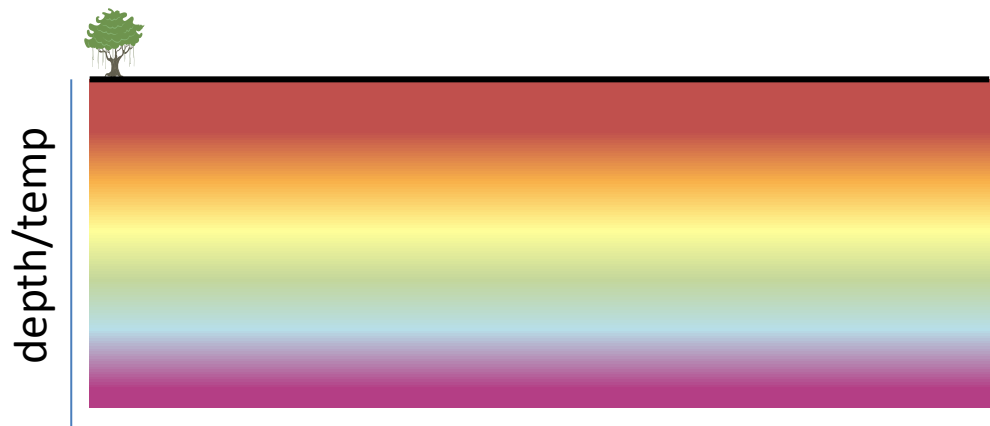
Martha A. House^{*†}, Brian P. Wernicke^{*} & Kenneth A. Farley^{*}

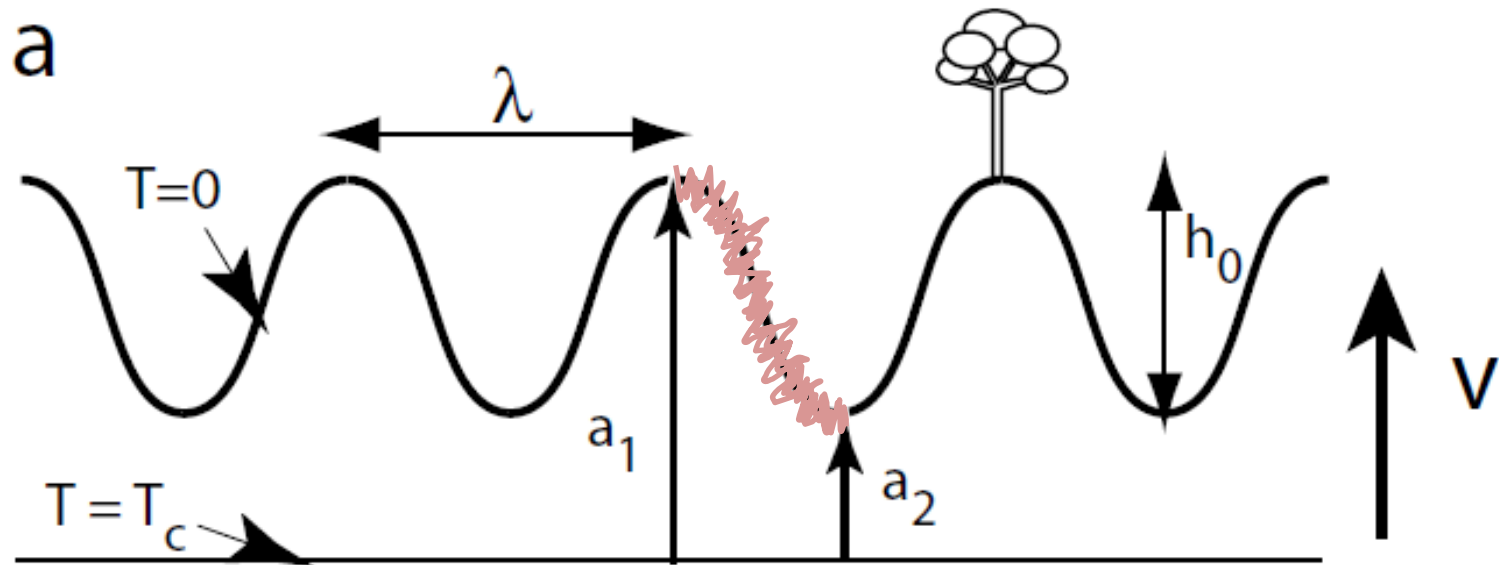
^{} Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, California 91125, USA*



How do isotherms behave under different topographic regions?
high topographic relief vs. low topographic relief?

Why pick a low temperature thermochronometer rather than a higher temp?



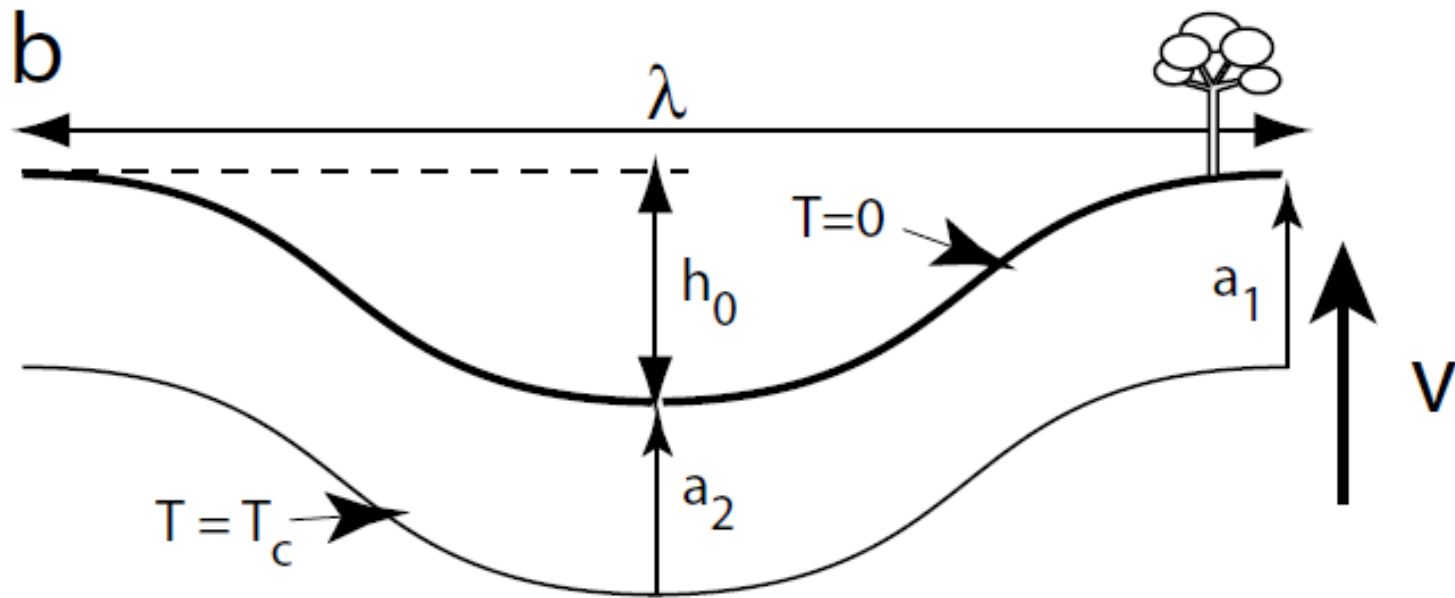


What is the relationship between topography and isotherm?

How does a_1 and a_2 compare?

When would this be a good topographic/isotherm setup?

What if you want to know about paleo-topography? High and rugged or mellow?

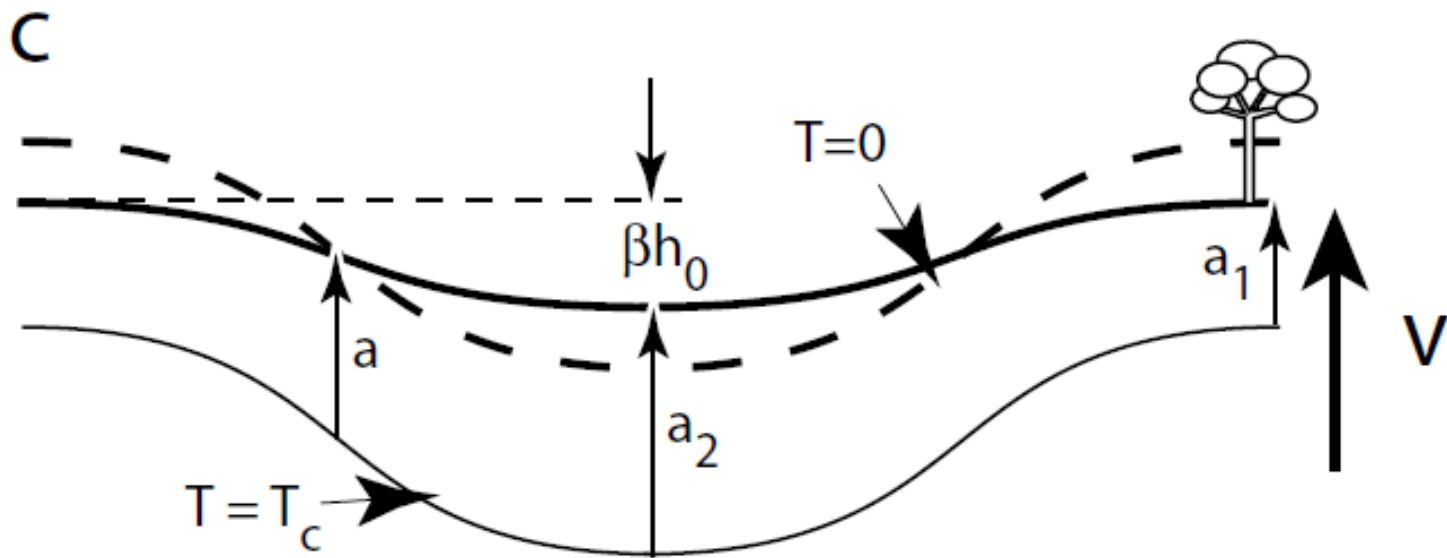


What is the relationship between topography and isotherm?

How does a_1 and a_2 compare?

How do you get differences between a_1 and a_2 ?

What if you want to know about paleo-topography? High and rugged or mellow?

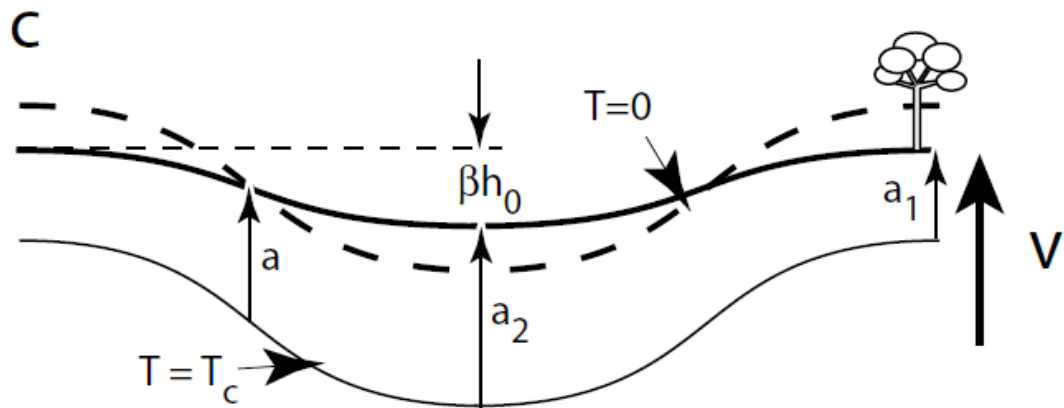
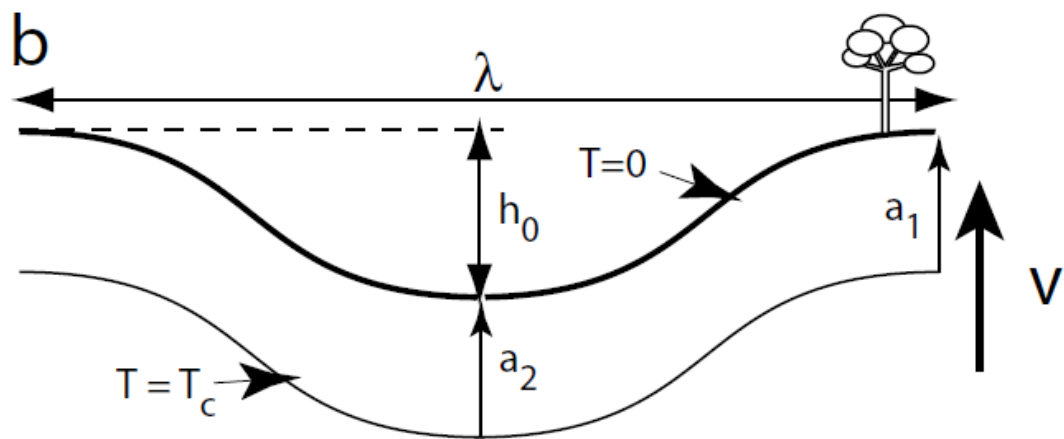
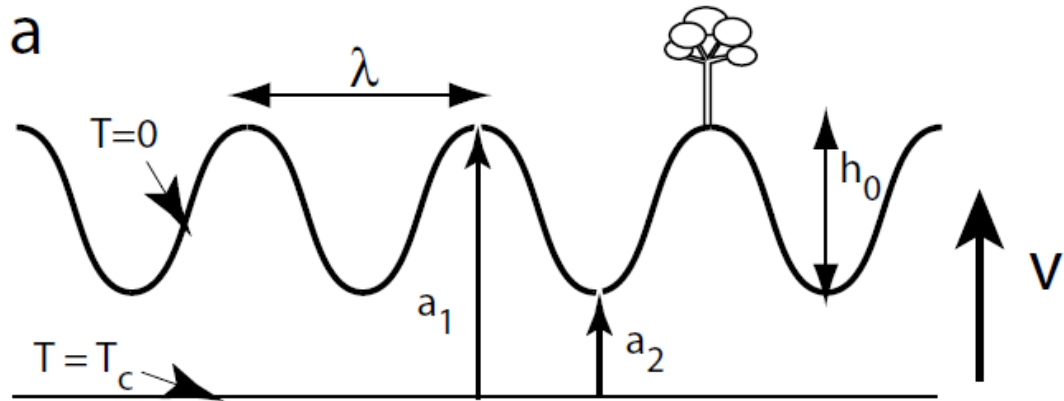


What is the relationship between topography and isotherm?

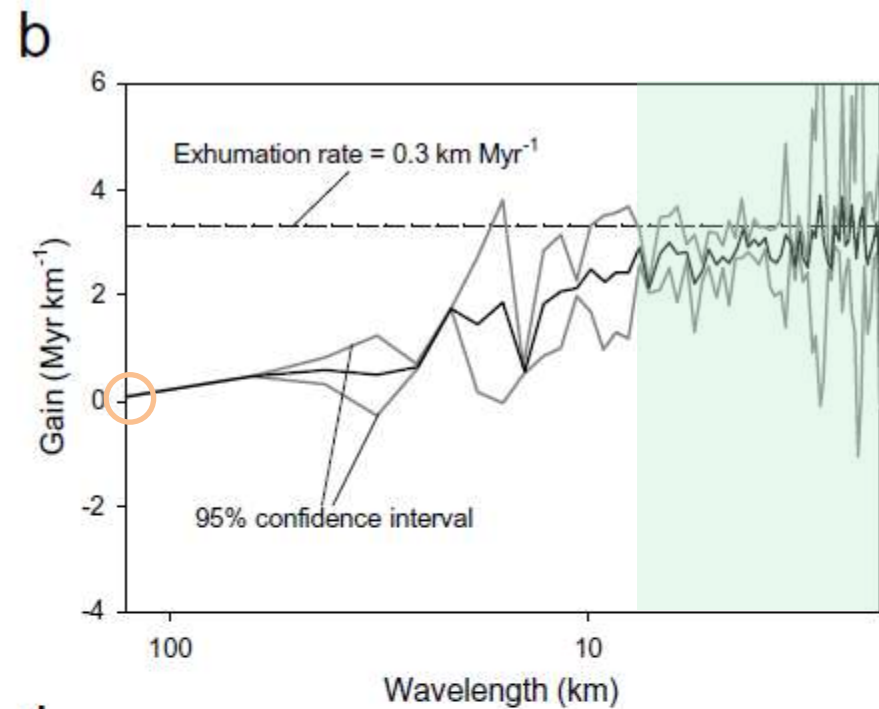
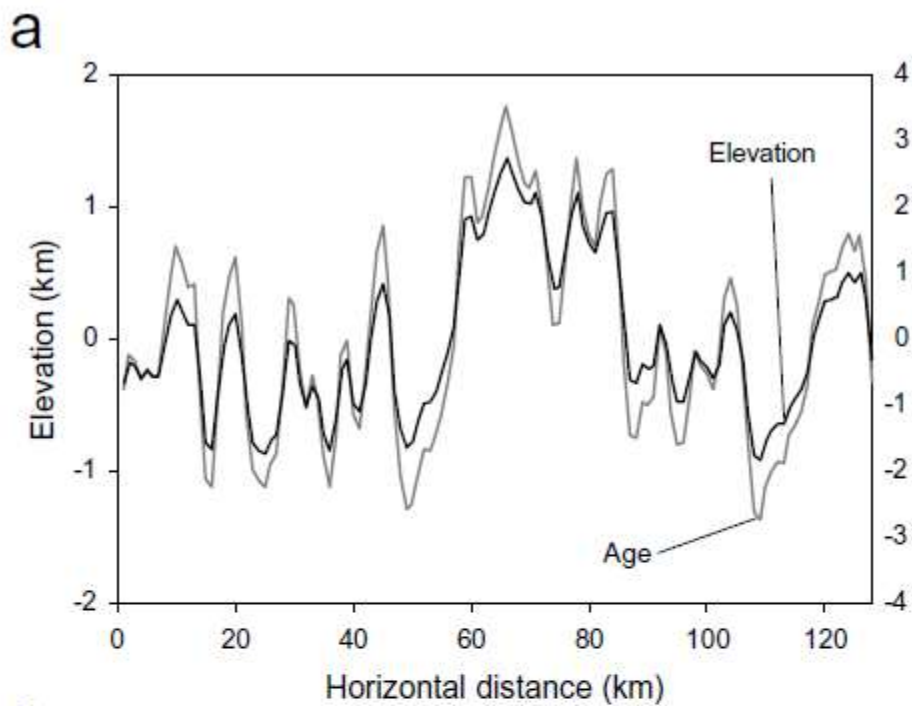
How does a_1 and a_2 compare?

How might the difference between a_1 and a_2 be achieved in a real landscape?

What does the age variation with elevation (a_1 and a_2) tell you about relief?

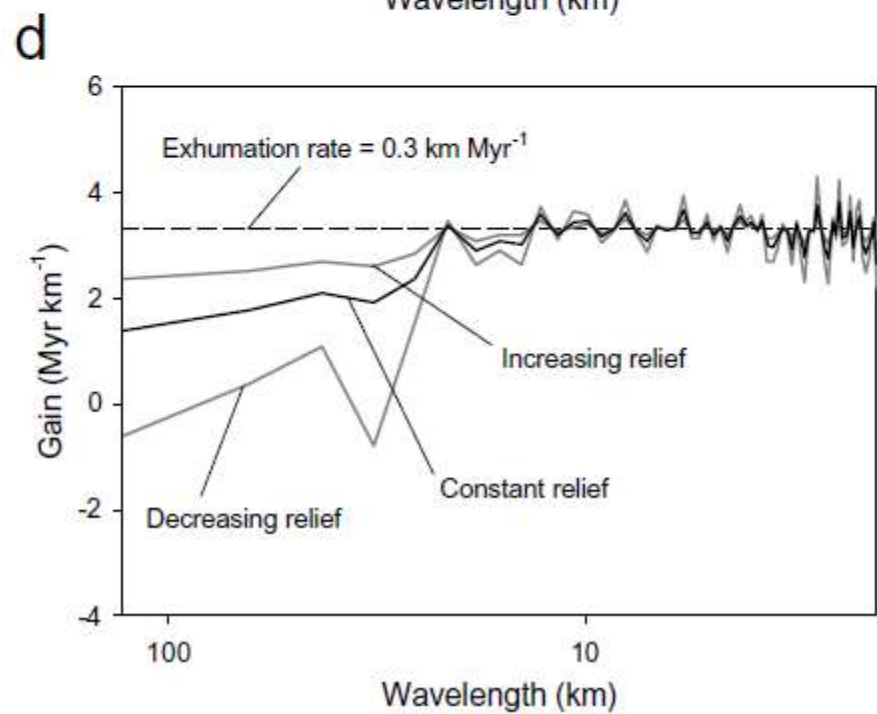
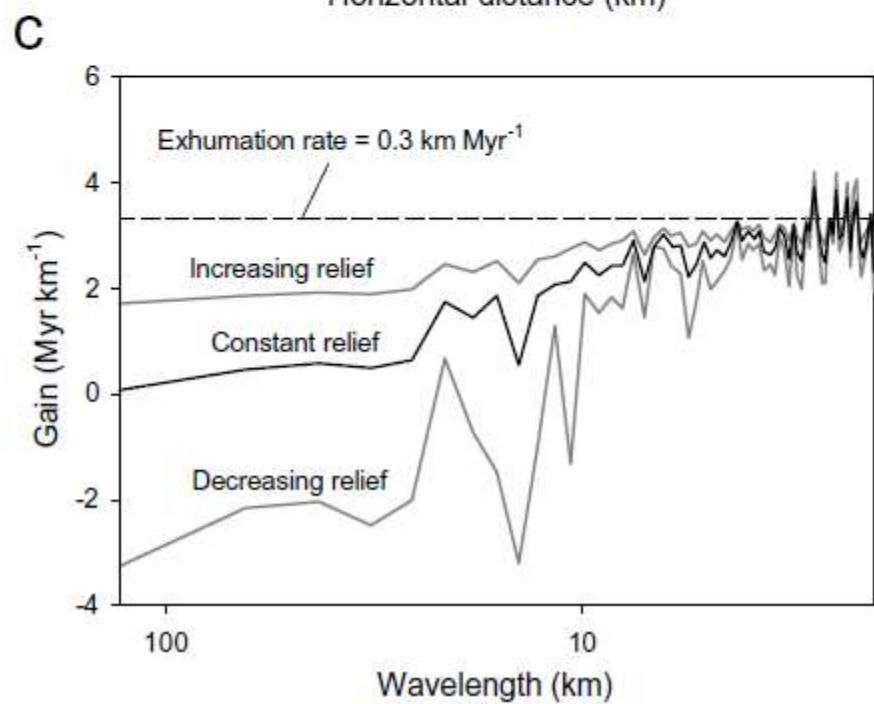
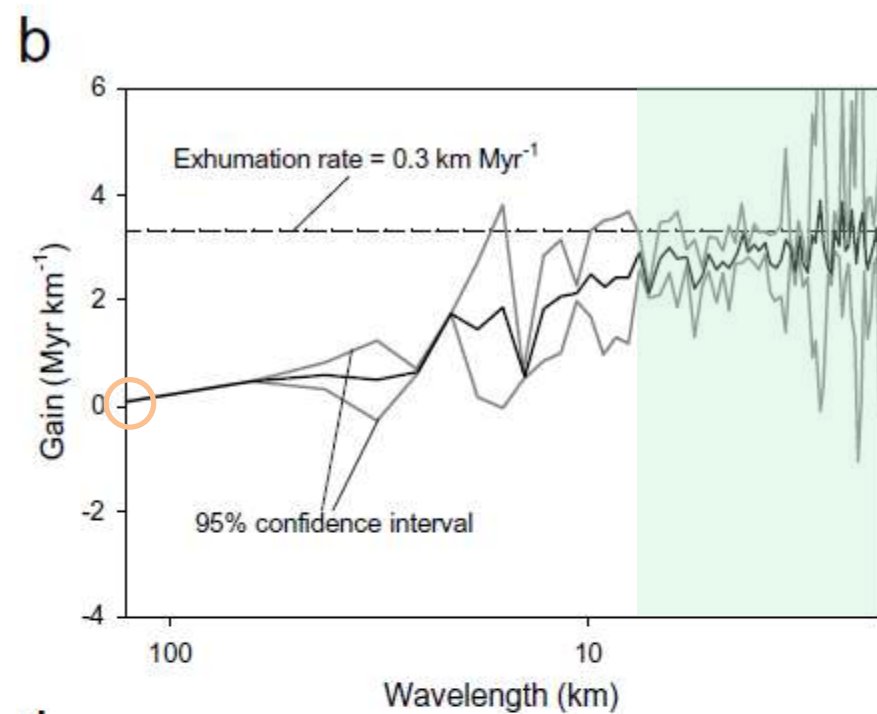
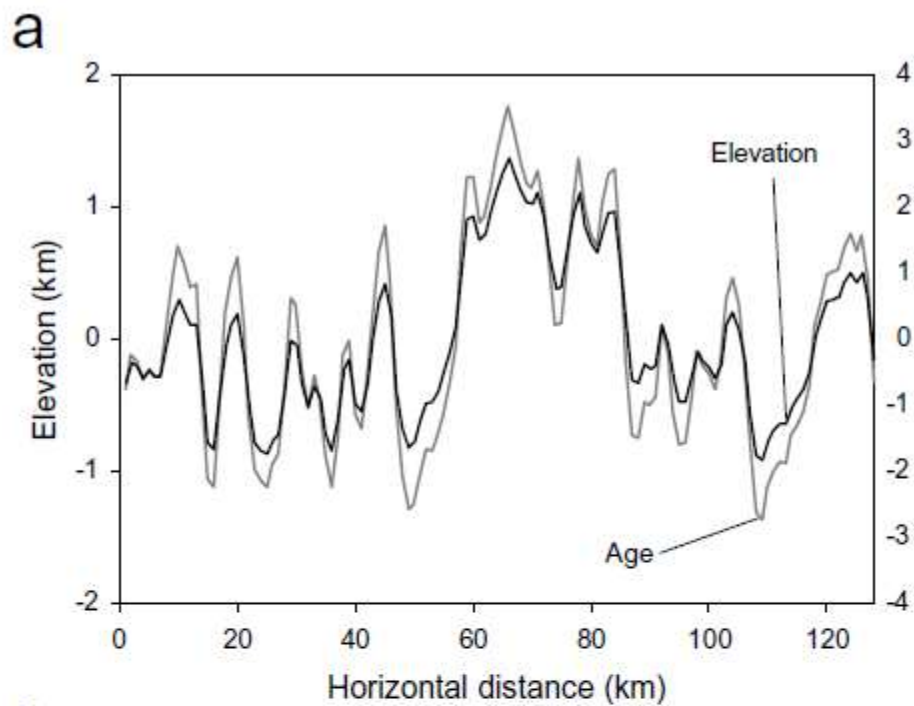


To the Math

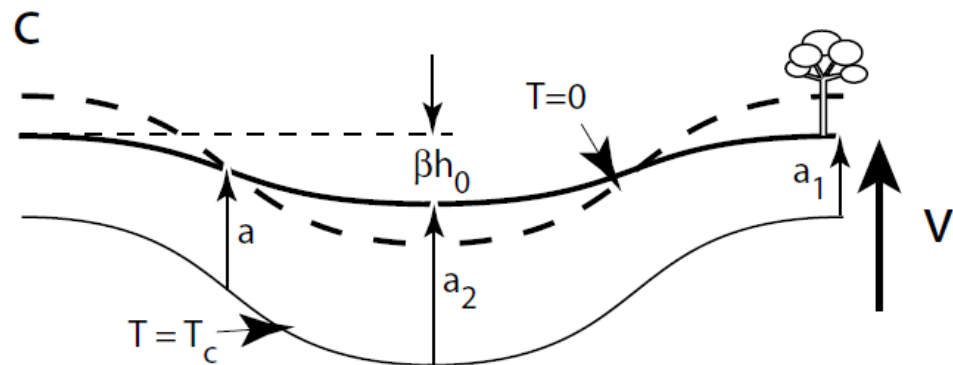


What is the meaning of the right side of the graph and how does that compare to the left?

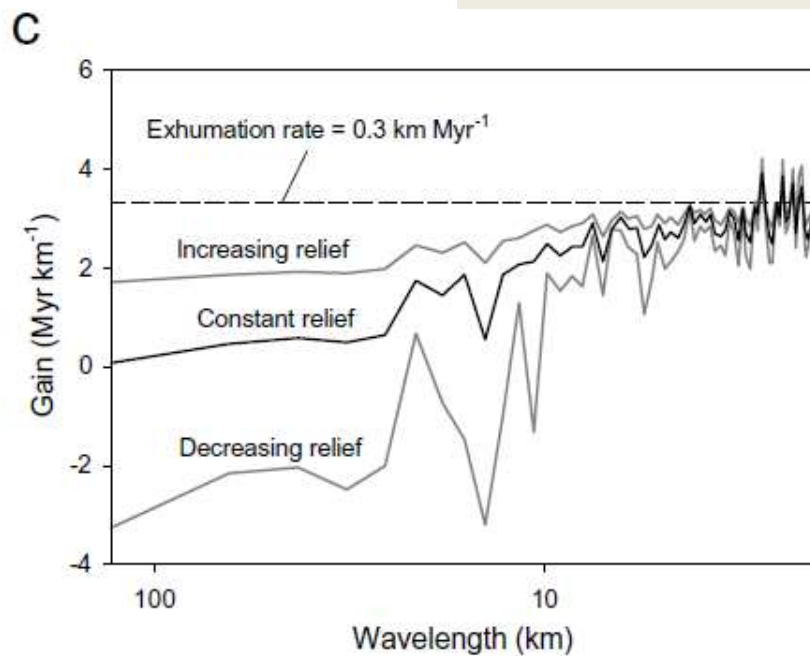
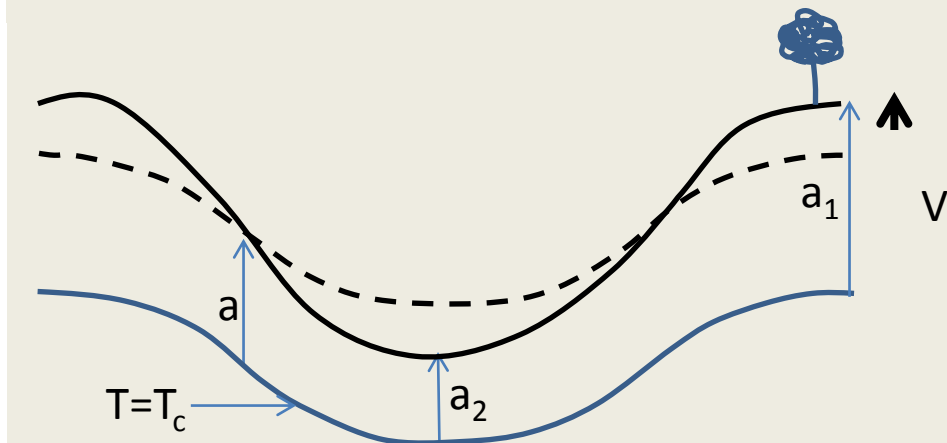
What does a gain value of 0 mean in terms of ages?



Decreasing Relief



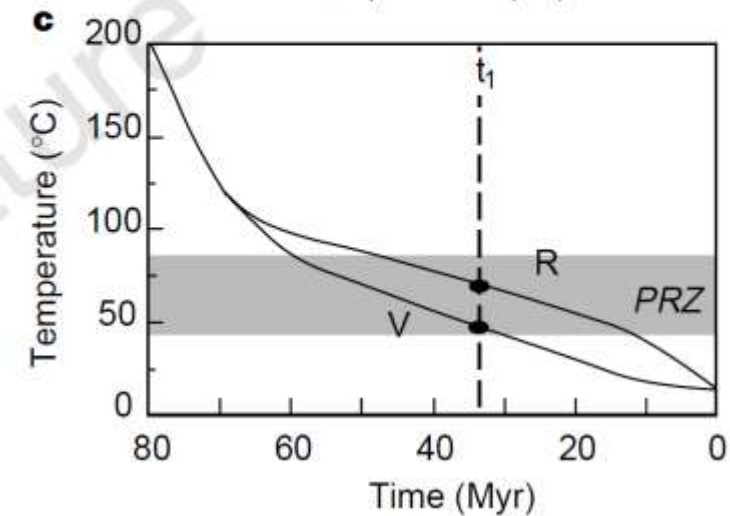
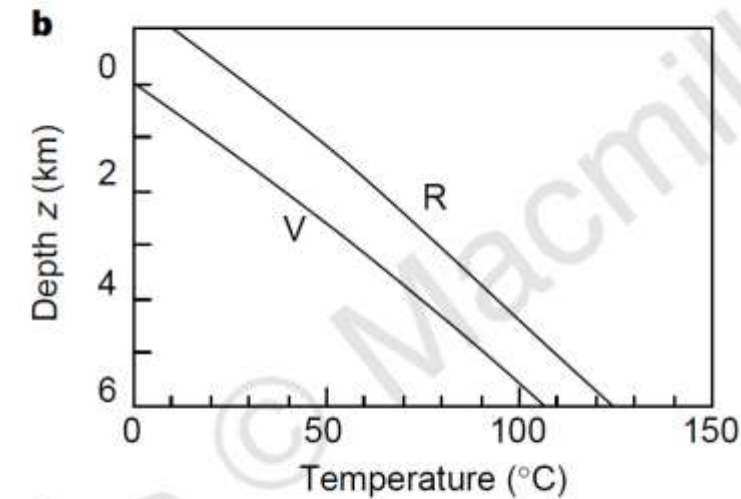
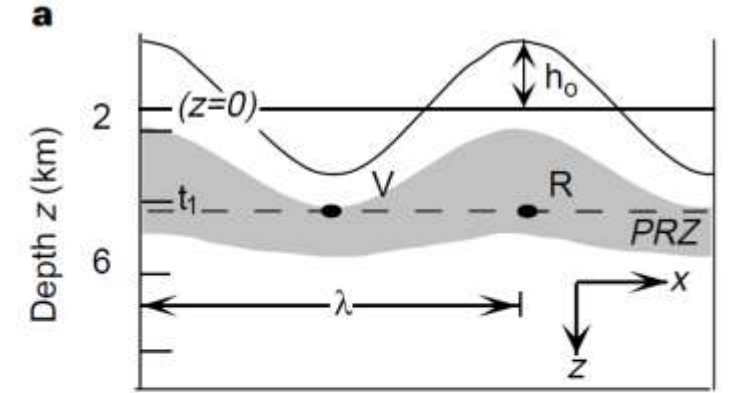
Increasing Relief



Sierra Nevada Case

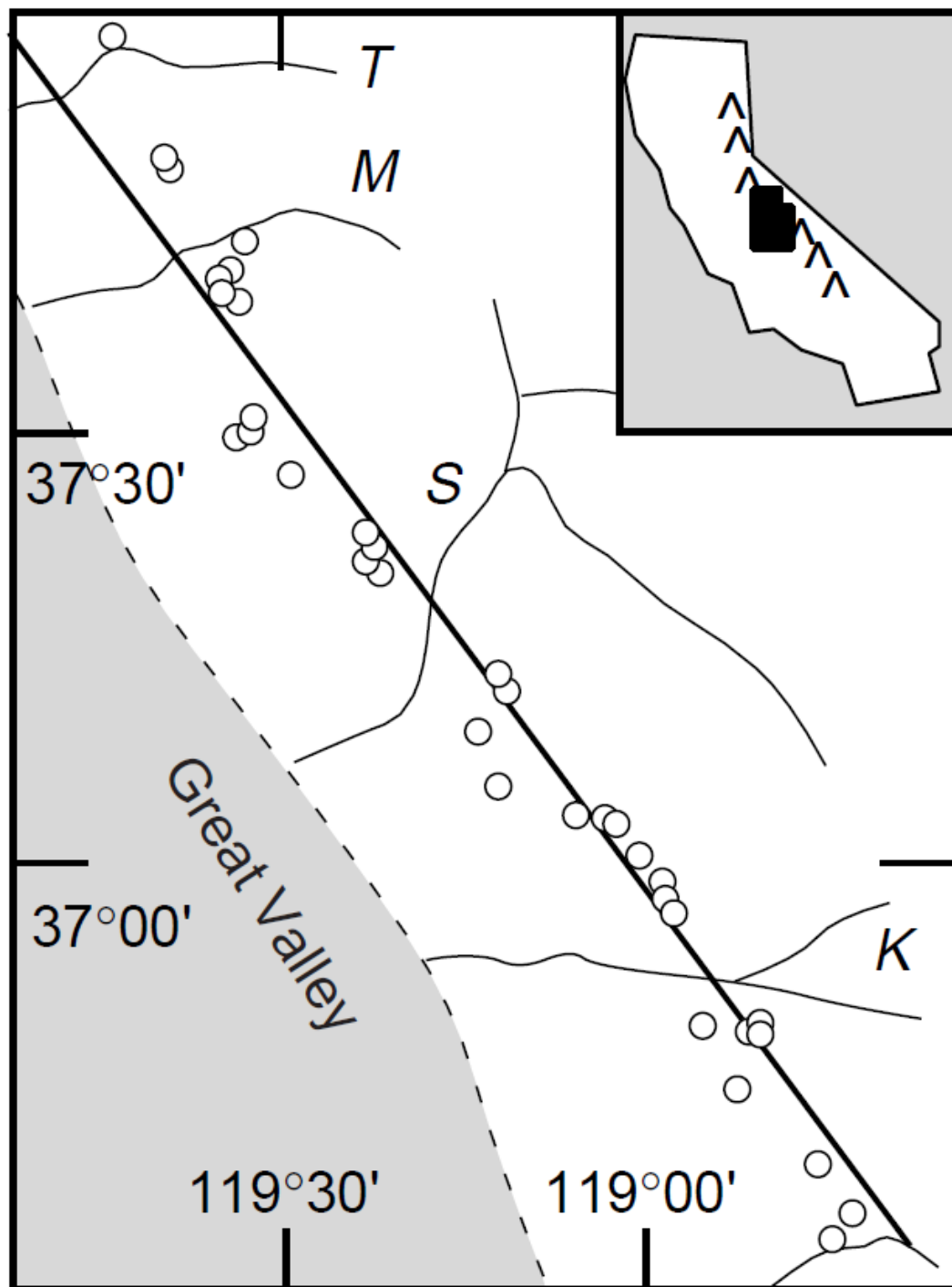
What is the meaning of the right side of the graph and how does that compare to the left?

What does a gain value of 0 mean in terms of ages?

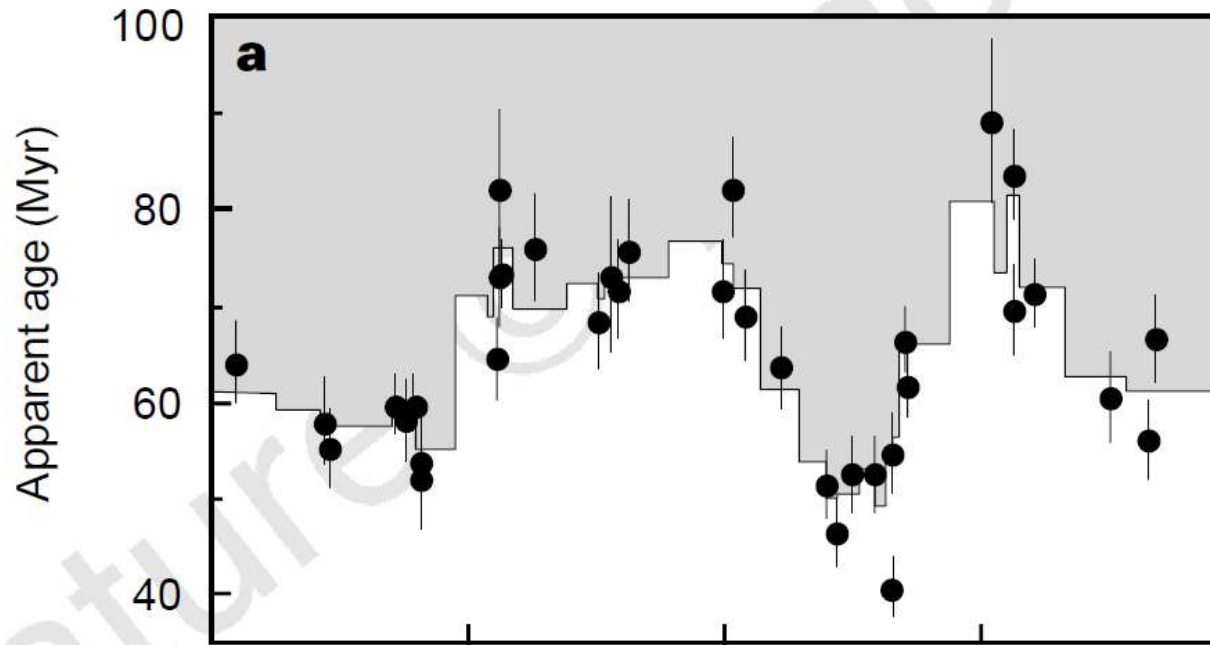


Sierra Nevada Case

Let's describe their methodology...



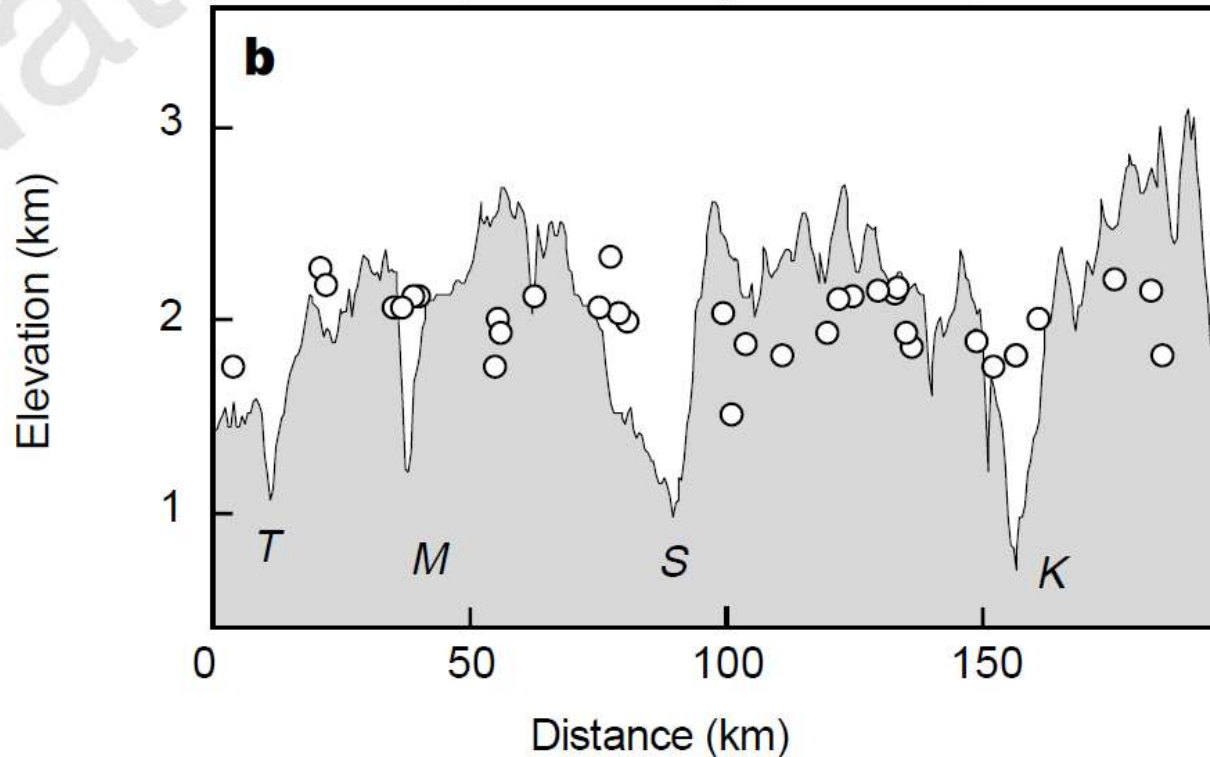
Sierra Nevada Case



Suggestive of increasing or decreasing relief?

How do you interpret if data set was all same age?

Conclusions about the Sierra Nevada

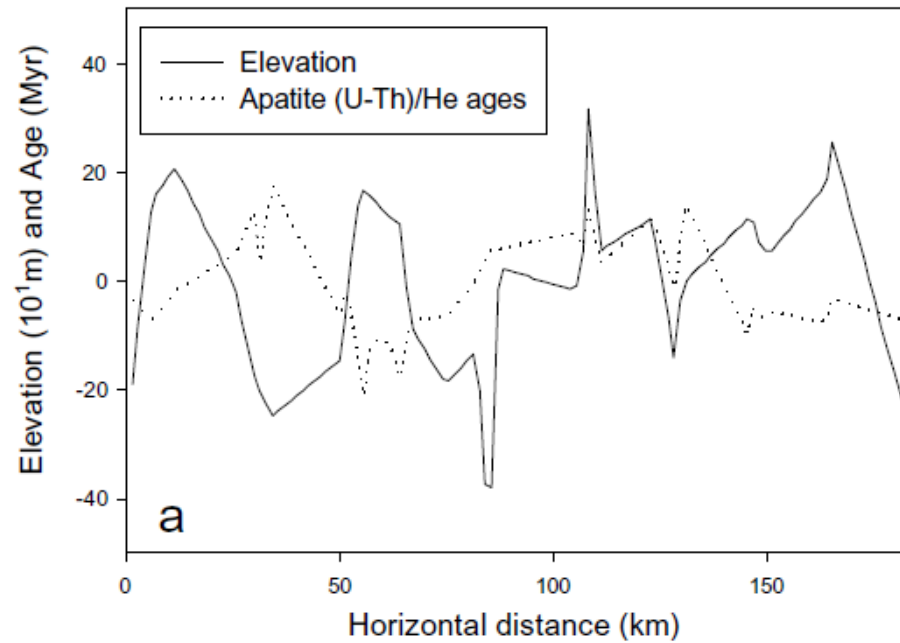


(1) Andean –scale topography set between ~185 -70 Myr

(2) ~70-80 Myr San Joaquin and Kings drainages deeply incised

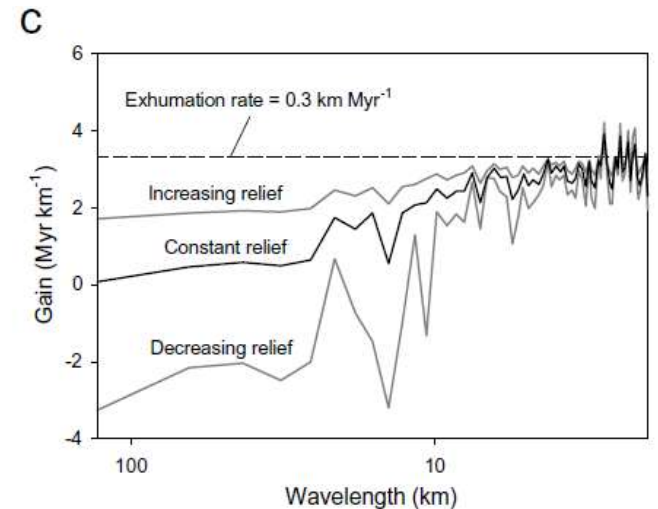
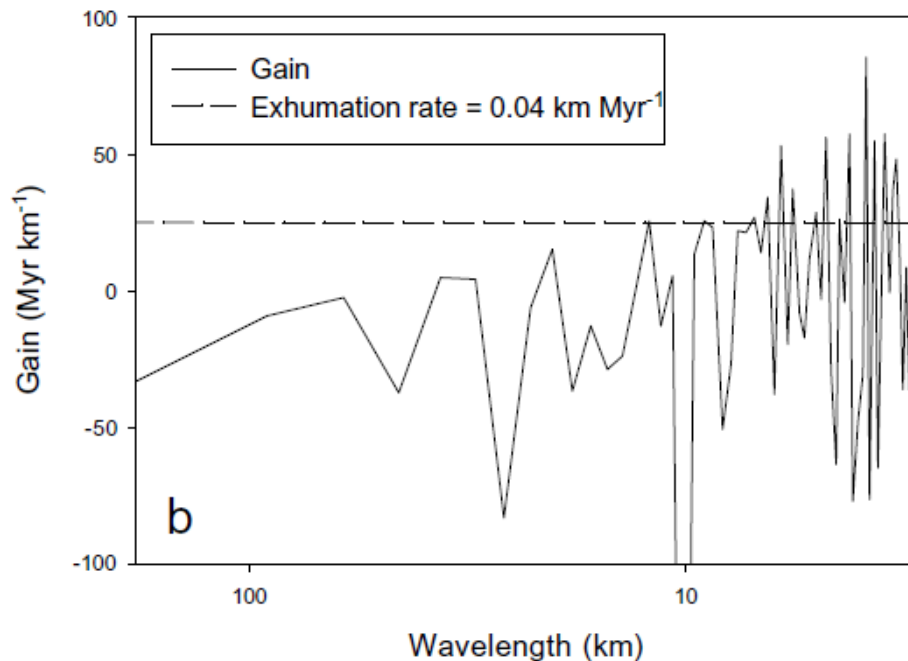
(3) Gradual reduction of elevation from ~4.5km to 2.8km

Re-interpreting Sierra Nevada age-elevation data (J. Braun)



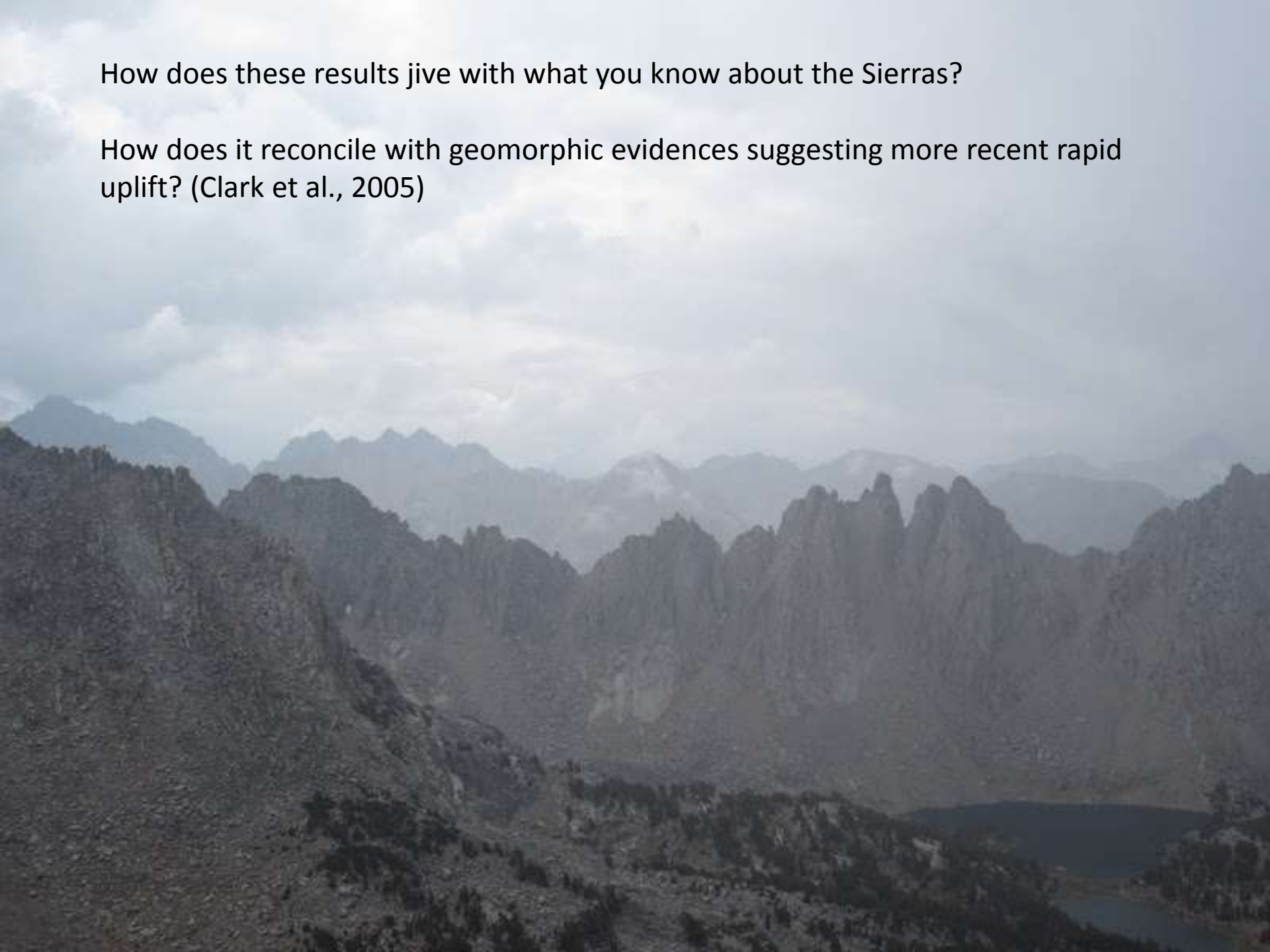
How was the exhumation rate determined?

How does the spectral analysis support the work of House?



How does these results jive with what you know about the Sierras?

How does it reconcile with geomorphic evidences suggesting more recent rapid uplift? (Clark et al., 2005)



Our approach is motivated by the work of House and others (1997, 1998, 2001), who use nearly the same cooling age data to argue for late Cretaceous topographic relief aligned with, but exceeding, modern river canyons. However, our comprehensive analysis shows very little local relief in the Cretaceous. The steady state, 2-dimensional estimates of House and others (1997, 1998, 2001) are evidently insufficient to capture the evolution of local relief, perhaps in part due to the sampling strategy (Braun, 2002). While we do not preclude Cretaceous transverse drainages, we find no evidence for local relief at the scale suggested by House and others (1997, 1998, 2001) to be aligned with modern canyons. For the mid-Cenozoic Sierra Nevada, our results are in agreement with more sophisticated investigations into stream morphology. Clark and others (2005) identify two uplift and incision events since 32 Ma. Pre-Pliocene relief documented by Stock and others (2004, 2005) was likely incised beginning at around this time.

- McPhillips and Brandon 2012 AJS



Temperature as a function of depth, how do we calculate that?

Let's start with heat flow through a slab within the earth and out the surface of the earth

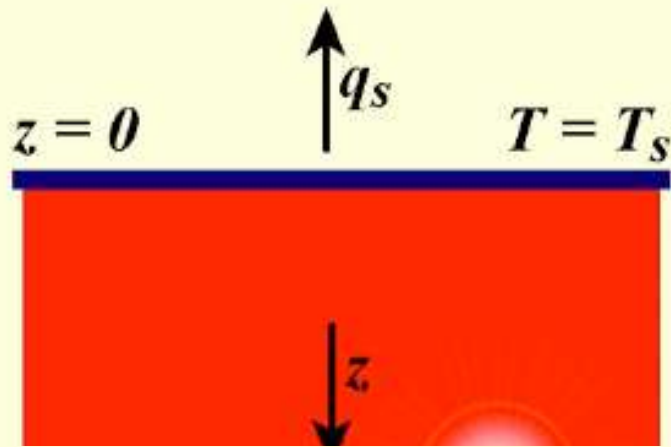
Define heat flow:

$$q = -k \frac{dT}{dx}$$

“heat flow is proportional to the rate of change of temperature with depth”

So, q depends on the thermal conductivity of rocks, and the geothermal gradient

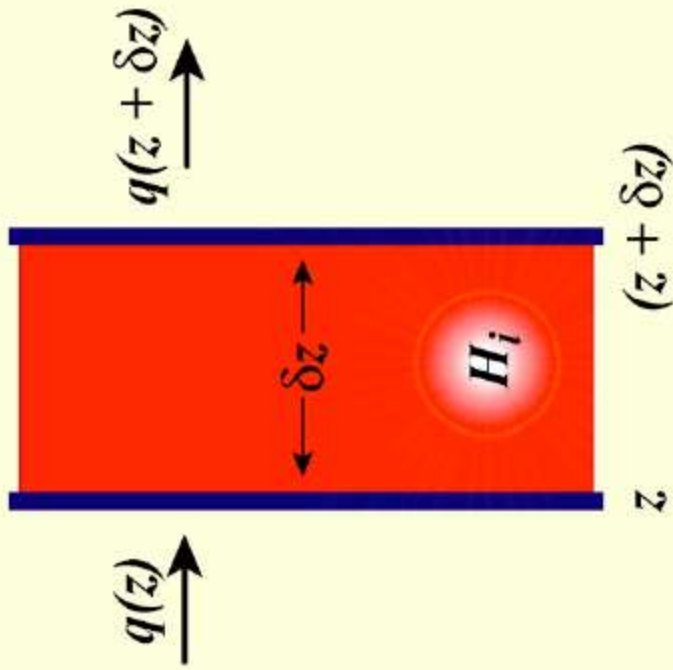
Now we'll look at heat flow through a slab within the earth and out the surface of the earth



At the surface we have boundary conditions we'll use later,

$$z = 0, T = T_s$$

Also surface heat flow q_s and depth z



Below the surface we have slabs with a certain heat production H_i and infinitesimally small sections (dz) we will add up later

Heat flux in: $q(z)$

Heat flux out: $q(z + dz)$

What processes produce heat within the Earth?

An approximation of $q(z+dz)$ can be made with a Taylor Expansion:

$$q(z + \delta z) = q(z) + \delta z \left(\frac{dq}{dz} \right) + \dots$$

$$q_{\text{net}} = q(z+dz) - q(z)$$

$$= \cancel{q(z)} + dz(dq/dz) - \cancel{q(z)}$$

$$= \delta z \left(\frac{dq}{dz} \right)$$

Plugging in: $q = -k \frac{dT}{dx}$

Fourier's Law

$$q_{\text{net}} = \delta z \frac{d}{dz} \left(-k \frac{dT}{dz} \right)$$

$$= \delta z \left(-k \frac{d^2 T}{dz^2} \right)$$

The condition where temperature is not changing, and heat flow out is the same as heat production is called steady-state

$$\delta z \left(-k \frac{d^2 T}{dz^2} \right) = \rho H_i \delta z$$

$$0 = k \frac{d^2 T}{dz^2} + \rho H_i$$

From here we integrate twice and use boundary conditions to find the constants of integration

Then, solve for T

$$T = T_s + \frac{q_s}{k} z - \frac{\rho H_i}{2k} z^2$$

$$T = T_s + \frac{q_s}{k} z - \frac{\rho H_i}{2k} z^2$$

This equation uses a heatflux from the surface,
and uniform heat production below the surface

Comments on that?

Most people use an exponential decrease in heat production with depth:

$$H = H_s e^{-z/z_r}$$

We can see why later.

So we'll go back a few steps to the steady state heat flow

$$0 = k \left(\frac{d^2 T}{dz^2} \right) + \rho H_s e^{-z/z_r}$$

We'll integrate

$$-k \frac{dT}{dz} = -\rho H_s z_r e^{-z/z_r} - C_{iii}$$

And if we let z go to infinity, we'll use Fourier's Law to get C_{iii} .

$$q_m = C_{iii}$$

Now we'll integrate this:

$$-q_m - \rho H_s z_r e^{-z/z_r} = -k \frac{dT}{dz}$$

and use our boundary conditions to get the constant of integration: $T = T_s, z = 0$

$$C_4 = -kT_s - \rho H_s z_r^2$$

$$T = T_s + \frac{zq_m}{k} + (1 - e^{-z/z_r}) \left(\frac{\rho H_s z_r^2}{k} \right)$$

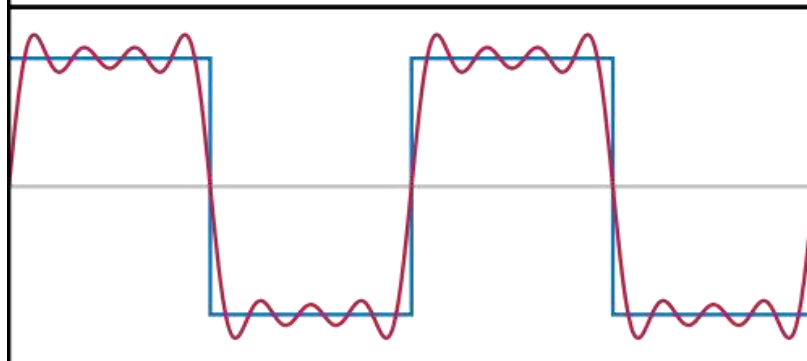
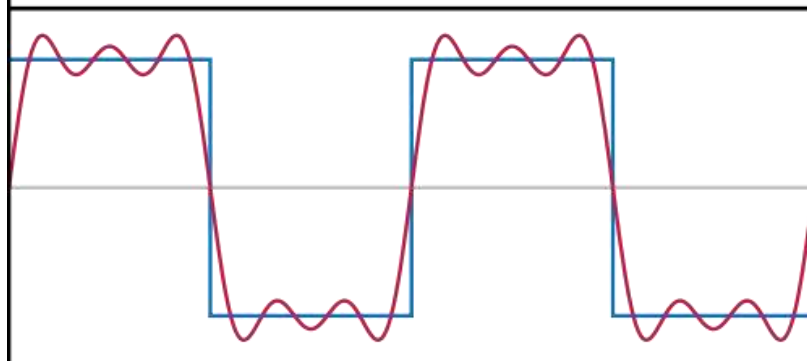
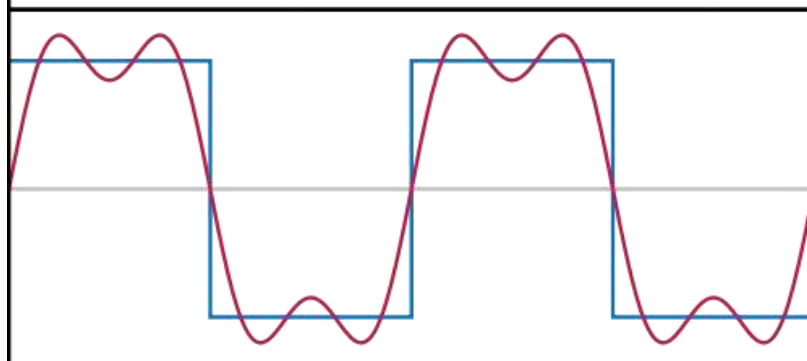
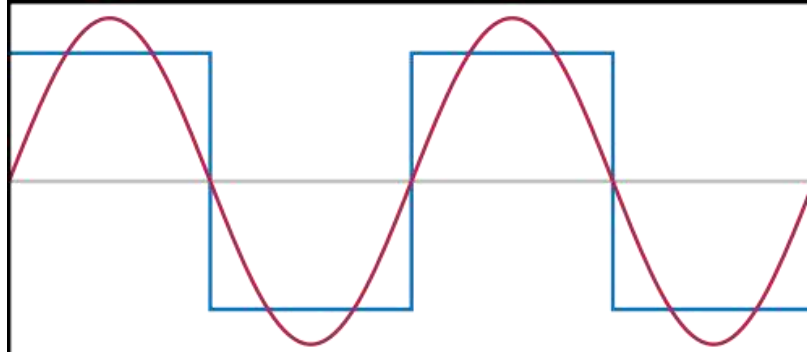
Fourier Transforms of a function

Take a wave form, like some topography, or music

Premise of Fourier Analysis: all complex waves are the sum of other simpler waves.

Calculate the frequencies that are present (Fourier Coefficients), (valley/ridges, or notes)

The Fourier Transform recreates a smooth version of the input wave form using those Fourier Coefficients



We wish to compare to periodic spectra that change over a distance

Elevation



Calculate the dominant frequencies
(Fourier Coefficients)



$$\mathcal{F}\{g(t)\} = G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi ift} dt$$

Age



Calculate the dominant frequencies
(Fourier Coefficients)



$$\mathcal{F}\{g(t)\} = G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi ift} dt$$

$G(f)$ = power (amplitude) of function at a frequency

Frequency and wavelength are related...

Gain is a function of the powers of the age and elevation functions

$$\text{Gain}(\lambda) \propto G_{\text{age}}(f)/G_{\text{elevation}}(f)$$

